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The Pricing Puzzle: The Default Term Structure of Collateralised Loan Obligations

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Abstract:

Ambivalence in the regulatory definition of capital adequacy for credit risk has recently steered the financial services industry to collateral loan obligations (CLOs) as an important balance sheet management tool. CLOs represent a specialised form of Asset-Backed Securitisation (ABS), with investors acquiring a structured claim on the interest proceeds generated from a portfolio of bank loans in the form of tranches with different seniority. By way of modelling Merton-type risk-neutral asset returns of contingent claims on a multi-asset portfolio of corporate loans in a CLO transaction, we analyse the optimal design of loan securitisation from the perspective of credit risk in potential collateral default. We propose a pricing model that draws on a careful simulation of expected loan loss based on parametric bootstrapping through extreme value theory (EVT). The analysis illustrates the dichotomous effect of loss cascading, as the most junior tranche of CLO transactions exhibits a distinctly different default tolerance compared to the remaining tranches. By solving the puzzling question of properly pricing the risk premium for expected credit loss, we explain the rationale of first loss retention as credit risk cover on the basis of our simulation results for pricing purposes under the impact of asymmetric information.

JEL Classification: C15, C22, D82, F34, G13, G18, G20

Keywords: Loan securitisation, CLO, structured finance

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1 INTRODUCTION

Over the recent past loan securitisation has proven its value as an efficient funding and capital management mechanism for financial institutions. As a structure of credit risk transfer (and loan sale) it could lead to a mitigation of required minimum capital when regulatory levels are proven to be overly conservative. By subjecting bank assets to market scrutiny loan securitisation facilitates prudent risk management and diversification as an effective method of redistributing credit risks to investors and broader capital markets through issued debt securities (MORRIS AND SHIN, 2001). Thus, loan securitisation is an expedient asset funding option in comparison with other means available to banks, since issuers are able to achieve a more precise matching of the duration of their managed assets and liabilities. However, the flexibility of issuers and sponsors to devise a particular transaction structure, i.e. choosing from a vast variety of methods to subdivide and redirect cash flows from an illiquid underlying reference portfolio, also bears the risk of severe misalignments of information between issuers and investors as to the term structure of default probability and loss severity of loan default.[1]

Ambivalence in the regulatory definition of capital adequacy for credit risk has recently stirred the financial services industry to advanced activism in loan securitisation. During the last years the securitisation of corporate and sovereign loans has attracted an increased following from both banks and financial service companies in order to obtain capital relief and gain liquidity or to exploit regulatory capital arbitrage opportunities. In the wake of increasing popularity of securitisation for reasons mainly to be found in regulatory and economic arbitrage, collateral loan obligations (CLOs) have evolved into an important balance sheet management tool. CLOs represent a specialised form of Asset-Backed Securitisation (ABS), with investors acquiring a structured claim on the interest proceeds generated from a portfolio of bank loans in return for a certain degree of collateral default they are willing to accept. Both interest spread and default losses are prioritised on the basis of contractual repartitioning according to investor seniority. Due to the inherently diverse and intransparent nature of bank loan portfolios the presence of asymmetric information imposes some sort of “lemon’s problem” á

[1] SKORA (1998) defines credit risk as the risk of loss on a financial or non-financial contract due to the counterparty’s failure to perform on that contract. Credit risk breaks down into default risk and recovery risk. Whereas default risk denotes the possibility that a counterparty will fail to meet its obligation, recovery risk is the possibility that the recovery value of the defaulted contract may be less than its promised

la AKERLOF (1970) on the economic workings of this securitisation process. Despite efficiency losses due to adverse selection and moral hazard, issuers of CLO transactions achieve gains from bundling assets and then further tranching them before they are sold into the capital markets. The dissemination of losses on theory of an agreed prioritisation warrants particular attention in determining the state-contingent pay-offs of investors in such a case of asymmetric information, since broad risk categories of structured transactions depend on the quality distribution of the collateral pool.

