No. 2001/03

Comparative Analysis of Alternative Credit Risk Models

– an Application on German Middle Market Loan Portfolios –

Markus Kern / Bernd Rudolph

Center for Financial Studies
an der Johann Wolfgang Goethe-Universität ▪ Taunusanlage 6 ▪ D-60329 Frankfurt am Main
Tel: (+49)069/242941-0 ▪ Fax: (+49)069/242941-77 ▪ E-Mail: ifk@ifk-cfs.de ▪ Internet: http://www.ifk-cfs.de

No. 2001/03

Comparative Analysis of Alternative Credit Risk Models

– an Application on German Middle Market Loan Portfolios –

Markus Kern / Bernd Rudolph

CFS Working Paper No. 2001/03

Comparative Analysis of Alternative Credit Risk Models
– an Application on German Middle Market Loan Portfolios –

Markus Kern* / Bernd Rudolph*

January 2001

Abstract: In recent years new methods and models have been developed to quantify credit risk on a portfolio basis. CreditMetrics™, CreditRisk+, CreditPortfolioView™ are among the best known and many others are similar to them. At first glance they are quite different in their approaches and methodologies. A comparison of these models especially with regard to their applicability on typical middle market loan portfolios is in the focus of this study. The analysis shows that differences in the results of an application of the models on a certain loan portfolio is mainly due to different approaches in approximating default correlations. That is especially true for typically non-rated medium-sized counterparties. On the other hand distributional assumptions or different solution techniques in the models are more or less compatible.

Zusammenfassung: Seit einigen Jahren finden sich in Wissenschaft und Bankpraxis neue Methoden und Modelle, um Risiken von Kreditportfolios zu messen. Zu den bekanntesten Vertretern gehören CreditMetrics™, CreditRisk+ und CreditPortfolioView™, welche sich auf den ersten Blick stark im Ansatz und in der Methodik unterscheiden. Im Mittelpunkt der vorliegenden Studie steht ein Vergleich dieser Modelle und zwar insbesondere hinsichtlich ihrer Anwendbarkeit auf ein typisches Portfolio aus mittelständischen Bankkrediten. Die Analyse zeigt, dass Unterschiede in den Ergebnissen zweier Modelle für ein und dasselbe Portfolio vor allem auf unterschiedliche Verfahren in der Approximation von Ausfallkorrelationen zurückzuführen sind. Dies gilt insbesondere für Kredite an nicht-geratete mittelständische Unternehmen.

Keywords: credit risk management, portfolio modelling, medium-sized debtors.

JEL classification: C15, G21, G33

This paper is an outcome of the Center for Financial Studies, Frankfurt/Main, industry project on "Credit Risk Management in Germany", a joint research project involving leading commercial banks in Germany (HypoVereinsbank, Deutsche Bank, DG Bank, Dresdner Bank, and Westdeutsche Landesbank) as well as finance faculties and Ph.D. students from the universities of Frankfurt, Mannheim and Munich. We are very grateful to their support and their comments. Special thanks go to Ralf Ewert, Jan Pieter Krahnen and Martin Weber.
* Ludwig-Maximilians-University Munich, Institute of Capital Market Research and Finance, Schackstrasse 4, 80539 Muenchen, Germany. e-mail: kern@bwl.uni-muenchen.de, rudolph@bwl.uni-muenchen.de.

1 Introduction

Since the mid-1980s, models geared to apply knowledge about market risk from portfolio selection theory to credit portfolios have been proposed in the relevant technical literature[1]. But it was not until the recognition of internal market risk models for regulatory purposes by the Basel Committee on Banking Supervision[2] in January, 1996 that modelling of credit portfolio risk with all of its particularities became a major focus of academics and practice. In the meantime, several methods and (software) products for measuring credit portfolio risk have been developed and become available. It is striking that most of the publications in current literature are still critiques and application tests referring to the four standard models from 1997 and 1998 or related approaches: CreditMetrics™ by J.P. Morgan[3], CreditRisk$^+$ by Credit Suisse Financial Products[4], CreditPortfolio-View™ by Wilson and McKinsey[5] and PortfolioManager™ by KMV[6].

Up to now, the Basel Committee on Banking Supervision has clearly rejected initial requests[7] for an explicit supervisory consideration of internal credit portfolio models. They understandably came to the conclusion that sufficient long term data is not available for an exact estimation of important input parameters of the models and that proper backtesting of model results is not possible due to the longer risk horizons of buy-and-hold credits.[8] In the consultative paper published in June 1999 on a revision of capital adequacy regulations[9] and with the publication of the "Principles for the Management of Credit Risk"[10], Basel made it clear, however, that methodically sound risk measurement and management at the portfolio level and thereby the recognizable consideration of diversification

1 cf. e.g., Bennett (1984).
2 cf. Basel Committee on Banking Supervision (1996).
3 See J.P. Morgan (1997).
4 See Credit Suisse Financial Products (CSFP, 1997).
5 See Wilson (1997a, 1997b, 1998).
6 See Kealhofer (1998).
7 cf. e.g., International Swaps and Derivatives Association (1998), p.14ff.
8 cf. Basel Committee on Banking Supervision (1999a), 47-54; a validation period of 250 days for market risk models would, for example, correspond to a test period of 250 years for credit risk models with a one year risk horizon.
9 cf. Basel Committee on Banking Supervision (1999b), p.56ff.

potential will positively influence the regulatory judgement within the context of the proposed “supervisory review process” already in the near and middle terms. It is therefore quite likely that in the long term, banks that internally have established a credit portfolio model even before an explicit regulatory recognition will gain a significant competitive advantage. Polls have shown that the large universal banks in Germany have recognized this challenge and have been working for some time on identifying, modifying, developing and implementing sophisticated credit risk models and the organizational context for a portfolio-orientated credit risk management. It should be noted that in most cases considerable effort is being made to install a model suitable for a bank's individual portfolio of assets subject to credit risk rather than a universal solution. Due to the German relationship banking system the greater portion of credit risk faced by domestic banks still stems from the classical business with medium sized debtors.[11]

The applicability of the four above-listed standard models for measuring credit risk has been addressed in various articles with reference to the different types of credit products, however, explicit considerations regarding their usage on a typical middle market[12] credit portfolio could not be found. The present study will focus on this criterion in an analysis comparing three of the above models. KMV’s PortfolioManager™ has been appreciated more for its approach of analyzing stand-alone credit risk than as a portfolio model. Furthermore, the portfolio part is related to CreditMetrics™ and hence will not be discussed separately here. Further implications for a choice between the models will also be drawn from already existing model analysis.

