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Abstract:
In this paper we assess the implications of sunk costs and product differentiation on the pricing decisions of the multinational firms. For this purpose we use a modified version of Salop's spatial competition. The model yields clear-cut predictions regarding the effects of exchange rate shocks on the market structure and on pass-through. The main results are following: shocks within the band of inaction do not affect market structure. The upper bound of this range rises as the industry ratio of sunk- to fixed costs increases. As fixed costs and product heterogeneity jointly increase, the lower bound drops. Outside of the range, depreciations cause one or several of those foreign brands closest to the home brand to leave. This decreases the overall responsiveness of prices to exchange rate shocks. Large appreciations induce entry and increase the elasticity of prices. This asymmetry implies larger positive than negative PPP deviations. When accounting for price changes in foreign markets, strategic pricing behaviour is no longer sufficient to generate real exchange rate variability. Incomplete pass-through obtains if and only if the domestic firms have a smaller market share abroad. With large nominal exchange rate shocks a hysteresis result obtains if and only if sunk costs are non-zero.

JEL Classification: C33, E31

Keywords: Market Structure, Exchange Rates, Hysteresis

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1 Introduction

For decades Purchasing Power Parity (PPP) theory was the key building block of the monetary model. It simply states that the symmetric price adjustment mechanism ensures that the real exchange rates are time invariant with respect to nominal exchange rate shocks. Empirical studies however are highly inconclusive. Two major controversial findings are as follows: Firstly, there is vast evidence that nominal exchange rate shocks often tend to lead to very persistent if not permanent PPP deviations.[1] Real exchange rates are very volatile and real exchange rate deviations are very persistent. (see e.g. Chari et al. (2000)). Even long time series or panel data applications do not provide convincing support in favor of the PPP. Although current research interprets PPP as a long run attractor evidence in favor is still rather poor. Secondly, there is causal evidence that price levels increase more easily than they decrease. If there is strong downward price stickiness real exchange rate adjustments can not necessarily be symmetric on both sides. Non linear tests such as of Enders and Dibooglu (2001) taking the French Franc and German DM as reference currencies provide some evidence that there is indeed asymmetry in the price adjustment process for positive vis-à-vis negative PPP deviations.

The pricing to market literature provides various theoretical explanations for incomplete short run symmetric price adjustment with respect to depreciations and appreciations. To name a few, product differentiation, currency denomination of exports and imports, size of the market, vertical trade all seem to contribute to short run deviations from the PPP. [2]Dornbusch (1987) for example uses Salop's circular model of spatial competition, among others, to analyze the adjustment of prices to exchange rate changes. This exercise yields some insights into the level of pass-through in differentiated product markets. Yet Dornbusch's discussion does not cover changes in market structure. His analysis essentially focuses on the short-run. For long run analysis, Baldwin (1988) was the first to point out that foreign firms, having invested heavily in marketing, R&D and the like to enter the US market, may find it profitable not to quit the US market even at very low exchange rates. By staying in the market, these firms anticipated to recover at least part of their sunk investment. Exchange rate shocks can change the structure of the market and the resulting changes in prices and trade volumes may persist even after the exchange rate returns to its previous value.[3] The new international macroeconomics literature incorporate some of these microfoundations for price stickiness.[4]

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[1] For this very controversial issue see for example Froot and Rogoff's (1995) literature review. For panel data evidence see Papell (1997) or O'Connell (1998).

[2] See among others Dornbusch (1987), Friberg (1998), Aksoy and Riyanto (2000) and Bacchetta and van Wincoop (2001). For a survey of vast empirical evidence which lends support for pricing to market see Menon (1995).

[3] This argument was further refined and extended by Baldwin and Krugman (1989) and Dixit (1989). For empirical evidence of the relevance of sunk costs see Roberts and Tybout (1997).

[4] See for example, Obstfeld and Rogoff (2000a,b). For explicit incorporation of sticky prices see Kollmann (1996) Betts and Devereux (1996, 2000) or Chari et al.(2000). For a useful survey

In this paper we suggest a dynamic integrated partial equilibrium approach that features product differentiation and endogenizes market structure at the same time. We substantially extend the Dornbusch-Salop model. There are two distinct features of our model. First the model can to account for the asymmetry in the price adjustment. We are not aware of a theoretical model that explains asymmetric adjustments for PPP deviations. Secondly, the integrated approach allows us to discuss an array of conditions where the short and long run monetary neutrality within the international context is violated. In other words, the model presents in detail under which conditions imperfect competition is able to generate persistent and volatile real exchange rate deviations. We also show that most of the predictions survive alternative market configurations.

Our main results can be summarized as follows. Firstly, small currency shocks which fall within a given range are unlikely to affect the structure of the domestic market. However, large and persistent devaluations may induce foreign firms to leave the market which underlies the hysteresis result in the literature. Similarly, sizeable appreciations can set off foreign entry into the home market.

Secondly, the model predicts asymmetry in the range of no-market-structure-change. Price adjustments with respect to appreciations and deprecations are not necessarily symmetric. The bounds of this range depend primarily on the persistence of shocks, the cost structure and the degree of product heterogeneity. As the persistence of the shock decreases, the foreign brand is willing to accept higher current losses and the devaluation threshold beyond which foreign brands quit the domestic market rises. Conversely, smaller appreciations suffice to trigger entry as the persistence increases. Furthermore, the devaluation threshold rises as the industry ratio of sunk- to fixed costs rises. Moreover, higher levels of product differentiation favor those firms that are at a competitive disadvantage. This implies that the lower bound of the range drops as differentiation increases. Entry at low exchange rates is simply less profitable for the entering foreign brand. Similarly, the upper bound rises since higher brand differentiation reduces the size of foreign brand losses at high exchange rates. Comparing the results across industries, this model predicts that larger appreciations are needed to trigger entry in industries characterized by high levels of heterogeneity and large fixed costs.

Thirdly, these structural market changes in turn affect pricing strategies. As the total number of players in the domestic market changes, the demand curves facing the firms shift and the extent of product differentiation is altered. This causes a fixed-size and across-the-board drop, in the case of an appreciation, or increase, in the case of a depreciation, of all brand prices. In addition, the domestic brands gain or lose market power as their relative market share changes. As a result, the responsiveness of prices to currency shocks is altered. All of this turns out to imply that pass-through is smaller for large depreciations and larger for large appreciations. Interestingly, this asymmetry implies larger, and possibly more persistent, positive than negative PPP deviations of the real

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of this literature see Lane (2001).

exchange rate.

Fourthly, a given currency shock has a larger impact on foreign brands facing direct competition from the domestic brand. This is where the spatial competition setup really starts to matter. The extent of product differentiation is the key to understanding the observed differences in price and quantity responses on the part of foreign firms. The foreign firms closest to the domestic brands are most likely to quit the market after a devaluation. These foreign brands, being relatively similar to their domestic counterparts, simply experience a larger decline in profit margins in response to a devaluation.

Finally, this model clearly shows that strategic pricing behavior as such is not sufficient to generate incomplete pass-through. The model predicts invariance of the real exchange rate with regard to 'small' nominal currency shocks if the market structure at home and abroad are fully symmetrical, that is if the domestic and foreign firms have identical market shares at home and abroad. Pass-through is incomplete if and only if the domestic brands have a smaller market share abroad. Remarkably, if some of the domestic firms operating abroad do not serve the domestic market, the model predicts that the real exchange rate moves counter to the nominal exchange rate.

The paper is organized as follows. Section 2 outlines the model and main underlying assumptions. Section 3 focuses on a stylized 4-incumbent version of the model. Here we show differential impact of large and small exchange rate changes on market structure. Section 4 provides some extensions on the stylized model. Finally, Section 5 concludes.

2 Outline of the Model

In his seminal paper Dornbusch (1987) applies the Salop (1979) model to analyze the adjustment of prices in response to exchange rate shocks in heterogeneous product markets. Dornbusch assumes that domestic and foreign brands are located equidistantly and alternatingly on a circle upon entry. Domestic and foreign brands differ in that the latter incur the costs of production in foreign currency. The analysis of the nature of price competition in this model yields novel insights into the pricing-to-market phenomenon. However, as such the model cannot deal with the effects of real exchange rate shocks on market structure since entry and exit destroy the symmetry of the model.

To allow for the possibility of market structure changes, Dornbusch's (1987) symmetrical setup is replaced by a clustered structure in which the markets covered by foreign and domestic brands touch at only two locations on the circle. In addition to its intuitive appeal, this asymmetrical setup proves quite tractable in dealing with entry and exit. It has to be noted, however, that this version of the model cannot be solved in general. This is why one needs to specify the number of foreign and domestic brands competing in the domestic market. Naturally, this entails some loss of generality. Below we illustrate that the main results are robust to changes in the number of firms.

In addition, the number of potential entrants is restricted. We concentrate our attention on entry and exit decisions in international trade. In other words, we do not consider the possibility of foreign direct investment in the presence of exchange rate shocks. Obviously, some foreign trade companies may get involved in domestic production activities rather than to take the stay, entry or exit decisions in their trading activities in the presence of exchange rate changes. For reasons of tractability we will exclude this relevant option. Generally speaking, changes in the exchange rate do not drive the creation of new firms in most heterogeneous goods industries (for instance, the automobile industry). Instead, exchange rate shocks mostly cause existing brands to enter or exit specific markets. It seems natural to assume that the number of potential entrants is fixed and bounded[5]. Furthermore, the market structure changes in response to exchange rate shocks in any given industry are triggered mainly by foreign brands. Accordingly, to focus the analysis on the relevant issues, assume there are no potential domestic entrants. Similarly, exit on the part of domestic incumbents is largely ruled out by the relative cost efficiency of domestic brands. This is discussed in detail below.

When considering the possibility of entry or exit in any given market, agents' exchange rate expectations naturally play a key role. In this model, agents are taken to have perfect foresight. Ideally, the exchange rate and the expectations thereof would be endogenously determined within the model. Our model adopts a partial equilibrium approach in the sense that the nominal exchange rate process is fully exogenous. This implies that endogenizing exchange rate expectations is not feasible.

Having outlined the main assumptions underlying the analysis, this section concludes by introducing the model itself.