The varying willingness of sponsors and issuers to securitise only part of selected loan portfolios has drawn increased attention to credit enhancement as a frequently observed feature of CLO securitisation. Banks commonly retain an equity claim on a portion of the collateral portfolio to provide capital cover for first losses by buying back the most junior securities issued by the conduit. Alternatively, such credit enhancement could also take the form of a standby letter of credit to the conduit, or by the sponsoring bank. In return for providing such credit enhancement, as well as the loan origination and servicing functions, the sponsoring bank lays claim to the residual spread between the yields on the underlying loans and the interest and non-interest cost of the conduit, net of any losses on pool assets covered by the credit enhancement. Capital market investors hold the remaining tranches of the securitisation transaction. This typical structure of a CLO transaction is subjected to a prioritisation of both default losses and interest income generated from the underlying collateral portfolio, which indicates the investment risk investors are willing to tolerate in the light of adverse asymmetric information about asset quality of the collateral. Credit enhancement represents an attempt of stymieing adverse selection to ensure incentive compatibility of issuers with investor expectations in a CLO transaction.[2] However, the concentration of all first losses in an equity tranche retained by the sponsor does result in a default rate different from the investors' default tolerance exhibited in the expected spreads on more senior mezzanine tranches. The security design of CLOs has it, the extent to which pricing of CLO transactions and the willingness of the issuer/sponsor to resort to asset retention mirrors the economic rationale of loan securitisation under information asymmetry. The modelling of the peculiar characteristic of default risk emanating from underlying loan collateral is critical in loan securitisation in the light of the high-yield structured finance market

[2] See also CALVO (1998) for a detailed discussion of the "lemons problem" in the context of financial contagion.

having witnesses distressed performance and inevitable adverse follow-on effects on the performing of loans in the underlying asset collateral over the recent past.

The manifestation of persistent downward rating drift have sparked interest in a diligent surveillance of asset performance. Especially the term structure of defaults in the underlying asset portfolio of securitisations has inadvertently drawn attention to the issue of credit enhancement whose assessment of covering first loss severity is coupled with the ability of the issuer to avert unexpected levels of substandard asset performance. Banks have identified particularly CLOs in curbing fears of rising levels of loan delinquencies jeopardising the rating levels of such transactions. In keeping with prudent collateral surveillance, this would eventuate a more careful contemplation of defaults (i.e. delinquencies and termination rates) of corporate loans as growing numbers of banks seriously considers securitisation as an expedient means of balance sheet restructuring through active credit portfolio management to the detriment of more agile interest-based bank business.

1.1 Objective

The following paper purports to a comprehensive examination of default risk and its ramifications on the security design of CLOs under symmetric information only, as the degree of transparency about the collateral pool quality does not have any bearing on the workings of the securitisation structure once it has been put in place. The paper aims at modelling the valuation of senioritised contingent claims on multi-asset reference portfolios underlying collateralised loan obligations (CLOs).

1.2 Method

In the course of proper asset pricing of structured finance transaction with a defined credit event, such as loan securitisation, extreme events enter very naturally, and as such, understanding the mechanism of loss allocation and the security design provisions governing them becomes essential. In absence of historical credit default data, we propose a Merton-based (1974) credit risk pricing model that draws on a careful simulation of expected loan loss based on parametric bootstrapping through extreme value theory (EVT). Given the number of stochastic variables and the complexity of the relationships no closed form solution for calculating the needed risk measures is available. Thus, the analysis is undertaken with a Monte

Carlo simulation model. In this way we simulate and measure credit risk, i.e. collateral default losses, of a loan portfolio for the issuing process of securitisation transactions under symmetric information.

The loss distribution function stems from portfolio-level statistics derived from deterministic assumptions about asset-specific properties as to the contribution of expected/unexpected credit loss by individual loans (see Figure 1 below).