10 cf. Basel Committee on Banking Supervision (1999c), p.21-22.

11 cf. e.g., Elsas and Krahnen (1998).

12 In this paper a "middle market" credit portfolio is a loan portfolio to medium sized companies. In Germany "medium sized" companies usually have sales lower than DEM 500 mio. which is also the case for the sample used here.

2 Portfolio Credit Risk Models

2.1 Measuring Credit Risk from Middle Market Loan Portfolios

All credit risk models share the goal of a complete description of the distribution of possible gains or losses from a credit portfolio. For a stand-alone credit you already get a skewed and non-continuous distribution due to the limitation on the profit side in the classical lending business and the – even if unlikely - possibility of a total loss of the unsecured exposure. This makes an aggregation at the portfolio level considerably more difficult in comparison with approximately normally distributed market risk positions. The "model performance" is hereby closely related to the trade-off between "model risk" and the complexity of the approach respectively inherent requirements for IT capacity. On the one hand, "model risk" is determined by the implications of simplifying assumptions onto overall results, on the other hand, by the quality or availability of required input parameters.

However, if the risk manager eventually were to succeed in calculating a trustworthy portfolio loss distribution, it would significantly enhance a bank's ability to manage and control credit risk. The risk potential effectively entered into can be quantified by the expected loss, the volatility of portfolio values, the calculation of a Credit-at-Risk analogous to the value-at-risk concept or, as well, the "expected shortfall" in order to more precisely describe the characteristic "fat tails".[13] Concentration risk and diversification opportunities can be identified by means of calculating marginal risk contributions of individual exposures to the overall portfolio risk. In their much respected study Froot / Stein (1998) point out that the price of non-tradable, marginal credit exposures would also have to depend on their value correlation with the already existing non-tradable risks in the bank's portfolio. In the long term, the contribution of any single loan to total portfolio risk should therefore be reflected in credit conditions, which to date might not have played a significant role, particularly in relationship banking.[14]

13 cf. regarding expected shortfall, e.g., Embrechts et al., (1997), chap. 6.
14 cf. Froot and Stein (1998), p.66.

Models for measuring credit portfolio risk require several input parameters - firstly, to quantify the loss risk from the individual positions and, secondly, to take the pairwise interdependencies, which are determined by joint risk drivers, into account at the portfolio level. Considering the loss that a bank can expect from a typical buy and hold transaction (assumption: no premature disposal is possible or attractive), it is obvious that such a loss is already made up of three uncertain components:

[1]

Expected Loss = Default Probability · (Outstanding Exposure · (1-Recovery Rate))

Usually estimation of the probability of default is initially based on an individual credit analysis (rating) but can vary considerably over the time horizon of the loan contract. The expected exposure at the time of default (discounted, outstanding interest and repayment) is likewise an uncertain value in the common case of unused lines of credit e.g. . Finally, the recovery rate is calculated as a percentage of outstanding nominal exposures and can depend on the future marketability of tangible collateral, hardly predictable work-out costs, etc. Therefore only to get the expected value of the (portfolio) loss distribution, actually, the product of three stochastic variables has to be calculated.

To determine the probability of joint default of two or more loans would actually require pairwise default correlations. Since loan defaults are (should be) very rare events, the joint default of loans at the same time happens even much more seldom. A direct historical estimate of default correlations for bank credits similar to an empirical estimate of stock price correlations on the basis of joint changes in stock prices e.g. is consequently not possible. Therefore, default correlations must be approximated using auxiliary variables. The reliability of the approximation plays a decisive role - as will be proved later - for the results of these credit risk models.

2.2 CreditMetrics™ – the market value model

In 1997, J.P. Morgan presented CreditMetrics™, as the credit risk counterpart to RiskMetrics™, and implemented it in the CreditManager™ software tool [15]. In its basic form, CreditMetrics™ is conceived for bond portfolios and is heavily relying on market values. Hence, credit risk arises not only from the danger of issuer default, but also from a potential (market) value loss due to a downgrade in the credit rating of the debtor.

CreditMetrics™ represents the asset value models that go back to Merton's 1974 work on the relationship between a company's capital structure and insolvency risk.[16] The risk driver is the change in firm value (asset value) over time. If that value falls below the book value of the liabilities, default results. However, CreditMetrics™ only makes use of the basic idea from this approach in order to deduce the changes in the credit rating of each bond from changes in asset values, which again are simulated as correlated standard-normally distributed random figures.

In the methodology of KMV's product, PortfolioManager™, which can likewise be assigned to the asset value models, Merton's option-pricing-theory approach is explicitly used for individual credit analysis. Probability of default and rating migration probabilities of each debtor are hereby dependent on the "distance to default", the difference between firm value - recursively derived via the option-pricing formula from the market value of equity - and the book value of liabilities.[17] In CreditMetrics™, these rating migration and default probabilities are approximated historically for each rating class and carried over into a so-called rating migration matrix. Rating migration matrixes of this type are an elementary input for many credit risk models and are published, for example, by rating agencies for publicly rated corporate bonds and companies. Table 1 gives an exemplary rating migration matrix by Standard&Poor's.

15 cf. J.P. Morgan (1997) in the following.
16 cf. Merton (1974).
17 cf. Crosbie (1999), 10-11, and Rudolph (2001).

Table 1:
Rating migration matrix for publicly rated corporate bonds
Probabilities of rating migration and default within a one-year horizon (%)

	Rating in t=1							
Rating t=0	**AAA**	**AA**	**A**	**BBB**	**BB**	**B**	**CCC**	**Default**
AAA	90,81	8,33	0,68	0,06	0,12	0	0	0
AA	0,70	90,65	7,79	0,64	0,06	0,14	0,02	0
A	0,09	2,27	91,05	5,52	0,74	0,26	0,01	0,06
BBB	0,02	0,33	5,95	86,93	5,30	1,17	1,12	0,18
BB	0,03	0,14	0,67	7,73	80,53	8,84	1,00	1,06
B	0	0,11	0,24	0,43	6,48	83,46	4,07	5,20
CCC	0,22	0	0,22	1,30	2,38	11,24	64,86	19,79

Source: Standard&Poor's Credit Week (April 15,1996)

For a middle market portfolio of loans to non-rated companies a bank would have to put up a migration matrix out of its historically generated internal ratings. Alternatively one might try to map the own rating scheme with "public" ratings which on the other hand seems problematic regarding the different migration and default characteristics of bond issues and typical bank loans. It can e.g. be presumed from a migration matrix that summarizes middle market credit data and especially the internal rating changes of five large German banks (table 2) that internal ratings are more often changed relative to the preceding evaluation than public ratings, and therefore there is less probability mass on the diagonals.[18]

Thereby every exposure gets (historically estimated) migration and default probabilities via its rating. Then for each bond possible market values at risk horizon[19] can be calculated using forward zero curves which can be obtained via bootstrapping from spot rates for each (new) rating category. With respect to the buy-and-hold character of German middle market loans one might be able to do

18 Data have been collected in course of the project "Credit Risk Management" being conducted by the "Center for Financial Studies", Johann Wolfgang Goethe University Frankfurt. See e.g. Elsas et al. (1999). Loans were re-evaluated abount every ten months on average.