2.1 Producers

Domestic and foreign firms incur fixed costs ($F^D$and F^F) when active in the domestic market. These include all recurrent expenses that are locked in for exactly one period such as brand name maintenance advertising and distribution costs (Baldwin, 1988). Firms have to incur these costs at the start of every period in order to stay in the market. Note that in our model, F does not include the actual costs of production. In fact, all firms have access to the same constant returns to scale production technology: $w^D = w^F$, where w denotes the marginal cost of production. Foreign brands are at a fixed cost disadvantage relative to the domestic brand[6]: $F^F > F^D$. This assumption will prove to be useful as we analyze the entry and exit decisions of the foreign firms in the domestic market.[7] In addition, entrants incur a sunk cost S before entering the

[5] This boundedness assumption reflects the scarcity of certain irreplacable inputs. It is hard to think of industries producing highly differentiated goods that have an unlimited supply of potential entrants.

[6] Foreign brands may incur higher fixed costs for a variety of reasons such as higher transportation, distribution and brand maintenance advertising expenditures.

[7] In spite of its intuitive nature this assumption may not always be true. Under certain circumstances some foreign firms may even have fixed maintenance cost advantage as compared to their domestic counter parts (as it may well be in the Central and Eastern European case,

domestic market. This includes all costs incurred while setting up a distribution network, establishing a brand name, etc. Once an incumbent has left the market, or, equivalently, not served the market for one period, re-entry requires spending of the entire sunk cost S again. Both the fixed (F) and sunk (S) costs are incurred in the domestic market, therefore in domestic currency, since these are not related to the actual production of the commodities. Production costs (w) are incurred in the country of origin, i.e. in foreign currency.

We exogenously impose the location of the firms in the product space. We assume that producers are located equidistantly on the circle upon entry. Rather than allowing the entrant to choose its location freely, we chose to maintain the equal spacing assumption. This assumption may impose some limitations on the analysis. Firstly, maximum differentiation obtains with quadratic transportation costs in the linear city. As shown in D'Aspremont *et al.* (1979) with the linear city example with two firms, the equilibrium has the two firms locating at the two extremes of the city.[8] For the circular city, Economides (1984) shows that the free entry symmetric equilibrium also obtains in locations and prices in the case of quadratic transportation costs. However, this result may not be general for different cost structures. Secondly, we assume simultaneity in the entry decisions of the firms to eliminate strategic aspects of product positioning (see Tirole (1994)). The analysis becomes more complicated if firms enter sequentially. Given that firms enter sequentially, the equilibrium pattern of location will be a function of the firms' anticipation of future locations in the product space. Furthermore, optimal timing of entry would be another matter of concern. To avoid further complication we choose for simultaneity in the entry decisions of the firms. Endogenizing the locational choice and allowing for sequential entry in this type of model, while technically demanding, may provide further insights into the effects of exchange rate shocks on market structure.

Naturally, some assumptions need to be made concerning the relative position of new entrants. In our model, a bidding game precedes the actual entry stage. Potential entrants engage in a bidding game with each of the foreign incumbents for the corresponding brand location. In equilibrium, the foreign incumbent bids an amount corresponding to the excess expected profits of this location relative to the location furthest from the domestic incumbent. This makes the entrant exactly indifferent between the latter location and the other locations on the circle. The entrant's optimal bid is zero and as a result she simply ends up in the location furthest away from the location of the domestic brand. Note that this situation constitutes the unique Nash equilibrium since the incumbents, not having to incur the sunk entry cost, can always outbid the entrants. As it turns out, foreign incumbents invariably maintain their brand's relative position in terms of proximity to the domestic brand and the entrant is forced to market the brand furthest from the domestic one. Note that this brand is the least profitable at real exchange rates below PPP. In what follows, the bidding game itself will not be explicitly analyzed any further. We simply

where Western companies may have access to better maintenance cost technologies).

[8] In other words, each firm tries to soften the price competition via locating (differentiating) as far as possible from each other (maximal differentiation).

assume that the new entrant ends up in the least profitable position.

2.2 Consumers

L domestic consumers are distributed uniformly on the circle. These consumers buy the brand closest to their preferred brand l^* which coincides with their position on the circle. Let c denote the utility cost per unit of distance. Consumers purchase one of the goods offered in the domestic market provided that:

$$\max_i [u - c\,|l_i - l^*| - p_i] \geq \widetilde{s} \tag{1}$$

where l_i denotes the position of brand i, $|l_i - l^*|$ refers to the shortest distance between l_i and l^* and $\widetilde{s}$ denotes the surplus derived from consuming the homogeneous outside good. Regardless of the exchange rate we only consider equilibria in what Salop (1979, p. 143) refers to as the competitive region. The competitive region is "composed of prices in which consumers are attracted who would otherwise purchase some other differentiated brand" (Salop, 1979, p.143). Simply put, the brands cover the entire market and engage in direct competition with each other. None of the consumers prefer the outside good[9].

3 Four- incumbent Model

To start the analysis, we analyze a particular version of the model that features 4 incumbents, three of which are foreign. Two of the foreign incumbents are situated adjacent to the domestic brand. These firms compete directly with the domestic brand and the non-adjacent foreign brand. Note that this simple 4-brand example features 3 different prices: the domestic brand's price, the non-adjacent- and the two adjacent foreign brands' prices. In addition, the maximum number of domestic entrants is fixed at 1, while the maximum number of potential foreign entrants is 4. This setup reflects all of the assumptions introduced in the previous section. The properties of this model will be explored under three different exchange rate scenarios.

This is a two-stage game. Initially the market is served by 4 incumbents. Before the start of the game the entire path of future exchange rates is observed by all players, including all potential entrants. In the first stage non-incumbent brands may decide to enter or incumbent brands may decide to leave the market. In the second stage these brands compete in prices. The two-stage game is solved by means of backward induction. Having solved for Bertrand-Nash equilibrium pricing strategies in the second stage, the equilibrium number of brands is derived from the reduced form profit functions.

In the second stage firms maximize current profits in deriving their optimal pricing strategies .[10] Let E_t denote expectations conditional on information at

[9] In other words, we assume $\widetilde{s}$ is relatively small.

[10] We assume that second order conditions are satisfied.

time t. R is the discount rate. First, consider the domestic firm's objective function :

$$E_t \left[\sum_{s=0}^{\infty} R^s \left[\left(p_{t+s}^{D} - w^{D} \right) \left[\frac{L}{c} (p_{t+s}^{FA} + \frac{c}{4} - p_{t+s}^{D}) \right] - F^D \right] \right], \tag{2}$$

where p_{t+s}^{FA} denotes the adjacent foreign brand. The domestic brand's demand curve was derived by locating the indifferent consumer L on both sides. Similarly, we derive the profit function of the adjacent foreign firm (denoted FA). Let e_{t+s} denote the exchange rate (units of domestic currency per unit of foreign currency). Note that these firms compete with two firms possibly charging different prices. Market shares are not necessarily equal on both sides. The adjacent foreign firm's objective function in terms of domestic currency is:

$$E_t \left[\sum_{s=0}^{\infty} R^s \left(p_{t+s}^{FA} - e_{t+s} w^{F} \right) \begin{bmatrix} \frac{L}{2c} \left(p_{t+s}^{D} + \frac{c}{4} - p_{t+s}^{FA} \right) + \\ \frac{L}{2c} \left(p_{t+s}^{FNA} + \frac{c}{4} - p_{t+s}^{FA} \right) \end{bmatrix} - F^F \right]. \tag{3}$$

Finally, we turn to the non-adjacent foreign firm (denoted FNA) facing identical prices on both sides:

$$E_t \left[\sum_{s=0}^{\infty} R^s \left(p_{t+s}^{FNA} - e_{t+s} w^{F} \right) \left[\frac{L}{c} \left(p_{t+s}^{FA} + \frac{c}{4} - p_{t+s}^{FNA} \right) \right] - F^F \right]. \tag{4}$$

Solving for the 3 optimal pricing strategies in period $t+s$ on the part of the incumbents yields:

$$p_{t+s}^{D} = \frac{c}{4} + \frac{1}{12} \left(7w^{D} + 5e_{t+s} w^{F} \right), \tag{5}$$

$$p_{t+s}^{FA} = \frac{c}{4} + \frac{1}{12} \left(2w^{D} + 10e_{t+s} w^{F} \right), \tag{6}$$

$$p_{t+s}^{FNA} = \frac{c}{4} + \frac{1}{12} \left(w^{D} + 11e_{t+s} w^{F} \right). \tag{7}$$

The derivation of optimal pricing strategies in the other market configurations is identical and will not be given. Several stylized exchange rate scenarios will be considered.

3.1 PPP Symmetric Equilibrium

First, consider the benchmark case in which the exchange rate is expected to remain indefinitely at its PPP value: $E_t e_{t+i} = e_{t+i} = 1, i = 1, 2, 3 \cdots$. Foreign and domestic firms produce at the same effective marginal cost in all future periods. In this particular case the standard fully symmetric equilibrium obtains in every future period. Domestic and foreign firms charge identical prices:

$$p = \frac{c}{n} + w, \tag{8}$$

where n denotes the number of incumbents. Turning to the first stage, it is simple matter to verify that no foreign brands enter or exit provided that:

$$n+1 > \sqrt{\frac{cL}{F^F + S(1-R)}} \text{ and } \sqrt{\frac{cL}{F^F}} \geq n. \tag{9}$$

To facilitate the analysis we assume below that the conditions in equation (9) are satisfied, meaning that no entry or exit occurs when the exchange rate is expected to remain indefinitely at its PPP level of 1. Note that the latter condition implies that:

$$\frac{L}{c}\left(\frac{c}{n}\right)^2 \geq F^F \geq F^D. \tag{10}$$

In our 4-brand model, this assumption implies that the symmetric equilibrium is sustainable if all players would initially expect the exchange rate to remain at PPP forever[11] and it will prove particularly useful below.

3.2 Tau-period PPP Deviation

Second, consider a fully anticipated PPP deviation that lasts for τ periods, after which the exchange rate reverts to its PPP value forever:

$$E_t e_{t+i} = e_{1,t+i} = e_{1,t}, i = 1, 2, 3 \cdots \tau \tag{11}$$

$$E_t e_{t+i} = e_{2,t+\tau+i} = e_{2,\tau}, i = 1, 2 \cdots . \tag{12}$$

Small shocks are likely to leave the structure of the market unchanged while large anticipated shocks may trigger entry or exit in the first stage of the game. This translates into a band of inaction, where neither new entry nor exit of foreign firms takes place. Whenever exchange rate shocks fall within this range, the market structure is unaffected by the exchange rate shocks.