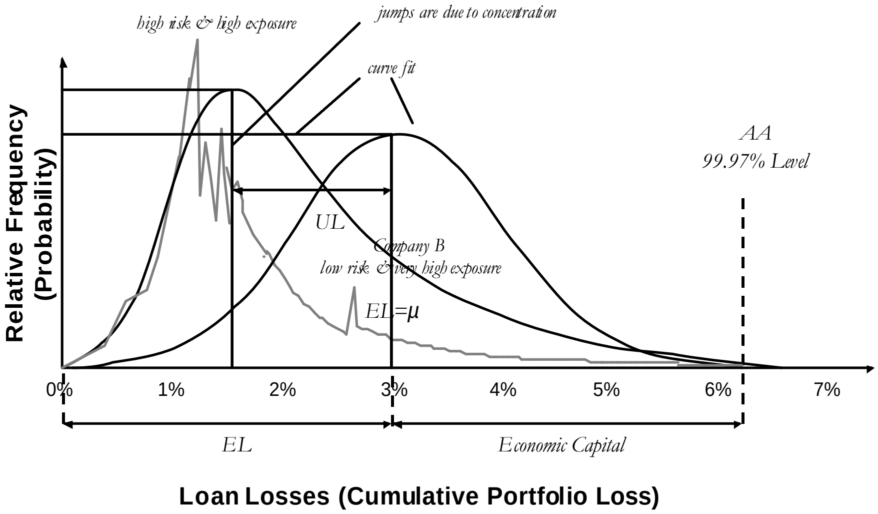

Figure 1. *Cumulative distribution function of credit portfolio losses.*

The allocation of periodic losses in proportion to the various constituent tranches, their evolution over time and their bearing on the pricing of the CLO tranches for risk-neutral investors and in comparison to zero-coupon bonds will be the focus of this examination. Hence, the suggested model serves as a blueprint for the adequate valuation of CLO transactions, if we render the value of contingent claims dependent on the totality of expected periodic credit loss of a multi-asset portfolio allocated to the tranches issued. The resulting tranche spreads are indicative of the default pattern of CLOs, which causes excess investor return expectations in view of information asymmetries. By assuming a uniform portfolio and a corresponding distributional approximation of stochastic loan losses (without taking into consideration prepayments and early amortisation triggers most common to CLOs), the analysis illustrates the dichotomous effect of loss cascading as the most junior tranche of CLO transactions exhibits a distinctly different default tolerance compared to the remaining tranches. Finally, we explain the rationale of first loss retention as credit risk cover on the basis of our simulation results for pricing purposes. As an extension to this paper, we propose to test the viability of the model by comparing simulated credit risk to historical probabilities of portfolio credit default, aside from the presented comparison of simulated risk-neutral returns on tranches and analytical bond prices.[3]

1.3 Scope of Research

We synthesise three major areas of research. Both professional and academic accounts appear with respect to (i) pricing credit risky instruments, credit risk measurement of portfolios, credit risk management (see CAOUETTE, ALTMANN AND NARAYANAN (1998) for an overview), (ii) contract theory, security design and asset liquidity (CLEMENZ, 1987; BOLTON AND SCHARFSTEIN, 1990; RAJAN, 1992; DEMARZO AND DUFFIE, 1997; HOLMSTRÖM AND TIROLE, 1998; BHASIN AND CAREY, 1999; PARK, 2000; WOLFE, 2000), (iii) financial intermediation and underwriting (DIAMOND, 1991; GORTON AND PENNACCHI, 1995; GANDE, PURI, AND SAUNDERS, 1999; DIAMOND AND RAJAN, 2000; and (iv) market structure and competition (KYLE, 1985; GLOSTE AND MILGROM, 1985; OLDFIELD, 2000) and (v) and loan securitisation (RIDDIOUGH, 1997; DUFFIE AND GÂRLEANU, 2001 and 1999; DUFFIE AND SINGLETON, 1998 and 1999). We follow the current literature in the combining credit risk and loan securitisation at portfolio level, as we model the asset returns of senioritised contingent claims on a multi-asset reference pool of loans in a CLO transaction. In order to value such claims we develop a variety of portfolio risk models to calculate the probability distribution potential credit losses for the guarantor's reference portfolio over a certain period of time. We consider systematic risk impacting on aggregate (uniform) portfolio default at a constant between-asset correlation, with individual risk being diversified in a pool of a sufficiently large number of independent risks. The parameters of simulated periodic default losses of the reference portfolio are translated into an economically meaningful concept, as we estimate the periodic loss severity for the CLO reference portfolio, which has a bearing on the pricing of the CLO tranches. We calculate the risk-free returns of CLO securities for both fixed and variable interest rates, required by investors to be sufficiently compensated for expected credit risk of the reference portfolio under the impression of a particular security design, i.e. the allocation

[3] See also BARNHILL AND MAXWELL (2002).

of default losses and their term structure. Adverse selection and moral hazard certainly are cardinal parameters in this regard.