19 In the following – as it is common standard in credit risk management - a risk horizon of one year is assumed.

without quantifying losses in (market) value from the change in ratings. If this is desired within CreditMetrics^{TM} potential future "values" of every loan would have to be calculated using "loan-forward-curves". These are especially determined by rating- and time-specific credit spreads which might diverge more or less from bond credit spreads due to the differing information structure. Those spreads therefore would have to be re-estimated by the bank once in a while.

Table 2:
Rating migration matrix for internally rated loans
Migration and default probabilities (%) for German middle market loans

	Rating in t=1						
Rating t=0	**1**	**2**	**3**	**4**	**5**	**6**	**Default**
1	51	40	9	0	0	0	0
2	8	62	19	8	2	1	0
3	0	8	69	17	6	0	0
4	1	1	10	64	21	3	0
5	0	1	2	19	66	12	0
6	0	0	0	2	16	70	12

Source: Machauer/Weber (1998), S. 1375.

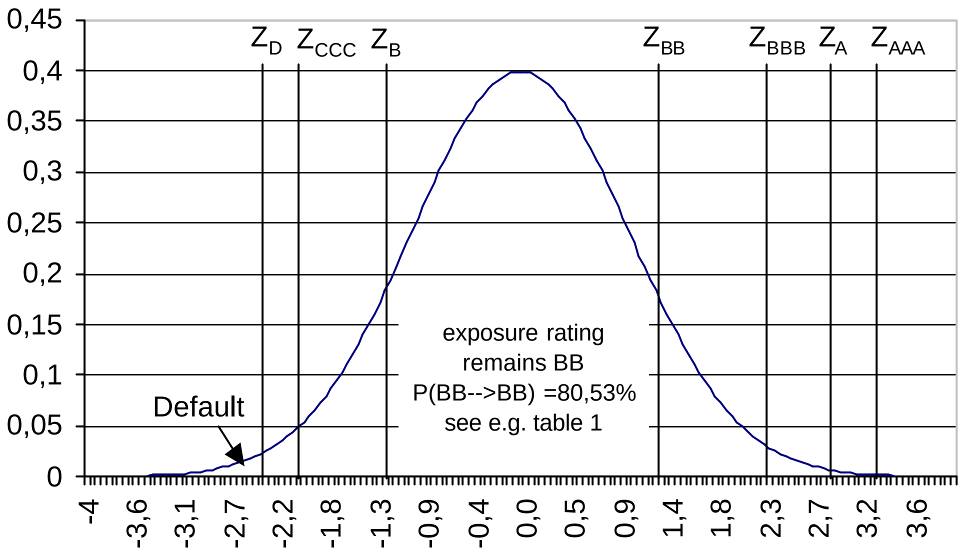
CreditMetrics^{TM} assumes a beta distribution for recovery rates in case of a default. For every "recovery class"[20] the beta distribution has to be determined by an expected value and its standard deviation. The assumption of the beta distribution takes into account the skewness of the real distribution of recovery rates from bank loans as e.g. discovered by Asarnow/Edwards (1995) in their empirical study. Asarnow/Edwards found a high dispersion in the respective rates and our own interviews with German credit managers confirmed that recovery rates for traditional loans are very difficult to quantify. They can be found over the whole range of 0%-100%. For middle market loans seniority is not as straightforward as for corporate bonds, therefore narrowing down the recovery problem into a certain distribution assumption is one critical factor for measurement results.

20 A „recovery class“ can be defined by seniority for corporate bonds, for traditional bank loans many different criteria (e.g. product specific or collateral specific) seem possible.

Having brought together all those input parameters firm value changes and consequently rating migrations and defaults are Monte Carlo simulated with CreditMetrics™ for any single bond or loan. Changes in firm value hereby are assumed to follow the normal distribution and the migration respectively default thresholds are taken from the migration matrix (probabilities as percentiles of the normal distribution). This simplified approach following Merton's intention is illustrated by graph 1:

Graph 1:

1.) Simulation of (standard-)normally distributed changes in firm value (assets) for any exposure in the credit portfolio.
2.) Mapping of random changes with new rating categories with respect to historical migration probabilities and thresholds Z_{Rating}



Exemplary migration thresholds for a bond / exposure with BB-Rating in t=0

As firm values do not move independently changes have to be simulated by drawing from multivariate normal distributions based on a $n \times n$ –correlation matrix. In the basic form of CreditMetrics™ correlations of firm (asset) values are approximated via correlations in stock prices as the former are not "observable" parameters. This already seems problematic regarding the call character of equity with respect to the firm value of leveraged companies. For non-listed medium sized companies considered here things even get worse. For them CreditMetrics™

proposes to decompose return on equity for every debtor in form of a factor model. Pairwise correlations of equity returns can then be calculated from the weights and the empirical correlations of assigned stock indices[21] as well as from the ex-ante specified portions of unsystematic risk. Graph 2 illustrates this stepwise approximation of migration respectively default correlations in CreditMetrics™ and the problematic nature of the inherent assumptions.

Graph 2 :
Approximation of migration and default correlations in CreditMetrics™

Required input: *migration / default correlations*

Approx.: → *correlation of firm values (asset value model)*
→ *correlations of equity returns*
→ *yearly correlations of certain country-industry stock indices*
→ *definition of a factor model*

precision?

For every single scenario of the simulation CreditMetrics™ generates a change in firm value for each counterparty resulting in n "new" ratings and market values for every credit exposure. Summing up losses (and gains for upgrades) over exposures gives the new portfolio value for every scenario, repeating the simulation ten thousands of times eventually results in the desired portfolio loss distribution.