The first subsection deals at length with the properties of the band of inaction and the effects of shocks within this particular range. The next subsection describes what happens when currency shocks exceed these bounds. In this section we simply assume the home country is relatively small with respect to the rest of the world, which means that fluctuations of the domestic currency have no effect on the prices of similar goods marketed abroad. This is equivalent to assuming that domestic firms are not active in foreign markets. Given this assumption, the real exchange rate behavior is governed by domestic prices and the nominal exchange rate.[12] In the next section, however, this assumption is dropped.

[11] Strictly speaking, this assumption conflicts with the perfect foresight assumption in the next subsection.

[12] For our purposes, the real exchange rate is defined as the average price of the goods sold abroad in domestic currency relative to the average price of goods sold at home.

3.2.1 The Band of Inaction

For any value τ an upper and lower bound can be derived on the size of currency shocks. Whenever currency shocks exceed these bounds, the structure of the market is altered. Let Δe_τ denote the size of the $\tau-$period PPP deviation ($\Delta e = 1 - e$). To keep the analysis tractable, we introduce some additional notation. Let

$$\overline{x}(n) = \left(\frac{c}{n} - \frac{1}{(1-R^\tau)} \sqrt{(1-R^\tau)\left(\frac{c}{L}\left(F^F\right) - R^\tau\left(\frac{c}{n}\right)^2\right)} \right) \tag{13}$$

and

$$\underline{x}(n) = \left(\frac{c}{n+1} - \frac{1}{(1-R^\tau)} \sqrt{(1-R^\tau)\left(\frac{c}{L}\left(F^F + (1-R)S\right) - R^\tau\left(\frac{c}{n+1}\right)^2\right)} \right), \tag{14}$$

where n is any non-negative integer. The conditions in equation (10) imply that $\overline{x}(n) \geq 0$ and $\underline{x}(n) \leq 0$. Also note that $|\underline{x}(n+1)| \geq |\underline{x}(n)|$ and $\overline{x}(n) \geq \overline{x}(n+1)$.

Proposition 1.a: When the size of the $\tau-$period currency shock falls in the interval $\left(\Delta\underline{e}_\tau^4, \Delta\overline{e}_\tau^4\right)$, there is a unique Subgame Perfect Nash Equilibrium (henceforth SPNE) of the two-stage game without entry and/or exit in the first stage. $\Delta\overline{e}_\tau^4$ and $\Delta\underline{e}_\tau^4$ denote the largest PPP deviations persisting for τ periods without affecting market structure:

$$\begin{aligned} \Delta\overline{e}_\tau^4 &= \frac{6}{w}\overline{x}(4), \\ \Delta\underline{e}_\tau^4 &= \frac{19}{w}\underline{x}(4). \end{aligned} \tag{15}$$

under suitable conditions[13].

Proof: See Appendix.

If the size of the shocks remains within the band of inaction, the two-stage game has a unique SPNE: in the first stage there is neither entry nor exit while in the second stage the 4 incumbent brands simply apply the pricing strategies in equations (5), (6) and (7).

The foreign adjacent incumbent in the 4-brand market determines the upper bound as this brand is most likely to leave after a depreciation. $\Delta\overline{e}_\tau^4$ is positive, or $\overline{e}_\tau^4 > 1$, whenever firms realize positive profits with the exchange rate at

[13] If $\left(\frac{c}{L}\left(F^F\right) - R^\tau\left(\frac{c}{n}\right)^2\right) < 0$ then the adjacent brand never leaves when facing a shock of duration τ as a result of the small fixed costs. Conversely, if $\left(\frac{c}{L}\left(F^F + (1-R)S\right) - R^\tau\left(\frac{c}{n+1}\right)^2\right) < 0$, no appreciation is needed to induce entry.

PPP[14]. In this case the foreign firms are willing to accept temporary losses as a result of the depreciation in anticipation of offsetting future profits. If the exchange rate depreciates beyond the $\overline{e}_{\tau}^{4}$ threshold, the initial 4-brand market structure can no longer be sustained in equilibrium. The lower bound of the range is determined by the potential entrants' strategy. It can be verified that $\Delta\underline{e}_{\tau}^{4}$ is negative, $\underline{e}_{\tau}^{4} < 1$, provided that the no-entry condition in the benchmark symmetric case is satisfied[15]. Assume all of these conditions hold.

The band of inaction is not symmetrical around zero. Three aspects of the model account for this asymmetry. Firstly, potential entrants naturally consider the expected entry profits in a 5-brand market. All things being equal, average profits are lower in a 5-brand market than in a 4-brand market. This tends to increase the size of the appreciation needed to induce entry relative to the size of the depreciation that triggers exit. Secondly, the entering brand is by assumption located at the position furthest from the domestic brand, its profits being less sensitive to the exchange rate as a result.[16] The incumbent which considers leaving is located next to the domestic brand making its payoff highly sensitive to the exchange rate. Finally, the entrant needs to incur a sunk cost upon entry, which pushes the lower bound down. This sunk cost by its very nature does not directly affect the incumbents' strategy. Consequently, the upper bound does not directly depend on the size of the sunk cost.

Next, we discuss the effects of variations in the persistence of shocks, the level of differentiation and the cost structure on the size of the band of inaction.

Corollary 1.1: Firstly, $\Delta\overline{e}_{\tau}^{4} \downarrow, \Delta\underline{e}_{\tau}^{4} \uparrow$ and the band of inaction $\left(\Delta\underline{e}_{\tau}^{4}, \Delta\overline{e}_{\tau}^{4}\right)$shrinks as persistence τ increases. Secondly, $\Delta\overline{e}_{\tau}^{4} \uparrow, \Delta\underline{e}_{\tau}^{4} \downarrow$ and the band of inaction $\left(\Delta\underline{e}_{\tau}^{4}, \Delta\overline{e}_{\tau}^{4}\right)$widens as the level of product differentiation c increases. Thirdly, $\Delta\overline{e}_{\tau}^{4} \downarrow, \Delta\underline{e}_{\tau}^{4} \downarrow$ as fixed costs F increase.

Proof: See Appendix.

The band of inaction shrinks as the persistence of shocks increases. An increase in persistence (τ) unambiguously lowers $\Delta\overline{e}_{\tau}^{4}$ and decreases $\Delta\underline{e}_{\tau}^{4}$ in absolute value under the assumptions in equation (10)(see the Appendix for a proof). Smaller appreciations induce entry as persistence increases while smaller depreciations suffice to cause one of the foreign brands to leave. The effects of changes in the extent of product differentiation $(\frac{c}{L})$ are perhaps less obvious.

The upper bound tends to increase as the level of product differentiation rises. Differentiation effectively protects the foreign brand from fierce price

[14] This is whenever $\left(\frac{c}{4}\right)^2 \geq \frac{F^F c}{L}$. To see this, note that $\frac{L}{c}\left(\frac{c}{4}\right)^2 - F^F$ denotes the foreign brands' profits in the symmetric equilibrium.

[15] See the second condition in eq (9) or equivalently $\left(\frac{c}{5}\right)^2 < \frac{c}{L}F^F + (1-R)S$.

[16] Prescott and Visscher (1977) provide examples of endogenous entry within models where firms enter sequentially and then determine simultaneously the equilibrium location and the equilibrium number of firms. For tractability reasons we impose the new entrant to locate furthest away from the domestic competitor making it least affected by exchange rate shocks at the same time least profitable in terms of product positioning.

competition at high exchange rates thereby reducing the size of its losses. Conversely, the lower bound drops because entry is not as profitable at low exchange rates[17]. As it turns out, increases in the size of fixed costs tend to widen and shift the band of inaction.

The direct effect of an increase in F is to lower $\Delta\overline{e}_\tau^4$ and increase $\Delta\underline{e}_\tau^4$ (in absolute value), as can easily be verified. The band of inaction shifts downwards. However, as fixed costs increase, the original number of incumbents most likely cannot not be sustained in the initial equilibrium at PPP as the conditions in equation (10) would be violated. To account for this effect, we explore the change in the bounds while keeping equilibrium PPP profits π[18] constant by increasing c accordingly (while L remains fixed). This comparative statics exercise will allow us to compare different industries with an identical number of incumbents which feature different degrees of product differentiation and different levels of brand maintenance and distribution costs. The net effect of an increase in F, combined with an offsetting increase in c, clearly is to decrease the lower bound of the band of inaction.

Corollary 1.2: *Keeping the initial number of incumbents fixed,* the lower bound of the band of inaction $\Delta\underline{e}_\tau^4 \downarrow$ drops as F and c rise such that $d\pi = 0$. The effect on the upper bound $\Delta\overline{e}_\tau^4$ cannot be signed in general.

Proof:

$$\frac{\partial \Delta\underline{e}_\tau^4}{\partial F}dF + \frac{\partial \Delta\underline{e}_\tau^4}{\partial c}dc = \left(\frac{\partial \Delta\underline{e}_\tau^4}{\partial F}\frac{L}{16} + \frac{\partial \Delta\underline{e}_\tau^4}{\partial c}\right)dc < 0, \tag{16}$$

where $d\pi = -dF + \frac{L}{16}dc = 0$, as can be verified from Corollary 1.1 and 1.2.

The sign of the effect on the upper bound cannot be determined unambiguously. This suggests that the market structure in industries characterized by both high levels of fixed costs and high levels of product differentiation is less susceptible to appreciations than in other industries. Finally, the effect of the sunk cost itself merits some attention. Naturally, the direct effect of an increase in the size of sunk costs is to increase the required appreciation of the domestic currency needed to induce entry. S does not affect the upper bound directly. However, an increase in S is likely to affect the number of initial entrants which we have assumed to fix at 4 firms.[19] To eliminate this effect, we keep net expected profits[20] π^n at PPP constant and explore the effect of a change in the industry's sunk to fixed cost ratio. This represents the long-run effect.

Corollary 1.3: *Keeping the initial number of incumbents fixed,* the upper bound of the band of inaction $\Delta\overline{e}_\tau^4 \uparrow$ rises as the ratio of sunk to fixed costs increases. The lower bound $\Delta\underline{e}_\tau^4$ is not affected.