CreditMetrics™ allows not only to calculate discounted portfolio losses from defaults but also from rating downgrades. But for this the model requires the extensive input of market data which usually is not fully available for middle market loan portfolios and therefore has to be approximated. This especially seems problematic with respect to the approximation of asset correlations[22]. Furthermore CreditMetrics™ is often criticized for relying too heavily on the

21 The indices represent the systematic risk as the "factors" in the model and are defined as country-industry indices.
22 cf. chapter 3.

migration matrixes which are usually generated from averaged historical data.[23] With this approach the model neither takes into account the current macroeconomic conditions for the debtors nor does it anyhow differentiate between debtors of the same rating category but different businesses. But this would especially be recommendable for medium sized companies which presumably show a greater heterogeneity than the comparably small group of publicly rated corporate bond issuers.

2.3. CreditRisk$^+$ - the actuarial risk model

CreditRisk$^+$ is a model that uses actuarial methods and offers the attractive feature of a closed form analytical solution in its basic version.[24] Only credit risk from defaults is considered and contrary to the asset value models potential reasons for a default are of no significance. Default rates are assumed to be stochastic and are the risk drivers themselves. Therefore CreditRisk$^+$ is also regarded as a representative of the "default rate models". Hereby the model takes the observation into account that default rates are not constant over time but can significantly fluctuate over the so-called credit cycle. Graph 3 illustrates this for the average default rate of German companies between 1972 and 1992.

CreditRisk$^+$ needs default rates per country-industry segment as input as well as (average) default rates for the individual credit exposures, again to be taken out of a migration matrix or to be generated by an internal credit analysis. Recovery rates are taken as constants or alternatively only exposures net of collateral are used for the calculation of losses. Then – for a big portfolio of n homogenous and independent loans with the same exposure and the same default rates – the probability q_d that *exactly d defaults* will happen in the portfolio approximately follows the Poisson distribution[25]:

23 cf. Crouhy/Galai/Mark (2000), p.66 e.g.

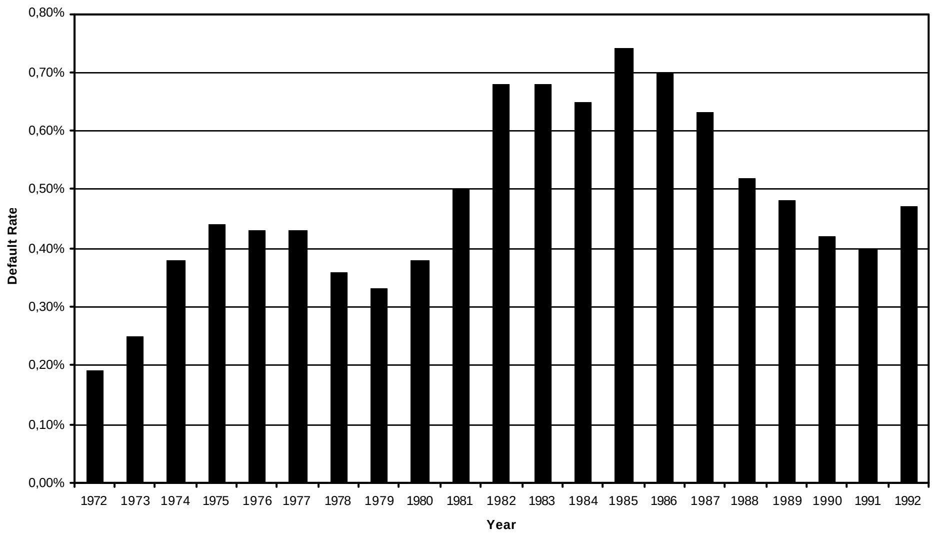
24 cf. Credit Suisse Financial Products (1997) in the following.

25 q_d is determined via actuarial technique in the basic model, approximately [2] is valid; see e.g. Schmid (1998), S.33.

$$q_d \approx \frac{e^{-m} \cdot m^d}{d!} \qquad , \tag{2}$$

with µ being the expected number of defaults or as well the sum over all stand-alone default probabilities in the portfolio. The approximation via the Poisson distribution looks intuitive as the stand-alone default probabilities are very small and n (the number of debtors in the portfolio) might be very large.

Graph 3: default rate volatility for German firms (West Germany; 1972-1992)



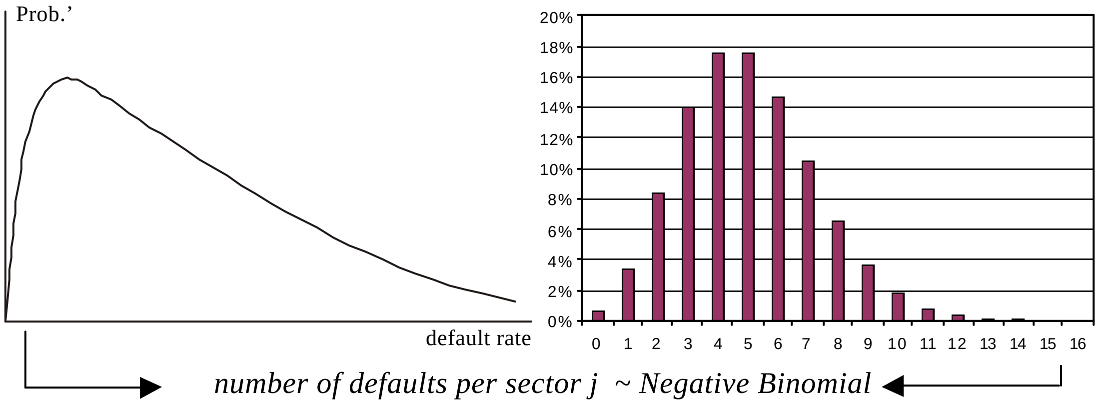
Source: Statistisches Bundesamt, Germany

In order to incorporate the joint and correlated default behaviour counterparties are assigned to different country-industry sectors for the systematic portion of their exposures similar to the procedure at CreditMetrics™. Thereby every exposure is divided into several sub-exposures each of which is allocated to exactly one sector. It is assumed that *default rates* follow the gamma distribution within any sector j. So sector-specific default rate distributions are fully described by their expected value μ_j and their standard deviation σ_j. The expected default rate of a sector (e.g. supply industry for automobiles, Germany) can be estimated historically or as the average (expected) default probability over all debtors actually being assigned to this sector. The volatility of the sector-specific default rate is to be determined in an analogous way as far as sufficient data is available. Default probabilities of any two counterparties fluctuate in a correlated way through this joint (even if only partial) affiliation to the same sector(s) and therefore due to the same macroeconomic influences characterizing this sector. In order to get to the distribution of the *number of defaults* within any sector the "independent" Poisson distribution now has to be combined with the sector-specific gamma distribution or to express it technically – the two distributions must be "folded". As an intermediate result a Negative Binomial distribution of defaults is obtained for every sector. Graph 4 illustrates the procedure.

Graph 4: twofold statistics with CreditRisk$^+$

sector-specific default rate ~ Gamma (μ_j, σ_j)

„independent" no. of defaults ~ Poisson (μ)



If all exposures were homogenous the sector-specific distributions of number of defaults could directly be translated into the portfolio loss distribution taking the weight matrix into account (that summarizes the dependence of the counterparties on different sectors). In the realistic case of exposures of differing size additionally a distribution of exposure sizes within any sector has to be defined by the user. Then the Negative Binomial distributions can be transferred sectorwise into the loss distributions again using actuarial technique.[26]

At first glance CreditRisk$^+$ looks very attractive for the fast analytical calculability of portfolio loss distributions. Furthermore the model only needs comparably few

26 more precise: a recursive procedure using a probability generating function is applied, cf. Credit Suisse Financial Products (1997), p. 46-49.

data input which accommodates with the lack of data in traditional credit business. Nevertheless it is questionable whether this "simplicity" of the approach might not be at odds with the aim of modelling the "complex" reality as accurately as possible.

Regarding that recovery rates for traditional bank loans are in no way constant but vary considerably (cf. 2.2.) the user might want to allow for stochastic recovery rates. Already with this modification an analytical solution is not feasible anymore and simulation methods have to be applied. In this context also the two fundamental distributional assumptions (Poisson and Gamma) have to be examined with respect to their implications for model results (cf. chapter 3). Finally it is important to note that in CreditRisk$^+$ the default correlation as the actually required input parameter is implicitly approximated over the affiliation of counterparties to sectors and the default volatilities within sectors. Thereby every sector represents one risk factor and the sectors are assumed to be independent from each other.

2.4 CreditPortfolioView^{TM} – the econometric model

The concept of CreditPortfolioView^{TM} can be seen somewhere in between CreditRisk$^+$ and CreditMetrics^{TM} [27]. As with CreditMetrics^{TM} losses from defaults and rating downgrades can be accounted for. As well a rating migration matrix constitutes the fundament of the model and has to be provided by the user. Yet default correlations are not approximated by stock data but the original migration matrix is "adjusted" according to the prevailing macroeconomic situation. Therefore default probabilities are not constant but volatile – as it is the case in CreditRisk$^+$. But while in the latter simply an expected value and a standard deviation of the default rate are assigned to each sector complete time series of default rates per sector are required in CreditPortfolioView^{TM}. Table 3 shows an example. Those time series are the most important data input for a complex econometric tool used by CreditPortfolioView^{TM} to (Monte Carlo-)simulate macroeconomic scenarios.

Table 3:
CreditPortfolioView™- data input:
country-industry-sectors and the time series of sector-specific default rates

Default Rates / Industry-Segment / Germany		**Year 1**	**Year 2**	**Year 3**	**Year 4**	**Year 5 etc.**
Source: "Statistisches Bundesamt", Germany						
Agriculture / Forestry / Fishery	p1	0,31%	0,40%	0,56%	0,56%	0,51%
Energy / Water Supply / Mining	p2	0,10%	0,05%	0,07%	0,07%	0,07%
Manufacturing Industry	p3	0,48%	0,64%	0,84%	0,86%	0,76%
Building Industry	p4	0,71%	1,04%	1,45%	1,31%	1,44%
Trade	p5	0,30%	0,40%	0,56%	0,56%	0,55%
Transportation / Communication	p6	0,41%	0,55%	0,74%	0,73%	0,62%
Financial Institutions / Insurance Ind.	p7	0,64%	0,60%	0,71%	0,80%	0,82%
Services / Others	p8	0,28%	0,36%	0,48%	0,50%	0,47%
All Sectors	**p**	**0,38%**	**0,50%**	**0,68%**	**0,68%**	**0,65%**

Example: default rates in the eight German main sectors (years: 1980 ff.)

In the first step – a well known procedure - a rating and an country-industry-segment have to be assigned to every credit exposure/debtor in the portfolio. Secondly macroeconomic variables have to be selected that might be suitable to represent the systematic risk of the default rates in the chosen country-industry-segments (e.g. unemployment rate in Germany, long term interest rate in the U.S., Euro-USD exchange rate, etc.).[28] Another preparatory work is to estimate auto-regressive (moving-average-) processes for these macroeconomic factors out of the respective time series. Subsequently for every country-industry-segment up to three macro variables are identified as the most suitable exogenous factors using a non-linear ordinary-least-squares (OLS) regression[29] and therefore as the best to explain past fluctuations of the default rate in this segment. This regression procedure can also be described as mapping the time series of the macro variables with the time series of the default rate per sector.

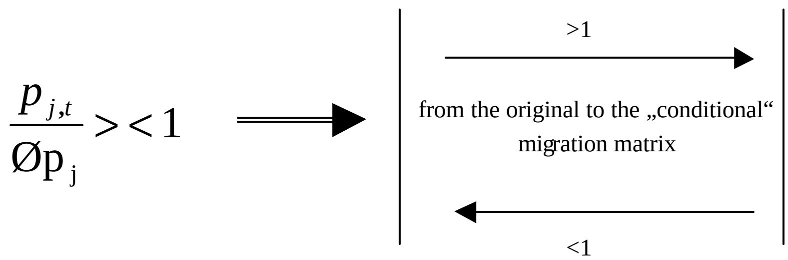
27 cf. Wilson (1997a, 1997b, 1998) in the following.

28 Country-industry-segments and macro variables can theoretically be defined by the user in any number. But for every segment and variable historical time series for the average default rate resp. the yearly realisation must be available.

29 A logistic transformation into a default probability is carried out.

After that the "new" realisations of every single macro variable for the next period (time until risk horizon) can be simulated using the historical auto-regressive patterns. Then those simulated realisations are directly translated into "current" default probabilities $p_{j,t}$ per sector j based on the causal connections identified in the OLS-regressions. If this simulated default probability turns out to be higher than the long term average $\varnothing p_j$ in this sector an "unfavourable" macro scenario prevails and the downgrade and default probabilities have to be marked up relative to their long term average. CreditPortfolioView™ hereby employs a so-called "shift-operator" that moves probability mass in the original migration matrix for each sector to the right or to the left dependent on whether $p_{j,t}/\varnothing p_j$ is bigger or smaller than one (graph 5).

Graph 5: The shift-operator in CreditPortfolioView™



Generating a migration matrix conditional on the macroeconomic status quo

When solely looking at defaults this means for instance that for $p_{j,t}/\varnothing p_j > 1$ in a certain sector the original default matrix is not valid anymore but the default probabilities are adjusted upwards for each rating category.