[17] "$\Delta\underline{e}_\tau^4$ drops" or "$\Delta\underline{e}_\tau^4$ increases in absolute value" will be used interchangeably below. Recall that we have assumed $\Delta\underline{e}_\tau^4 < 0$.

[18] $\pi = \frac{L}{c}\left(\frac{c}{4}\right)^2 - F^F$

[19] Violation of this assumption requires the calculation of initial entrants every time which would diverge our focus from our main objective of entry and exit decisions.

[20] $\pi^n(1-R) = \frac{L}{c}\left(\frac{c}{4}\right)^2 - F^F - S(1-R)$

Proof:

$$\frac{\partial \Delta \overline{e}_\tau^4}{\partial S} \mid_{d\pi^n=0} = -\frac{1}{1-R} \frac{\partial \Delta \overline{e}_\tau^4}{\partial F} > 0, \tag{17}$$

$$\frac{\partial \Delta \underline{e}_\tau^4}{\partial F} \mid_{d\pi^n=0} = 0. \tag{18}$$

As is to be expected, the market structure of industries characterized by a high ratio of sunk to fixed costs is less likely to be affected by a depreciation. Once the incumbents have incurred the sunk cost, they are willing to stay in the market even at very low exchange rates. The ratio of sunk to fixed costs has no effect on the minimum size of the appreciation that induces entry. Summarizing, industries featuring higher sunk entry costs have comparatively wider positive sections of the inaction band.

To drive this point home completely, consider the US experience of the 80's in a high sunk entry cost industry. Suppose the initial appreciation of the dollar exceeded the lower bound $\Delta \underline{e}_\tau^4$, causing exactly one foreign brand to make the required advertising expenditures and join the US market. If the $5-$brand equilibrium is viable at PPP, all of the brands remain in the market even after the exchange rate returns to its original value. In fact, a larger or more persistent depreciation than the original appreciation is needed to revert to the initial n-brand equilibrium when the ratio of sunk to fixed costs is high, as $\left|\Delta \underline{e}_\tau^4\right| \leq \Delta \overline{e}_\tau^5$ for large $\frac{S}{F^F}$.

Corollary 1.4: $\Delta \overline{e}_\tau^5 \geq \left|\Delta \underline{e}_\tau^4\right|$ for S relatively large to F.

Proof: see Appendix.

Naturally, if the entrant anticipates losses once the exchange rate has returned to its initial value at time $t+\tau$, he withdraws from the market at that point in time rather than incur losses. A different lower bound then obtains.

Proposition 1.b: When the entrant anticipates losses at $t+\tau$, the lower bound changes to:

$$\Delta \underline{e}_\tau^4 = \frac{19}{w}\left(\frac{c}{5} - \sqrt{\frac{c}{L}\left(F^F + \frac{(1-R)}{1-R^\tau}S\right)}\right), \tag{19}$$

for $\left(\frac{c}{5}\right)^2 < \frac{c}{L}F^F$.

Proof: see Appendix.

Having described the nature of the band of inaction, the behavior of prices and volumes in response to currency shocks remains to be discussed. As long as the exchange rate change does not exceed these bounds, firms simply follow the optimal pricing strategies in (5), (6) and (7) while market structure remains unaltered. Using these strategies, simple calculus yields the elasticities of prices and import volumes with respect to exchange rate changes and we derive the following corollaries.

Corollary 1.5: The price adjustment in the domestic market is incomplete with respect to the exchange rate shock when $\Delta e_\tau \in \left(\Delta \underline{e}_\tau^4, \Delta \overline{e}_\tau^4\right)$. The elasticity of prices with respect to changes in the exchange rates are:$\varphi^D = \frac{5}{12}\psi, \varphi^{FA} = \frac{10}{12}\psi$, $\varphi^{FNA} = \frac{11}{12}\psi$ and $\varphi^{Av} = \frac{9}{12}\psi$ where $\psi = \frac{1}{1+\frac{c}{4w}}$. The market share of imports drops below its initial value of 3 quarters in case of a depreciation when $\Delta e_\tau \in \left(\Delta \underline{e}_\tau^4, \Delta \overline{e}_\tau^4\right)$. The elasticity of the volume of imports w.r.t. the exchange rate in this setup is $-\frac{5}{9}\frac{w}{c}$.

Proof: For the first part return to equations (5), (6) and (7). For the second part see the Appendix.

The relative price of both imported brands (FA and FNA) increases in response to a depreciation. The non-adjacent imported brand raises its price by more than the adjacent imported brand, because it does not face direct competition from the domestic brand. It should be noted that the relative price of brand FA in terms of the domestic brand has an elasticity of $\frac{5}{12}\psi$, whereas the brand FNA has an elasticity of $\frac{1}{2}\psi$. [21] Unlike previous models, this particular model predicts substantial differences in pass-through depending on the market position of the foreign and domestic brands. Clearly, pass-through increases uniformly as the distance separating the foreign from the domestic brand increases. The average price-elasticity (in other words weighted by market share: $\varphi^{Av} = \frac{9}{12}\psi$) is a useful indicator of overall pass-through in a given industry.

Next, we evaluate the trade volumes. Note that the elasticity of imports declines as the disutility cost per unit of distance increases, or equivalently, as the substitutability of brands decreases. It increases as marginal costs increase, since the exchange rate shocks affect marginal costs. Since the underlying structure of the economy is not affected by small currency shocks, no hysteresis effects obtain. When PPP is restored in period $t + \tau$ ($e_{t+s} = 1$), all effects of these small devaluations gradually disappear. Hence, we can refer to $\left(\Delta \underline{e}_\tau^4, \Delta \overline{e}_\tau^4\right)$ as the band of inaction: in this range all exchange rate shocks are market structure neutral and do not affect prices and trade volumes in the long run.

3.2.2 Large Depreciations

Shocks outside of the range trigger structural changes. Note that the no-entry condition in the symmetric equilibrium (see the second condition in equation (9)) implies that appreciations cannot have any permanent effects when S tends to zero. For $S = 0$, the entrant simply leaves at $t + \tau$ as the 5-brand equilibrium is not sustainable at PPP. In case of a depreciation, re-entry at $\tau + t$ will be

[21] These price elasticities decline as the substitutability of brands decreases ($c \uparrow$).

effectively deterred only if the sunk entry cost is relatively large. By contrast, if foreign entry is profitable at PPP[22], the market reverts to its original 4-brand equilibrium. If not, the 3-brand equilibrium prevails and the effects of the depreciation become permanent. For $S = 0$, entry is always profitable (see the second condition in equation (9)) and naturally no hysteresis effects obtain.

Remark 1: Large nominal currency shocks have hysteresis effects if and only if the sunk entry cost is non-zero.

First, we analyze the effects of large depreciations. These cause one or several of the foreign brands to quit serving the domestic market. Recall that entry by an additional domestic firm is ruled out by the restriction imposed on the number of potential domestic producers. The domestic incumbent always stays irrespective of the foreign brands' strategies: re-entry is a dominant strategy for this firm, as it anticipates strictly positive profits in every future period. The following propositions describe a number of SPNE in which one or more of the foreign incumbents quit in the first stage of the game.

If the size of the depreciation does not exceed a given threshold, the non-adjacent foreign brand stays regardless of the other brands' strategies. In this case the theory allows for accurate predictions about which brand(s) will actually leave. If the size of the depreciation exceeds this bound, multiple equilibria obtain. In some of these the non-adjacent brands leave.

Proposition 2.a: When the size of the $\tau-$period currency depreciation falls in the interval $\left(\Delta\overline{e}_{\tau}^{4}, \Delta\overline{e}_{\tau}^{4,NA}\right)$, at least one of the adjacent foreign brands leaves the market in the first stage in both of the SPNE of the two-stage game.

$$\Delta\overline{e}_{\tau}^{4,NA} = \frac{12}{w}\overline{x}(4), \tag{20}$$

for $\frac{F^F c}{L} \geq R^{\tau}\left(\frac{c}{4}\right)^2$ and $\left(\frac{c}{4}\right)^2 \geq \frac{F^F c}{L}$.

Proof: see Appendix.

Proposition 2.b: When the size of the $\tau-$period depreciation shock falls in the interval $\left(\Delta\overline{e}_{\tau}^{4}, \Delta\overline{e}_{\tau}^{3}\right)$ and $\Delta\overline{e}_{\tau}^{3} \leq \Delta\overline{e}_{\tau}^{4,NA}$, exactly one of the adjacent foreign brands leaves the market in the first stage in both of the SPNE of the two-stage game.

$$\Delta\overline{e}_{\tau}^{3} = \frac{5}{w}\overline{x}(3), \tag{21}$$

[22] $\frac{L}{c}\left(\frac{c}{4}\right)^2 - F - S(1-R) \geq 0$

for $\frac{F^F c}{L} \geq R^\tau \left(\frac{c}{3}\right)^2$ and $\left(\frac{c}{3}\right)^2 \geq \frac{F^F c}{L}$.
When the size of the $\tau-$period currency depreciation falls in the interval $\left(\Delta\overline{e}_\tau^{4,NA}, \Delta\overline{e}_\tau^3\right)$ and $\Delta\overline{e}_\tau^3 > \Delta\overline{e}_\tau^{4,NA}$, exactly one of the foreign brands, but not necessarily the adjacent one, leaves the market in the first stage in both of the SPNE of the two-stage game.

Proof: see Appendix.

This decrease in the number of foreign brands has two distinct effects on prices. Firstly, all brands are subject to a once-off and uniform price increase, the size of which does not vary with the size of the exchange rate change. The increased extent of product differentiation due to the decrease in the total number of brands only affects absolute price levels: all brands charge a uniformly higher price as a result of the decrease in the total number of brands. Consumers therefore end up with a lower surplus. Secondly, and more importantly, the relative number of foreign brands will have decreased. As a result, foreign firms will face tougher price competition from the domestic brand when the real exchange rate falls below PPP. As a consequence they are prevented from raising their prices by as much when the exchange rate rises[23]. The price elasticities are smaller than the ones in the small devaluation case. The percentage increase in domestic and imported brand's absolute prices may be either larger or smaller than in the 4-brand case, depending on the relative size of these two effects.

Corollary 2.1: When $\Delta e_\tau \in \left(\Delta\overline{e}_\tau^4, \Delta\overline{e}_\tau^3\right)$, all prices are subject to a uniform price increase (of 1/3). In addition, firms price to the market and the elasticity of prices with respect to changes in the exchange rates are:$\varphi^D = \frac{2}{5}\widetilde{\psi}, \varphi^F = \frac{4}{5}\widetilde{\psi}$ and $\varphi^{Av} = \frac{2}{3}\widetilde{\psi}$ where $\widetilde{\psi} = \frac{1}{1+\frac{c}{3w}}$. On the other hand, large exchange rate devaluations have proportionately larger effects on imports than small devaluations. When $\Delta e_\tau \in \left(\Delta\overline{e}_\tau^4, \Delta\overline{e}_\tau^3\right)$, the elasticity of the volume of imports with respect to the exchange rate is $-\frac{3}{5}\frac{w}{c}$.

Proof: From equations (22) and (23).

The percentage increase in the relative price of imported goods is unambiguously lower in the large devaluation case: $\frac{2}{5}$. The relative price elasticity is entirely determined by the second effect. Large devaluations trigger smaller relative import price increases. The average price elasticity drops from $\frac{3}{4}\frac{1}{1+\frac{c}{4w}}$ to $\frac{2}{3}\frac{1}{1+\frac{c}{3w}}$, clearly indicating an overall decrease in pass-through due to the decrease in foreign brand market power.

[23] In a 3-brand market the optimal pricing strategies for domestic and foreign brands are:

$$p^D = \frac{c}{3} + \frac{1}{5}(3w^D + 2ew^F), \tag{22}$$

and

$$p^F = \frac{c}{3} + \frac{1}{5}(w^D + 4ew^F). \tag{23}$$

Next, we turn to trade volumes. Two mutually strengthening effects determine the response of import volumes to the exchange rate change. Firstly, there exists a lump-sized *market structure* effect: the drop in the number of foreign firms causes the market share of imports to drop by a given percentage whenever the currency devaluation exceeds the upper bound $(\Delta\overline{e}_\tau^4)$.[24] Secondly, there is the *direct effect* of the real exchange rate change on import volumes. The latter turns out to be larger than the percentage change in response to small exchange rate changes (this can be verified by returning to Corollary 1.2). This means that the percentage decline in imports is unambiguously larger in response to large exchange rate changes.

Note that large devaluations have a stronger impact on imports than small devaluations because the resulting decrease in the relative number of foreign firms toughens price competition with the domestic firm as the non-adjacent foreign firm becomes an adjacent foreign firm.

Finally, we examine what happens when the exchange rate returns to its original value. Recall that non-incumbents incur a sunk cost S. Re-entry will only be deterred when this sunk cost is relatively large.

Corollary 2.2: When $\Delta e_\tau \in \left(\Delta\overline{e}_\tau^4, \Delta\overline{e}_\tau^3\right)$ the 3-brand market structure is preserved at time $t+\tau$ when PPP is restored in all of the SPNE of the game if and only if

$$4 > \sqrt{\frac{cL}{F^F + S(1-R)}} \tag{24}$$

Proof: Consider the expected profits at PPP to derive this condition.

Large temporary exchange rate changes can have lasting effects on prices and trade volumes if entrants incur a sufficiently large sunk cost. If so, all brands charge a uniformly higher price ($p = \frac{c}{3} + w$) from $t+\tau$ and imports only account for 66% of total sales in the domestic market, as opposed to 75% before the depreciation.

This subsection concludes by briefly considering the effects of an even larger depreciation.

Proposition 2.c: When the size of the $\tau-$period currency depreciation falls in the interval $\left(\Delta\overline{e}_\tau^3, \Delta\overline{e}_\tau^2\right)$ and $\Delta\overline{e}_\tau^2 \leq \Delta\overline{e}_\tau^{4,NA}$, both of the adjacent foreign brands leave the market in the first stage in both of the SPNE of the two-stage game.

$$\Delta\overline{e}_\tau^2 = \frac{3}{w}\overline{x}(2), \tag{25}$$

[24] In our numerical example the lump-sized percentage increase in the volume of imports equals 6%.

for $\frac{F^F c}{L} \geq R^{\tau}\left(\frac{c}{2}\right)^2$ and $\left(\frac{c}{2}\right)^2 \geq \frac{F^F c}{L}$.
When the size of the $\tau-$period currency shock falls in the interval $\left(\Delta\overline{e}_{\tau}^{4,NA}, \Delta\overline{e}_{\tau}^{2}\right)$ and $\Delta\overline{e}_{\tau}^{2} > \Delta\overline{e}_{\tau}^{4,NA}$, exactly one of the foreign brands, not necessarily the adjacent one, leaves the market in the first stage in both of the SPNE of the two-stage game.

Proof: see Appendix.

Corollary 2.3: When $\Delta e_{\tau} \in \left(\Delta\overline{e}_{\tau}^{3}, \Delta\overline{e}_{\tau}^{2}\right)$, all prices are subject to a uniform price increase (of 1/2). In addition, firms price to the market and the elasticities of prices with respect to changes in the exchange rates are: $\varphi^D = \frac{1}{3}\widetilde{\psi}, \varphi^F = \frac{2}{3}\widetilde{\psi}$ and $\varphi^{Av} = \frac{1}{2}\widetilde{\psi}$ where $\widetilde{\psi} = \frac{1}{1+\frac{c}{2w}}$. On the other hand, import volumes are more responsive to even larger currency shocks. When $\Delta e_{\tau} \in \left(\Delta\overline{e}_{\tau}^{3}, \Delta\overline{e}_{\tau}^{2}\right)$, the elasticity of the volume of imports with respect to the exchange rate is $-\frac{2}{3}\frac{w}{c}$.

Proof: From equations (22) and (23).

The relative price elasticity of imported goods $\left(\frac{1}{3}\right)$ is even lower in this case. Large devaluations trigger smaller relative import price increases.

3.2.3 Large Appreciations

Large appreciations cause one or several of the non-incumbent foreign brands to enter the market. In addition, the domestic incumbent may consider leaving its home market if it anticipates high losses as a result of the appreciation. Casual observation suggests that the main driving forces behind the market structure effects of exchange rate shocks are entry and exit on the part of foreign firms. Domestic brands generally do not quit serving their home markets in the face of low real exchange rates. This may be due to the relative fixed cost efficiency of domestic brands. In our model foreign brands incur higher fixed costs while serving the home market ($F^D < F^F$).

Proposition 3.a: The home brand invariably leaves the market in the first stage of the two stage game if the appreciation exceeds $\Delta\underline{e}_{\tau}^{4,D}$:

$$\Delta\underline{e}_{\tau}^{4,D} = -\frac{12}{5w}\overline{x}^D(4), \tag{26}$$

for $\frac{F^D c}{L} \geq R^{\tau}\left(\frac{c}{4}\right)^2$, $\left(\frac{c}{4}\right)^2 \geq \frac{F^D c}{L}$ and $\overline{x}^D(4) = \overline{x}(4)$ where F is adjusted accordingly.

Proof: see derivation of Proposition 2.a.

Naturally, if the domestic brand leaves, pass-through is complete since the remaining n foreign brands price according to:

$$p^F = ew + \frac{c}{n} \tag{27}$$

Note that the domestic brand never leaves, regardless of the size of the appreciation provided that $\frac{F^D c}{L} \leq R^\tau \left(\frac{c}{4}\right)^2$. In order to concentrate on foreign entry and exit, it simply needs to be assumed that this condition holds for all shocks Δe_τ considered in this last section[25].

Proposition 3.b: When the size of the $\tau-$period currency appreciation falls in the interval $\left(\Delta \underline{e}_\tau^5, \Delta \underline{e}_\tau^4\right)$, one foreign brand enters the market in the first stage of the SPNE of the two-stage game.

$$\Delta \underline{e}_\tau^5 = \frac{45}{w} \underline{x}(5) \tag{28}$$

Proof: for $\left(\frac{c}{6}\right)^2 \geq \frac{c}{L}\left(F^F\right)$ and $\frac{c}{L}\left(F^F + (1-R)S\right) \geq R^\tau \left(\frac{c}{6}\right)$.

If the 5-brand configuration is not sustainable at PPP[26], the entering firm's optimal strategy is to leave at $t+\tau$. Hence, a new bound obtains:

$$\Delta \overline{e}_\tau^5 = \frac{45}{w}\left(\frac{c}{6} - \sqrt{\frac{c}{L}\left(F^F + \frac{(1-R)}{1-R^\tau}S\right)}\right). \tag{29}$$

If the appreciation exceeds $\Delta \underline{e}_\tau^5$, 2 foreign brands will enter. Recall that the total number of potential foreign brands serving the domestic market was fixed at five. No additional entry is feasible.

Proposition 3.c: When the size of the $\tau-$period currency appreciation exceeds $\Delta \underline{e}_\tau^5$, two foreign brands enter the market in the first stage of the SPNE of the two-stage game.

Proof: see Appendix.

As before, this change in the number of foreign brands has two distinct effects on prices. Firstly, the total number of brands has increased. The price mark-ups are subject to a once-off and uniform cut, independent of the size of the exchange rate appreciation. Secondly, the relative number of domestic brands has decreased. As a result, foreign firms will not face as much price competition from the domestic brand.

[25] In other words, we assume F^D is sufficiently small.
[26] That is if $\left(\frac{c}{6}\right)^2 \leq \frac{c}{L}\left(F^F\right)$.

Corollary 3.1: When $\Delta e_\tau \in \left(\Delta \underline{e}_\tau^5, \Delta \underline{e}_\tau^4\right)$, all prices are subject to a uniform price decrease. In addition, firms price to the market and the elasticity of prices w.r.t. changes in the exchange rates are:$\varphi^D = \frac{8}{19}\widetilde{\psi}, \varphi^{FA1} = \frac{16}{19}\widetilde{\psi}$, $\varphi^{FNA} = \frac{18}{19}\widetilde{\psi}$ and $\varphi^{Av} = \frac{4}{5}\widetilde{\psi}$, where $\widetilde{\psi} = \frac{1}{1+\frac{c}{5w}}$. On the other hand, volume of imports increases proportionally higher with large appreciations than with small appreciations. When $\Delta e_\tau \in \left(\Delta \overline{e}_\tau^4, \Delta \overline{e}_\tau^3\right)$, the elasticity of the volume of imports w.r.t. the exchange rate is $-\frac{10}{19}\frac{w}{c}$.