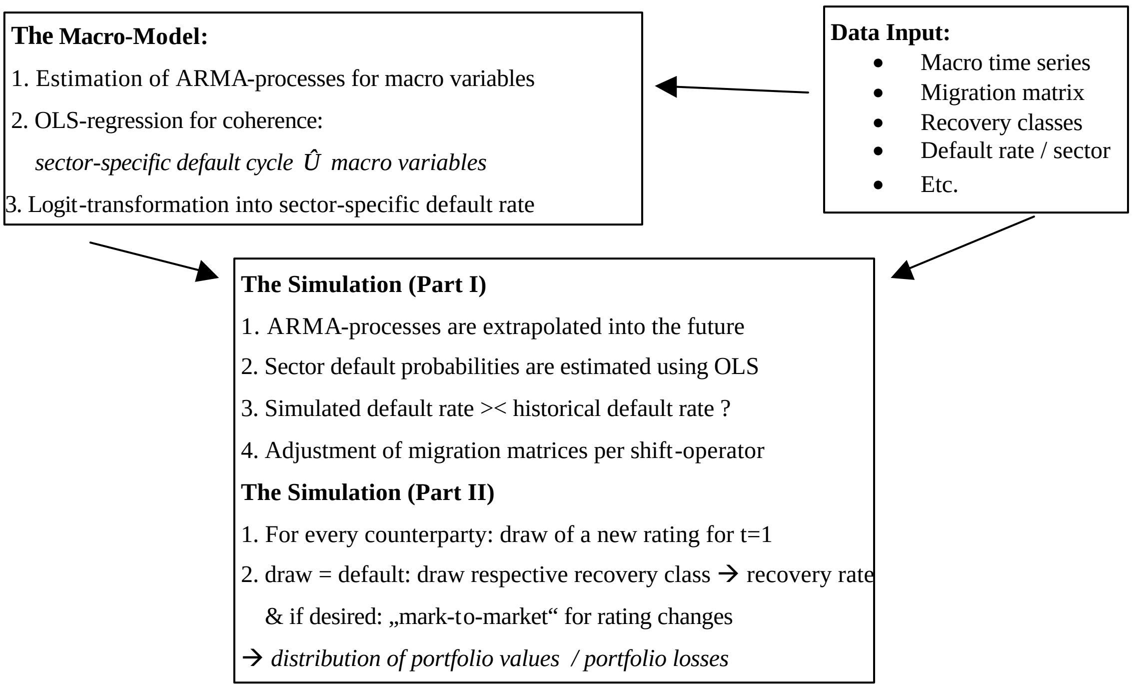
Finally CreditPortfolioView™ draws new ratings (and defaults) for every counterparty in the portfolio and for every simulation scenario out of those "conditional" sector-specific migration matrices. Analogous to CreditMetrics™ "mark-to-market" valuations can be performed for liquid credit exposures that have not been drawn as "defaulted" and mature later than the risk horizon.[30] For defaults net losses are simulated after having assigned every exposure in the

30 As well for this CreditPortfolioView™ additionally requires spot rates and rating-specific credit spreads as data inputs (cf. 2.2, forward zero curves).

portfolio to a certain recovery rate distribution.[31] Performing many thousands of Monte-Carlo-simulations eventually leads to the portfolio loss distribution.

Thence in CreditPortfolioView™ similar to CreditRisk$^+$ correlations between the single country-industry-segments are not taken into account. It is the joint dependency on macroeconomic risk drivers that results in correlated rating migrations and defaults. But other than in CreditRisk$^+$ (cf. 2.3) it is not a type of a single factor model but every sector-specific default rate is dependent on several (exogenous) macroeconomic factors. Graph 6 summarizes the different modules in CreditPortfolioView™.

Graph 6: the CreditPortfolioView™ modules



Because of its complex econometric approach to model sector-specific default rates CreditPortfolioView™ needs a lot of historic data. At first e.g. it is up to the user's economic intuition to identify the potentially relevant macro variables for the different groups of debtors in the portfolio and to supply the respective time series. It is true that altogether CreditPortfolioView™ is a much more complex model than for instance CreditRisk$^+$, but thereby its economic intuition and the

31 The user can prespecify any number of different recovery rate distributions that might for instance be correspondent to certain credit product types or seniorities.

transparent causality between the macroeconomic environment and the default behaviour in the different segments are convincing.

But also for CreditPortfolioView^{TM} it is questionable how well actual default correlations can be approximated. Miscellaneous studies showed in this context that for speculative grade exposures default rate fluctuations could be explained quite well by the economic cycle. Investment grade counterpartys remained more or less unaffected.[32] Prima facie CreditPortfolioView^{TM}'s extensive data requirements might not seem attractive for the usage on a middle market credit portfolio. However, as the user will concentrate on the credit risk from defaults for a buy-and-hold portfolio, they do not seem to be unrealisable. Especially for internationally diversified credit portfolios adequate macro variables and their historical values as well as a reasonable country-industry-segmentation should be identifiable and determinable. Furthermore CreditPortfolioView^{TM} allows its users many degrees of freedom for data entry in such a way to enable them to modify diverse model components.

3 Comparison of Models and Implications for an Implementation

At first sight the discussed models seem to show elementary differences in accordance with the trade-off between the "simplicity" of a model and the "complex" reality mentioned in chapter 2.1. . Those differences apply to the "risk definition" (default only vs. rating downgrades and defaults), the modelling "technique / methodology" (distributional assumptions, calculation methods), the necessary "data input" and the required information technology. In fact the user will obtain very different distributions and Credit-at-Risk values with a "naive" application of the three models on the bank-specific portfolio – even if only credit risk from defaults is considered in CreditMetrics™ and in CreditPortfolioView™ as assumed in the following.

Though looking at the basic structures of the models it can be seen that they all have remarkable similarities. All three models tie the "conditional" default probability of a rating class or a segment to states of the world respectively (systematic) risk factors. CreditMetrics™ assumes a normal distribution of the risk driver "asset value". By this and by the simulation of correlated asset returns it comes to an implicit transformation resulting in the "conditional" distribution of default rates (actually being constant) in the model per rating category. CreditPortfolioView™ as well assumes a normal distribution for risk drivers in the residual term of the auto-regressive processes, and there is an explicit Logit-Transformation into a "conditional" default probability per segment. CreditRisk+ doesn't put up any distributional assumptions for the risk drivers but defines the "conditional" default probability per segment as gamma distributed right from the outset.[33]

In all three models the joint influence of the same risk factors on two counterparties replaces an explicit consideration of default correlations. As the transformation takes place dependent on the state of the world the default behaviour of any exposure itself can be regarded as "independent" and therefore as binomially distributed. In CreditMetrics™ and in CreditPortfolioView™

32 cf. Müller-Groeling / Niethen (2000), p.10 e.g.
33 cf. Koyluoglu / Hickman (1998), p.58.

binomially distributed random numbers are generated directly with the default probabilities from the migration matrices. CreditRisk$^+$ approximates the binomial distribution via the Poisson distribution.[34] By aggregating the "conditional" default distributions over all possible states of the world the user finally arrives to the total portfolio loss distribution. For this purpose CreditMetrics^{TM} and CreditPortfolioView^{TM} employ Monte-Carlo simulation techniques, CreditRisk$^+$ "folds" the gamma and the Poisson distribution analytically into the Negative Binomial distribution.