Proof: From equations (22) and (23).Firstly, the across-the-board price cut of all prices amounts to 20%. Secondly, the average price elasticity increases to $\frac{4}{5}\frac{1}{1+\frac{c}{5w}}$, which reflects the foreign brands' relative increase in market share power[27]. As a result, pass-through is clearly larger in response to large appreciations. This result continues to hold more generally. It does not depend on our assumption about the cost structure of domestic and foreign firms and the resulting tendency of the domestic incumbent not to quit even at relatively low exchange rates. In fact, if the domestic brand leaves in response to an appreciation, pass-through is complete.

Next, we turn to trade volumes. Two mutually strengthening effects determine the response of import volumes to the exchange rate change. Firstly, there is a lump-sized *market structure* effect: the increase in the number of foreign firms causes the market share of imports to rise by a given percentage whenever the currency appreciation exceeds the lower bound $(\Delta \underline{e}_\tau^4)$.[28] Secondly, there is the *direct effect* of the real exchange rate change on import volumes. Large exchange rate appreciations have proportionately smaller direct effects (about a 30% drop) on imports than small ones, because the resulting increase in the relative number of foreign firms makes prices more responsive to exchange rate changes when the real exchange rate drops below PPP. The total percentage increase in imports reflects the combined impact of both of the above and cannot be unambiguously determined.

Small and large appreciations have significantly different effects on prices in this version of the model. Pass-through is larger in response to large appreciations. For given foreign prices, this implies that real exchange rate deviations are larger in response to large negative shocks than in response to large positive shocks. In contrast, the responses to small shocks are symmetrical.

Remark 2: Pass-through is smaller for $\Delta e_\tau > \Delta \underline{e}_\tau^4$ shocks than for $\Delta e_\tau \in \left(\Delta \underline{e}_\tau^4, \Delta \overline{e}_\tau^4\right)$. Pass-through is larger for $\Delta e_\tau < \Delta \underline{e}_\tau^4$ shocks than for $\Delta e_\tau \in \left(\Delta \underline{e}_\tau^4, \Delta \overline{e}_\tau^4\right)$.

[27] Note that the relative import price elasticies for the adjacent brands ($\frac{8}{19}$ for the FA brand and $\frac{10}{19}$ for the FNA brand)) are slightly higher than the corresponding relative price elasticities in the case of small appreciations ($\frac{5}{12}$ for the FA brand and $\frac{6}{12}$ for the FNA brand).

[28] In our numerical example the lump-sized percentage decrease in the volume of imports equals 12%.

The extent of pass-through increases even further as an additional foreign brand enters. The average price elasticity increases to $\frac{5}{6}\frac{1}{1+\frac{c}{6w}}$. In addition, the market structure effect of entry on the part of these two brands increases the volume of imports by about 50%. On the other hand, the volume of imports becomes less responsive to exchange rate changes as a result of the foreign brand's increased market share.

Corollary 3.2: When $\Delta e_\tau \geq \Delta \underline{e}_\tau^5$, all prices are subject to a uniform price decrease. In addition, firms price to the market and the elasticity of prices with respect to changes in the exchange rates are:$\varphi^D = \frac{19}{45}\widetilde{\psi}, \varphi^{FA1} = \frac{38}{45}\widetilde{\psi}$, $\varphi^{FA2} = \frac{43}{45}\widetilde{\psi}, \varphi^{FNA} = \frac{44}{45}\widetilde{\psi}$ and $\varphi^{Av} = \frac{5}{6}\widetilde{\psi}$ where $\widetilde{\psi} = \frac{1}{1+\frac{c}{6w}}$. On the other hand, when $\Delta e_\tau \geq \Delta \underline{e}_\tau^5$, the elasticity of the volume of imports with respect to the exchange rate is $-\frac{38}{75}\frac{w}{c}$.

3.3 Autocorrelation

To illustrate the more general scope of the above results, we explore the impact of a shock that dies out gradually. The model and all the assumptions introduced in the previous section are maintained unless indicated otherwise. Most importantly, all of the agents again are assumed to have perfect foresight. As before, an upper and lower bound can be derived. Shocks within this band of inaction do not affect market structure. The size of this range depends on the specifics of the time series process.

Let us consider an AR(1)-process:

$$e_t = \sum_{s=0}^{\infty} \rho^s \varepsilon_{t+s}. \tag{30}$$

We can write the upper and lower bounds of inaction as:

$$\overline{x}^\rho(n) = \left(\frac{c}{n} - \sqrt{\left(\frac{F^F c}{L}\frac{1}{1-\rho^2 R} - \left(\frac{c}{n}\right)^2 \frac{R(1-\rho)^2}{(1-R)(1-\rho R)^2}\right)}\right), \tag{31}$$

and

$$\underline{x}^\rho(n) = \left(\frac{c}{n+1} - \sqrt{\left(\frac{\left(F^F+(1-R)S\right)c}{L}\frac{1}{1-\rho^2 R} - \left(\frac{c}{n+1}\right)^2 \frac{R(1-\rho)^2}{(1-R)(1-\rho R)^2}\right)}\right), \tag{32}$$

where n is any non-negative integer. Given this definition of $\overline{x}^\rho(n)$ and $\underline{x}^\rho(n)$, all of the above results carry over to this adjusted version of the model. For the sake of brevity, we only mention the 1st proposition again for a 4 firm set-up.

Proposition 4 When the size of the $\tau-$period currency shock falls in the interval $\left(\Delta \underline{e}_\tau^4, \Delta \overline{e}_\tau^4\right)$, there is a unique SPNE of the two-stage game without entry and/or exit in the first stage, where $\Delta \overline{e}_\tau^4$ and $\Delta \underline{e}_\tau^4$ denote the largest PPP deviations persisting for τ periods without affecting market structure:

$$\Delta\overline{e}^4 = \frac{6}{w}\overline{x}^\rho(4),$$
$$\Delta\underline{e}^4 = \frac{19}{w}\underline{x}^\rho(4), \quad (33)$$

assuming the 5-brand-equilibrium is profitable: $\frac{L}{c}(\frac{c}{5})^2 - F \geq 0$.

Proof: Analogous to previous derivations.

3.4 Baldwin Hypothesis

The same exercise can be carried out for more general time series processes. One could easily introduce a second-order autoregressive process

$$e_t = (1 - \rho_1 L - \rho_2 L^2)^{-1}\varepsilon_t, \quad (34)$$

to describe the hump-shaped time pattern - large rise and subsequent fall - of the US dollar centered around 1985 and, as was done for the AR(1), derive the bounds of the band of inaction. As it turns out, the general predictions of the model are entirely consistent with the US experience in the mid-80's.[29] Consider an initial shock Δe to the US dollar that falls below the appropriately defined lower bound $\Delta\underline{e}^n$, where n denotes the number of incumbents in the industry. This triggers entry by one or several of the foreign brands. All domestic US prices decrease by a given percentage (depending on the exact number of firms in the industry) as a result of the decreased mark-up. In addition, the appreciation has a larger direct impact on brand prices because of the increased elasticity of US prices with respect to the dollar (see Corollary 3.2 and Remark 1). As the dollar depreciates back to its original value ($e = 1$), some of the m foreign entrants remain in the US market and US firms do not regain all of their lost market shares. In fact, if the $n + m$-brand symmetric equilibrium is profitable $\left(\frac{L}{c}(\frac{c}{n+m})^2 - F \geq 0\right)$, none of the entrants leave. As the number of firms does not change, the depreciation has no mark-up effect on US prices. There is only a direct exchange rate effect. Even if a number of the m entrants leave after the exchange rate has returned to its original value, the mark-up effect would still be smaller than the effect of the appreciation, unless all m left. In addition, as foreign brands leave, average pass-through declines and the direct effect decreases . Hence, the model predicts US prices do not rise by as much during the depreciation as they fell during the appreciation, which is exactly what happened (see Baldwin, 1988).

[29] Note that during the 1980's the European trade balance was more responsive to exchange rate fluctuations as compared to the United States or Japan. For some European, in particular German, evidence consult Gagnon and Knetter (1995), Feenstra *et al.* (1996) or Feenstra and Kendall (1997).

4 Home and Abroad

So far, in analyzing pass-through, we have maintained the small country assumption which holds that domestic currency shocks were not allowed to affect the foreign prices of the same commodity. This section puts it all together. The foreign market is introduced into the analysis and examines the combined effect of pricing responses at home and abroad on the real exchange rate. Coming back to the US experience of the 80's, the dollar rise and subsequent fall is likely to have affected European and Japanese prices as well in most industries, given the strong presence of US enterprises abroad. Second, the exact numerical results in the previous sections naturally depend on the relative number of domestic vis-a-vis foreign incumbents and the absolute number of brands in the market.[30] This section's numerical examples serve to illustrate the more general scope of the above results.

Firstly, the basic results discussed above are shown to reproduce themselves in slightly different market settings. Table 1 summarizes the effects of variations in the relative number of foreign versus domestic market players. All of the basic assumptions of the model above continue to hold. Recall that domestic and foreign brands are clustered.

Also, consider the price and import elasticities reported in Table 1 and notice how the price elasticities decline as the relative number of domestic market players increases, while the import volume elasticities increase. The elasticity of import prices tends to unity as the number of foreign firms increases relative to the number of domestic firms.[31] Logically, higher levels of pass-through prevail in foreign dominated markets. In fact, it is obvious from Table 1 and 2 that the average price elasticity simply equals the proportion of foreign brands operating in the market (times a constant which increases uniformly with the absolute number of brands).[32]

Remark 3: The average elasticity of prices with respect to exchange rates equals the market share of foreign brands times a constant ψ, where $\psi = \frac{1}{1+\frac{c}{nw}}$.

The closest neighboring foreign brands of the domestic brand invariably have lower absolute and relative import price elasticities. As domestic brand concentration increases, import volumes become more responsive to devaluations, while the price-cost margins of foreign brands are subject to a larger drop for a given size of the devaluation. As a result of this, foreign brands leave the market at

[30] Also see Dornbusch (1987).

[31] However, relative import price elasticities (import prices in terms of domestically manufactured commodities) are nearly invariant to changes in the relative number of domestic vs. foreign brands. Let x denote the initial number of domestic brands, while y is the number of foreign brands. Switching from a xD-yF to a yD-xF market does not change the relative import price elasticities, as can readily observed in Tables 1 and 2. .