At this point one can already suspect that fundamental reasons for divergent model results might be due to differences in modelling the joint default behaviour of any two debtors. This presumption directly follows from the relatedness in the remaining modelling "techniques/methodologies", i.e. from the relatedness of the binomial and the Poisson distribution and the approximation goodness of Monte Carlo simulations with a sufficient number of simulation runs. Gordy (1998) and Wahrenburg/Niethen (2000) verify this by reducing CreditMetrics^{TM} to a version comparable with CreditRisk$^+$ [35] and by carrying out further simplifications until only one default probability for the portfolio, its volatility (CreditRisk$^+$) and one explicit pairwise asset correlation for all exposures (CreditMetrics$^+$) are left as input data.[36] Remaining differences in results of portfolio calculations then can only be attributed to an inconsistent approximation of default correlations, in CreditRisk$^+$ happening implicitly through the default volatility, in CreditMetrics^{TM} through the asset correlation. Exemplary calculations with an empirically estimated asset correlation and default rate volatility in fact lead to heavily diverging loss distributions. Marginal tests hereby show that the "fat tail" in CreditRisk$^+$ reacts especially sensitively to changes in the default rate volatility. For this Gordy demonstrates in his study that the gamma distribution even aggravates the approximation errors of the Poisson distribution by folding them.[37]

34 For many credit exposures and very small stand-alone default probabilities in the portfolio the Poisson distribution is a very good approximation for the binomial distribution, c.f. Gordy (1998), S.3, e.g..

35 Assumptions: only defaults are considered, recovery rates are constant, homogenous exposures, one rating category, one sector.

36 cf. Gordy (1998) and Wahrenburg/Niethen (2000).

37 see Gordy (1998), p.20-23.

But other studies also show for CreditMetrics™ and CreditPortfolioView™ how sensible model results are with respect to different asset correlations or to the coefficients in the macroeconomic modelling (CPV).[38]

However – if the user calibrates the input parameters being critical for the approximation of default correlations in CreditMetrics™, CreditRisk$^+$ and CreditPortfolioView™ in a way that they are analytically "consistent" between the models you get very similar results for the three calculations.[39] Unfortunately the suchlike calibrated input parameters often appear to be unrealistic and differ strongly from empirically generated values (see examples in table 4):

Table 4: empirically estimated vs „consistent" parameters

empirical value	*„model-consistent" value*	
r =0,5	*r =0,016*	*estimated for CreditMetrics™,* *calibrated for s =0,0026 CR$^+$-value*
s =0,0026 *(µ=0,0122)*	*s =0,04*	*estimated for CreditRisk$^+$,* *calibrated for r =0,5 CM™-value*

Example: sector "building industry", Germany, 1980-1994; share price correlation r and default rate volatility s ; source: Wahrenburg/Niethen (2000), p.252/253.

Therefore differences in model results are to a lesser extent due to the model methodology or distributional assumptions but rather to different ways of approximating the default correlations that are empirically hardly available. The choice of the critical data inputs, i.e. the asset or stock price correlations in CreditMetrics™, the default volatilities in CreditRisk$^+$, and the respective regression coefficients in CreditPortfolioView™ determines the model results to a high degree. With respect to a realistic assessment of portfolio risk it is hereby especially problematic that errors in aggregating stand-alone risks even reinforce the effect of errors in estimating the expected loss from the individual positions (see equation [1], 2.1).

38 see. AMS (1999), p.6/7 and Bucay/Rosen (1999), p.56ff.
39 cf. Koyluoglu / Hickman (1998), p.61 e.g.

Against this background also the choice or the design of a certain model should be decided. For a portfolio consisting of loans to non-listed, medium sized companies an approximation of default correlations through the pairwise, joint influence of macro variables will be more reasonable than through the implicit correlations from a share return factor model. Time series of default rates for specific country-industry-segments can be quite easily generated out of the official statistics. Default rate volatilities can then be directly computed if CreditRisk$^+$ is chosen. If the bank's risk management decides in favour of an econometric model like CreditPortfolioView^{TM} in order to thereby take account of the current macroeconomic situation, then additionally for every country-industry-segment the dependencies on certain macro variables have to be determined.

An own study provides interesting outcomes if this shall likewise be reached via a regression model with three exogenous factors. It could namely be shown that for seven of the eight main sectors in Germany good model specifications – looking at the adjusted R^2 – can be reached with the same three regressors.[40] The respective model to explain the sector-specific default rates with the realisations of macroeconomic variables in the same period t looks as follows:

[3] $$p\ (\text{default})_j = f\ (\Delta \text{GDP(GER) real, Unempl. Rate (GER), DEM/USD})$$

p(default)$_j$ = default rate in sector j;

ΔGDP(GER) real = real change in gross domestic product, Germany;

Unempl. Rate (GER) = unemployment rate, Germany;

DEM/USD= exchange rate German Mark / US-Dollar.

40 cf. Kern / Reitzig (2000) in the following. Insolvency rates for the eight main sectors (sector classification by "WZ 1979") were considered between 1965 and 1992 (Source: Statistisches Bundesamt). Time series of diverse macro variables were captured over the same period. Regression estimates were then conducted for all possible permutations of the available macro

Tests showed that in most cases the sector-specific gross value added (SGVA) can be used alternatively to the real change in gross domestic product. Table 5 exemplarily shows the results of the estimation for the default rate in the sector "trade".

Table 5:
Regression for default rate in the sector "trade"
(results with correction of type Cochrane-Orcutt)

independent variable	**coefficient (S.D.)**
sector-specific gross value added (SVGA)	-0.39* (0.23)
unemployment rate (UER)	0.05*** (0.005)
DEM/USD- exchange rate	0.01 (0.02)
F-test	47,12 (P<=.001)
adjusted-R^2	0.84

Source: Kern/Reitzig (2000), p.16.