[32] This result turns out not to depend on the clustering of domestic and foreign brands. In fact, Dornbusch's (1987) alternating/symmetric setup yields an average price elasticity of 1/2 ψ, which is exactly the market share of foreign firms.

increasingly lower levels of the exchange rate: the upper bound decreases and the band of inaction shrinks. Note from Table 1 that the range in the 3D-1F setup is about one third of the 1D-3F range. A similar decrease obtains in the 6 brand case. Conversely, the lower bound rises (i.e. decreases in absolute value) as domestic concentration increases. This allows the entering foreign brand to realize larger profits in case of an appreciation as the home brands' prices respond very little to the appreciation. Summarizing, as domestic concentration rises, the band of inaction shrinks and ultimately disappears altogether.

It is easy to verify that the basic results derived in Propositions 1,2 and 3 and the Corollaries continue to hold as the relative number of foreign and domestic brands changes.

Remark 4: The band of inaction shrinks uniformly as domestic concentration increases. It all but disappears as the domestic brands' market share tends to unity.

Next, we drop the small country assumption. First, consider small shocks, i.e. those that fall within the band of inaction. Consider a world consisting of two countries, home and foreign. From Table 1 and 2, it is easy to verify that the real exchange rate is not affected by small nominal shocks if domestic firms and foreign firms have identical market shares at home and abroad. If domestic firms have a higher market share at home than abroad, then pass-through is less than complete and the real exchange rate tracks the nominal exchange rate, albeit less than one-for-one. Finally, pass-through exceeds 100% whenever domestic firms jointly have a higher market share abroad than at home. Given that intra-industry trade accounts for an increasing portion of the total trade volume, this qualification of our previous result is quite important.

Consider for instance the case where 1 domestic and 3 foreign brands compete both at home and abroad. In response to a 'small' 1% depreciation, the average domestic price increases by .75% and the average foreign price decreases by .25%, leaving the real exchange rate unchanged. By contrast, if the domestic firm is not active abroad, the real exchange depreciates by .25% in response to a 1% nominal depreciation. Finally, in the unlikely case that 2 domestic firms operate abroad (in a 4-brand market), the real exchange rate appreciates by .25 %.

Remark 5: The real exchange rate is invariant with respect to small nominal shocks if the market structures at home and abroad are fully symmetrical. If the domestic brands have a smaller market share abroad than at home, pass-through is incomplete. The real exchange rate variability increases as the difference between the domestic and foreign market shares increases. If the domestic brands have a larger market share abroad than at home, the real exchange rate moves in the direction counter to the nominal exchange rate.

To conclude, we briefly consider what this implies for relinquishing the national currencies as in the case of EMU. If the volume of intra-industry trade within Europe grows and market structures in different European countries slowly become more alike as a result of the economic integration process, nominal currency shocks are less likely to have significant effects on the real exchange rate. This means that (ex-post) the cost of giving up the national currencies as a tool to improve competitiveness will be very small. In fact, in the limiting case in which the market structures in different European countries are completely identical, both the real exchange rate and the current account are invariant with respect to nominal shocks. This would clearly reduce the importance of the exchange rate as a tool of macroeconomic policy. On the other hand, if the process of economic integration favors regional concentration, the opposite result obtains and the loss of monetary autonomy may (ex-post) be quite costly.

Table 1: **Four Brands: Domestic and Foreign Brands Clustered**

Setup	Upper Bound	Lower Bound	Price Elasticity	Average Price El.	Import El.
$n = 4$	$\Delta\overline{e}_{\tau}^{4}$	$\Delta\underline{e}_{\tau}^{4}$			
1D&3F	$\frac{6}{w}\overline{x}(n)$	$\frac{19}{w}\underline{x}(n)$	$\varphi^{D} = \frac{5}{12}\psi$ $\varphi^{FA} = \frac{10}{12}\psi$ $\varphi^{FNA} = \frac{11}{12}\psi$	$\frac{3}{4}\psi$	$-\frac{5w}{9c}$
2D&2F	$\frac{4}{w}\overline{x}(n)$	$\frac{19}{2w}\underline{x}(n)$	$\varphi^{D} = \frac{3}{12}\psi$ $\varphi^{F} = \frac{9}{12}\psi$	$\frac{2}{4}\psi$	$-\frac{w}{c}$
3D&1F	$\frac{12}{5w}\overline{x}(n)$	$\frac{19}{5w}\underline{x}(n)$	$\varphi^{DNA} = \frac{1}{12}\psi$ $\varphi^{DA} = \frac{2}{12}\psi$ $\varphi^{F} = \frac{7}{12}\psi$	$\frac{1}{4}\psi$	$-\frac{5w}{3c}$

5 Concluding Remarks

This paper focused on the short and long term aspects of nominal exchange rate shocks. We analyzed the effects of nominal currency shocks on prices and market structure in differentiated goods markets served by domestic and foreign brands. Nominal shocks that fall within a given range do not affect market structure and have no hysteresis effects. The bounds of this range are asymmetric. The size of the asymmetry depend on the size and persistence of the nominal exchange rate shock, the nature of product heterogeneity and the relative size of the sunk entry cost. More heterogeneity favors the brands that are at a competitive disadvantage. As a result, it discourages entry of foreign brands at low exchange rates. Similarly, it makes foreign brands less likely to leave at high exchange rates. This implies that the band of inaction widens as the level of product differentiation increases. On the other hand, sunk costs invariably protect the incumbents. If sunk costs are relatively large, foreign incumbents may stay in the market even at low exchange rates. As the ratio of sunk- to fixed costs increases, the upper bound of the band of inaction rises. High sunk cost industries are less susceptible to large depreciations. Finally, this range shrinks as the domestic brands' market share rises.

Table 2: **Six Brands: Domestic and Foreign Brands Clustered**

Setup	Price Elasticity	Average Price El.	Import Elasticity
$n = 6$			
1D&5F	$\varphi^{D1} = \frac{19}{45}\psi$ $\varphi^{FA1} = \frac{38}{45}\psi$ $\varphi^{FA2} = \frac{43}{45}\psi$ $\varphi^{FNA} = \frac{44}{45}\psi$	$\frac{5}{6}\psi$	$-\frac{38w}{75c}$
2D & **4F**	$\varphi^{DA} = \frac{4}{15}\psi$ $\varphi^{FA1} = \frac{12}{15}\psi$ $\varphi^{FA2} = \frac{14}{15}\psi$	$\frac{4}{6}\psi$	$-\frac{4w}{5c}$
3D & **3F**	$\varphi^{DNA} = \frac{1}{9}\psi$ $\varphi^{DA} = \frac{2}{9}\psi$ $\varphi^{FA} = \frac{7}{9}\psi$ $\varphi^{FNA} = \frac{8}{9}\psi$	$\frac{3}{6}\psi$	$-\frac{10w}{9c}$
4D & **2F**	$\varphi^{DA1} = \frac{1}{15}\psi$ $\varphi^{DA2} = \frac{3}{15}\psi$ $\varphi^{FA} = \frac{11}{15}\psi$	$\frac{2}{6}\psi$	$-\frac{8w}{5c}$
5D & **1F**	$\varphi^{F} = \frac{26}{45}\psi$ $\varphi^{DA1} = \frac{7}{45}\psi$ $\varphi^{DA2} = \frac{2}{45}\psi$ $\varphi^{DNA} = \frac{1}{45}\psi$	$\frac{1}{6}\psi$	$-\frac{38w}{15c}$

Outside of this range, shocks trigger entry or exit. In this case the industry's pricing and pass-through parameters are permanently altered. Pass-through turns out to grow larger when appreciations fall below the lower bound. It decreases when depreciations exceed the upper bound. Taking foreign prices as given, this asymmetrical response implies large and persistent positive than negative real exchange rate deviations. On the other hand we argue that hysteresis result can obtain if and only if sunk costs are non-zero.

When accounting for the change in foreign prices, strategic pricing behavior as such is not always sufficient to generate real exchange rate fluctuations. In fact, if foreign and domestic brands have identical market shares in both markets, pass-through is complete and the real exchange rate is invariant with respect to small nominal shocks. More generally, pricing-to-market yields substantial variability of the industry real exchange rate if and only if the domestic brand's foreign market shares fall considerably short of its domestic market shares. This suggests that in industries characterized by substantial inter-industry trade, pricing-to-market alone may not suffice as an explanation of observed real exchange rate behavior.

6 Appendix

Proof of Proposition 1: First, the upper bound on the exchange rate is derived from the objective function of the adjacent foreign brand in equation (3) by inserting the values of all future exchange rates. The foreign adjacent firm opts to leave the market if its expected payoff is negative, i.e. if:

$$\begin{aligned} v^{FA}(e) &= \sum_{s=0}^{t-1} R^s \left(\frac{L}{c} \left(\frac{2}{12} w(1 - e_{t+1+s}) + \frac{c}{4} \right)^2 - F^F \right) \\ &\quad + \sum_{s=t}^{\infty} R^s \left(\frac{L}{c} \left(\frac{c}{4} \right)^2 - F^F \right). \end{aligned} \tag{35}$$

This is equivalent to:

$$\begin{aligned} v^{FA}(e) = \frac{1 - R^\tau}{1 - R} \left(\frac{L}{c} \left(\frac{2}{12} w(1 - e) + \frac{c}{4} \right)^2 - F^F \right) \\ + \left(\frac{L}{c} \left(\frac{c}{4} \right)^2 - F^F \right) \frac{R^\tau}{1 - R} \leq 0. \end{aligned} \tag{36}$$

Solving this equation for the root of e on the positive section of the demand curve yields:

$$\bar{e}_\tau^4 = 1 + \frac{6}{w} \left(\frac{c}{4} - \frac{1}{1 - R^\tau} \sqrt{(1 - R^\tau) \left(\frac{F^F c}{L} - R^\tau \left(\frac{c}{4} \right)^2 \right)} \right), \tag{37}$$

for $\left(\frac{F^F c}{L} - R^\tau \left(\frac{c}{4} \right)^2 \right) \geq 0$. If $\left(\frac{F^F c}{L} - R^\tau \left(\frac{c}{4} \right)^2 \right) < 0$ the foreign brand never quits irrespective of the actual size of the depreciation. On the other hand, if $\left(\frac{F^F c}{L} - R^\tau \left(\frac{c}{4} \right)^2 \right) < 0$, the foreign brand does not quit when facing a $\tau-$period PPP deviation regardless of its actual size. The existence of a real root requires $\left(\frac{F^F c}{L} - R^\tau \left(\frac{c}{4} \right)^2 \right) \geq 0$, $\left(\frac{c}{L} \left(F^F + (1 - R)S \right) - R^\tau \left(\frac{c}{5} \right)^2 \right) \geq 0$ and $\left(\frac{c}{5} \right)^2 \geq \frac{c}{L} \left(F^F + (1 - R)S \right)$(see equation 10). Assume these conditions are satisfied.

Second, in case of an appreciation ($e < 1$) an additional foreign firm decides to enter if its expected payoff, derived by plugging the future exchange rates in equation (4), covers the sunk entry cost. Assuming that the entrant remains in the market after $t + \tau$ ($\left(\frac{c}{5} \right)^2 \geq \frac{c}{L} \left(F^F \right)$) , its entry payoff becomes:

$$\begin{aligned} v^{FNA}(e) = \frac{1 - R^\tau}{1 - R} \left(\frac{L}{c} \left(\frac{1}{19} w(1 - e) + \frac{c}{5} \right)^2 - F^F \right) \\ + \left(\frac{L}{c} \left(\frac{c}{5} \right)^2 - F^F \right) \frac{R^\tau}{1 - R} - S \geq 0. \end{aligned} \tag{38}$$

Again, solving for the root of e on the positive section of the demand curve one obtains the lower bound for the exchange rate:

$$\underline{e}_{\tau}^{4} = 1 + \frac{19}{w}\left(\frac{c}{5} - \frac{1}{1-R^{\tau}}\sqrt{(1-R^{\tau})\left(\frac{c}{L}\left(F^{F} + (1-R)S\right) - R^{\tau}\left(\frac{c}{5}\right)^{2}\right)}\right), \tag{39}$$

for $\left(\frac{c}{L}\left(F^{F} + (1-R)S\right) - R^{\tau}\left(\frac{c}{5}\right)^{2}\right) \geq 0$. If the latter condition is violated, no τ-period appreciation, regardless of its size, triggers entry.

Suppose to the contrary that the entrant is located at one of the adjacent positions. It is easy to verify from the objective function in equation (3) that this gives rise to a new lower bound:

$$\underline{e}_{\tau}^{4} = 1 + \frac{19}{3w}\left(\frac{c}{5} - \frac{1}{1-R^{\tau}}\sqrt{(1-R^{\tau})\left(\begin{array}{c}\frac{c}{L}\left(F^{F} + (1-R)S\right) \\ -R^{\tau}\left(\frac{c}{5}\right)^{2}\end{array}\right)}\right). \tag{40}$$

If the entrant anticipates losses once the exchange rate has returned to its initial value at time $t+\tau$ (i.e. if $\left(\frac{c}{5}\right)^{2} < \frac{c}{L}F^{F}$), he leaves the market at that point in time rather than incur losses. In this case the correct lower bound can be derived by finding the root of the non-negativity condition on the expected entry payoff:

$$v^{FNA}(e) = \frac{1-R^{\tau}}{1-R}\left(\frac{L}{c}\left(\frac{1}{19}w(1-e) + \frac{c}{5}\right)^{2} - F^{F}\right) - S \geq 0, \tag{41}$$

Proof of Corollary 1.1:

Consider the effects of an increase in τ. Taking partial derivatives yields:

$$\begin{aligned}\frac{\partial \overline{e}_{\tau}^{4}}{\partial \tau} &= -\frac{6}{w}R^{\tau}\ln R\left(\frac{\sqrt{(1-R^{\tau})\left(\frac{F^{F}c}{L} - R^{\tau}\left(\frac{c}{4}\right)^{2}\right)}}{(1-R^{\tau})^{2}} - \frac{R^{\tau}\ln R\left(R^{\tau}\left(\frac{c}{4}\right)^{2} - \frac{F^{F}c}{L}\right)}{2\sqrt{(1-R^{\tau})\left(\frac{F^{F}c}{L} - R^{\tau}\left(\frac{c}{4}\right)^{2}\right)}}\right) \\ &= \frac{6}{w}R^{\tau}\ln R\frac{\left(\left(\frac{c}{4}\right)^{2} - \frac{F^{F}c}{L}\right)}{2(1-R^{\tau})^{2}\sqrt{(1-R^{\tau})\left(\frac{F^{F}c}{L} - R^{\tau}\left(\frac{c}{4}\right)^{2}\right)}},\end{aligned} \tag{42}$$

which is negative if the conditions in equation (10) hold and if $R < 1$. Consider the effects of an increase in τ on the lower bound. Taking partial derivatives yields:

$$\begin{aligned}\frac{\partial \underline{e}_{\tau}^{4}}{\partial \tau} &= -\frac{19}{w}R^{\tau}\ln R\left(\frac{\sqrt{(1-R^{\tau})\left(\frac{F^{F}c}{L} - R^{\tau}\left(\frac{c}{5}\right)^{2}\right)}}{(1-R^{\tau})^{2}} - \frac{R^{\tau}\ln R\left(R^{\tau}\left(\frac{c}{5}\right)^{2} - \frac{F^{F}c}{L}\right)}{2\sqrt{(1-R^{\tau})\left(\frac{F^{F}c}{L} - R^{\tau}\left(\frac{c}{5}\right)^{2}\right)}}\right), \\ &= \frac{19}{w}R^{\tau}\ln R\frac{\left(\left(\frac{c}{5}\right)^{2} - \frac{\left(F^{F} + S(1-R)\right)c}{L}\right)}{2(1-R^{\tau})^{2}\sqrt{(1-R^{\tau})\left(\frac{F^{F}c}{L} - R^{\tau}\left(\frac{c}{5}\right)^{2}\right)}}.\end{aligned} \tag{43}$$

which is positive if the conditions in equation (10) hold and if $R < 1$.

Next, apply the envelope theorem to derive the effect of an increase in c on the adjacent brand's profits:

$$\frac{\partial v^{FA}(e)}{\partial c} = \frac{1-R^{\tau}}{1-R} - \frac{L}{c^2}\left(\frac{2}{12}w(1-e)\right)\left(p^{FA} - we\right) > 0 \ for \ e < 1. \tag{44}$$

As before, apply the envelope theorem to derive the effect of an increase in c on the non-adjacent brand's expected payoff:

$$\frac{\partial v^{FNA}(e)}{\partial c} = \frac{1-R^{\tau}}{1-R} - \frac{L}{c^2}\left(\frac{1}{19}w(1-e)\right)\left(p^{FNA} - we\right) < 0 \ for \ e > 1. \tag{45}$$

To see the effect of an increase in F verify only that the terms within the square root of equations (13) and (14) are non-negative.

Proof of Corollary 1.4: By comparing $\Delta\overline{e}_{\tau}^{5}$ and $\Delta\underline{e}_{\tau}^{4}$ derive the condition on S :

$$\frac{S(1-R)}{F} = \frac{4}{9}\frac{L}{c}\left(\frac{c}{5}\right)^2(1-R^{\tau}) + \frac{8}{9} - \frac{L}{9F}\sqrt{(1-R^{\tau})\left(\begin{array}{c}\frac{c}{L}\left(F^F\right)\\ -R^{\tau}\left(\frac{c}{5}\right)^2\end{array}\right)} \tag{46}$$

Proof of Corollary 1.5: Note that the total volume of imports is:

$$\frac{L}{c}\left(\frac{3c}{4} + \frac{5}{12}(1-e)w\right). \tag{47}$$

Calculate the elasticity for $e = 1$.

Proof of Proposition 2.a: Use the objective function of the non-adjacent brand in equation (3) to derive this brand's expected payoff.

$$v^{FNA}(e) = \frac{1-R^{\tau}}{1-R}\left(\frac{L}{c}\left(\frac{1}{12}w(1-e) + \frac{c}{4}\right)^2 - F^F\right) + \left(\frac{L}{c}\left(\frac{c}{4}\right)^2 - F^F\right)\frac{R^{\tau}}{1-R}. \tag{48}$$

If this is strictly positive, the non-adjacent brand's dominant strategy obviously consists in staying. This can be solved to obtain:

$$\Delta\overline{e}_{\tau}^{4,NA} = \frac{12}{w}\left(\frac{c}{4} - \frac{1}{1-R^{\tau}}\sqrt{(1-R^{\tau})\left(\frac{F^F c}{L} - R^{\tau}\left(\frac{c}{4}\right)^2\right)}\right), \tag{49}$$

Given that the non-adjacent brand stays in the first stage, at least one of the adjacent foreign brands leaves in the first stage of both of the pure strategy SPNE.

Proof of Proposition 2.b: In the 3-brand market the foreign brand's expected payoff is:

$$v^F(e) = \frac{1-R^\tau}{1-R}\left(\frac{L}{c}\left(\frac{1}{5}w(1-e)+\frac{c}{3}\right)^2 - F^F\right) + \left(\frac{L}{c}\left(\frac{c}{3}\right)^2 - F^F\right)\frac{R^\tau}{1-R}. \tag{50}$$

Solve for the root of this equation to obtain:

$$\Delta\bar{e}^3_\tau = \frac{5}{w}\left(\frac{c}{3} - \tfrac{1}{1-R^\tau}\sqrt{(1-R^\tau)\left(\tfrac{F^F c}{L} - R^\tau\left(\tfrac{c}{3}\right)^2\right)}\right), \tag{51}$$

If this payoff is strictly positive, one of the foreign adjacent brands stays given that the other brand leaves.

Proof of Proposition 2.c: In the 3-brand market the foreign brand's expected payoff is:

$$v^F(e) = \frac{1-R^\tau}{1-R}\left(\frac{L}{c}\left(\frac{1}{3}w(1-e)+\frac{c}{2}\right)^2 - F^F\right) + \left(\frac{L}{c}\left(\frac{c}{2}\right)^2 - F^F\right)\frac{R^\tau}{1-R}. \tag{52}$$

Solve for the root of this equation to obtain:

$$\Delta\bar{e}^2_\tau = \frac{3}{w}\left(\frac{c}{2} - \tfrac{1}{1-R^\tau}\sqrt{(1-R^\tau)\left(\tfrac{F^F c}{L} - R^\tau\left(\tfrac{c}{2}\right)^2\right)}\right). \tag{53}$$

If this payoff is strictly positive, one of the foreign adjacent brands stays given that the other brand leaves.

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