As can be intuitively expected there's a negative coherence between the default rates in all sectors and the real change in the GDP / the SGVA. As well it is not surprising that an increase in the unemployment rate occurs at the same time as higher default rates.

If such a model is chosen the implicit correlations result from the proportioning of the exposures to the different country-industry-segments and therefore from the pairwise joint influences via the three coefficients $b_{GDP/SGVA}$, b_{UER} and $b_{DEM/USD}$.

variables as regressors. As could be expected the default rate in the sector "energy / water supply/ mining" turned out to be relatively insensitive to the economic cycle.

So far the type of approximation of default correlations has been identified as the most critical model element for credit portfolio results. Nevertheless it must not be forgotten that of course many other factors (e.g. the assumption of constant or stochastic recovery rates) can have a significant impact on model results in terms of a Credit-at-Risk value.

4 Summary and Outlook

As has been shown in this paper all three models are very similar in their basic structure and in principle they are all implementable on a portfolio of traditional bank loans. Every model has its “pros” and “cons” of which the most important have been mentioned in the respective chapters.

Presently it is still one of the most important aspects that the user finds confidence in the particular approximation technique for default correlations. The choice of methodology hereby adds substantially to differences in model results. There’s still no assured knowledge about which of the three models at best approximates actual default correlations. Fur this purpose first of all a thorough backtesting of the single models would have to be conducted – but for risk horizons of six months or a year there simply does not exist enough performance history yet. Thus it has to be seen as the primary task for further progresses in credit risk measurement to consistently estimate or approximate default correlations despite of the lack in empirical data. Of course that’s even more difficult for traditional bank loans than for corporate bonds. It has been indicated at the end of this paper how such an approximation could be modelled based on the coherence between sector-specific default rates and the macroeconomic environment.

References

AMS Incorporation (1999): A credible performance, Credit Risk Special Report, Risk Magazine, April 1999, p.4-7.

Asarnow, Elliott / Edwards, David (1995): Measuring Loss on Defaulted Bank Loans: A 24-Year Study, Journal of Commercial Lending 77, March 1995, p.11-23.

Basel Committee on Banking Supervision (1996): Amendment to the Capital Accord to Incorporate Market Risks, Manuscript, January 1996.

Basel Committee on Banking Supervision (1999a): Credit Risk Modelling: Current Practices and Applications, Manuscript, April 1999.

Basel Committee on Banking Supervision (1999b): A New Capital Adequacy Framework, consultative paper, June 1999.

Basel Committee on Banking Supervision (1999c): Principles for the Management of Credit Risk, consultative paper, July 1999.

Bennett, Paul (1984): Applying Portfolio Theory to Global Bank Lending, in: Journal of Banking & Finance 8, 1984, p.153-169.

Buca, Nisso / Rosen, Dan (1999): Das Kreditrisiko eines internationalen Wertpapierportfolios: Eine Fallstudie, Solutions 03/04, 1999, p. 47-69.

Credit Suisse Financial Products, CSFP (1997) : CreditRisk$^+$: A Credit Risk Management Framework – Technical Documentation, 1997.

Crosbie, Peter J. (1999): Modeling Default Risk, Manuscript, KMV Corporation, January 1999.

Crouhy, Michel / Galai, Dan / Mark, Robert (2000): A comparative analysis of current credit risk models, Journal of Banking & Finance 24, 2000, p. 59-117.

Elsas, Ralf / Ewert, Ralf / Krahnen, Jan Pieter / Rudolph, Bernd / Weber, Martin (Elsas et al., 1999) : Risikoorientiertes Kreditmanagement deutscher Banken, Die Bank, 3, 1999, p.190-199.

Elsas, Ralf / Krahnen, Jan Pieter (1998): Is relationship lending special? Evidence from credit-file data in Germany, Journal of Banking & Finance 22, 1998, p. 1283-1316.

Embrechts, Paul / Klüppelberg, Claudia / Mikosch, Thomas (1997): Modelling Extremal Events for Insurance and Finance, Springer Verlag, Berlin u.a., 1997.

Froot, Kenneth A. / Stein, Jeremy C. (1998): Risk management, capital budgeting, and capital structure policy for financial institutions: an integrated approach, The Journal of Financial Economics, Vol. 47, 1998, p. 55-82.

Gordy, Michael B. (1998): A Comparative Anatomy of Credit Risk Models, Manuskript, Conference on Credit Modeling and the Regulatory Implications, London, September 1998.

International Swaps and Derivatives Association, ISDA (1998): Credit Risk and Regulatory Capital, Manuscript, March 1998.

J.P. Morgan (1997): Credit Metrics ™ – Technical Document, New York, 1997.
Kealhofer, Stephen (1998): Portfolio Management of Default Risk, Manuskript, KMV Corporation, February 1998.

Kern, Markus / Reitzig, Markus (2000): „Zur Erklärung von Insolvenzquoten verschiedener Branchen in Deutschland mithilfe makroökonomischer Indikatoren“, Arbeitspapier.

Koyluoglu, Ugur H. / Hickman, Andrew (1998): Reconcilable Differences, Risk Magazine, Vol. 11, No.10, S.56-62, 1998.

Machauer, Achim / Weber, Martin (1998): Bank Behavior Based on Internal Credit Ratings of Borrowers, Journal of Banking & Finance 22, 1998, p. 1355-1383.

Merton, Robert (1974): On the Pricing of Corporate Debt: The Risk Structure of Interest Rates, Journal of Finance, Vol. 29, 1974, p.449-470.

Müller-Groeling, Axel / Niethen, Susanne (2000): Rating transition matrices conditional on the business cycle, Working Paper, 2000.

Rudolph, Bernd (2001): Kalkulation von Risikokosten auf Basis des Optionspreismodells, in: Rolfes / Schierenbeck / Schüller (Edit.): Handbuch Bankcontrolling, 2. Aufl., Gabler Verlag, Wiesbaden, appears: spring 2001.

Schmid, Bernd (1998): Quantifizierung von Ausfallrisiken, Solutions, 01/98, p. 27-38.

Wahrenburg, Mark / Niethen, Susanne (2000): Vergleichende Analyse alternativer Kreditrisikomodelle, in Kredit und Kapital, 33. Jg. Heft 2, p. 235-257.

Wilson, Thomas (1997a,b): Credit Portfolio Risk (I, II), in: Risk Magazine, September / October 1997, p.111-117 & p.56-61.

Wilson, Thomas (1998): Portfolio Credit Risk, FRBNY Economic Policy Review, October 1998, S.71-82.