With respect to the security design of CLOs three important factors enter this paper as important considerations. First, what are the loss sharing effects of the extent to which banks are willing to bear part of the losses stemming from the non-performance of the reference portfolio in expedient of partial retention of a part of the portfolio? As issuers pass on remaining asset claims to capital market investors, any credit enhancement establishes a particular degree of "collateralisation" of the securitisation transaction with corresponding changes to the expectation of possible losses to be borne by outstanding debt securities issued on the loan portfolio. Second, the assumed structure of these outstanding debt securities has to be in compliance with the current security design in the securitisation market of corporate loans, i.e. the model assumptions about the specificities of the CLO tranches match the empirical observations of standard ranking of asset claims in CLO transactions. Third, the market value of the securitised collateral portfolio is subjected to gradual erosion through loan default (excluding effects of amortisation through prepayment). Default loss is accounted for only at the end of a single period t, such that the reference portfolio is assumed to be 1 at time t.

The objective of illustrating the default term structure and pricing of CLO tranches based on a loss cascading model breaks down in four distinct sections. First, we project the estimated default expectations of holders of tranches in the securitisation transaction by simulating the expected and unexpected losses on a loan portfolio with standard assumptions as to the portfolio risk parameters. Since we are at a loss of sufficient historic data about loan defaults extreme value theory will be employed to determine proxies of default loss on a loan portfolio with a constant, increasing and decreasing forward rate of default probability. Subsequently, we derive approximate credit ratings for the CLO tranches by means of benchmarking the term structure of default rates of expected losses with expedient zero-coupon bonds as upper und lower bounds, before the risk-neutral spreads on the various tranches of a CLO transaction are calculated from cumulative default loss per tranche. Finally, the obtained results are contemplated in a post-simulation assessment of the incentive of both issuers/sponsors of the transaction and investors to acquire certain tranches on the theory of remedying information asymmetries. These findings have important implications for portfolio models to explain the implications of credit risk on structured finance.

2 LITERATURE REVIEW

Our approach follows the present credit risk literature in the valuation of contingent claims and the analysis of loan default as to the issuer's ability and incentive to meet payment obligations in terms of credit risk cover for expected losses. This area of asset pricing sits well with prominent structural models of credit risk (MERTON, 1974; BLACK AND COX, 1976; BRENNAN AND SCHWARZ, 1978; LELAND, 1994 and 1998). We also resort to WALL AND FUNG (1987), IBEN AND BROTHERON-RATCLIFFE (1994) and DUFFEE (1996), who discuss credit risk as it applies to portfolio risk management. We offer an original contribution to this field of research as we focus on extending traditional models of valuing multiple underlying assets in the context of security design of loan securitisation and the impact of maturity on asset valuation (LELAND AND TOFT, 1996). Despite the abundance of structural models, pricing methodologies of the CLO mechanism are still found wanting (FIDLER AND BOLAND, 2002), leave alone multi-asset approaches to reliably represent the cost and risk associated with the partial asset retention as credit enhancement as a vital requisite of CLO transactions. One of the main reasons for a lack of appropriate asset pricing techniques is that the valuation of multi-asset contingent claims defies a closed-form solution. We address this issue by modifying the multi-asset process of credit losses of loan reference portfolios by means of extreme value theory (EVT) simulation in order to bootstrap parameters estimates for the determination of credit loss as a basis for the valuation of claims.

The reason for EVT as a methodology is straightforward. In the course of proper asset pricing of structured finance transaction with a defined credit event, such as loan securitisation, extreme events enter very naturally, and as such, understanding the mechanism of loss allocation and the security design provisions governing them becomes essential. In absence of historical credit default data, the set-up of this CLO pricing model makes an explicit analytic formulae in the spirit of a closed form solution to give way to careful simulation, i.e. a sufficiently iterative process of deterministic credit default in order to generate extreme events of portfolio distress (ASMUSSEN, 1999). In advent of imminent regulatory change (BASEL

COMMITTEE, 1999a) this aspect should prove to be particularly instrumental in resolving the puzzling pricing mechanism underpinning CLO spreads. We believe that our novel approach of asset-pricing CLO tranches is not limited to robust valuation of CLOs. It also complements a variety of valuation models, based on multiple underlying uncertainties stemming from time-varying contractual contingencies of asset origination and financial intermediation.

One major strand of research explores the interdependence of the asset structure of the firm and the organisational form that supports the separation between the firm's assets as a crucial underpinning of securitisation. GREENBAUM AND THAKOR (1987) analysed the adverse selection effects of asset structure of financial institutions. According to their study private information held about the quality of originated assets would induce financial institutions to prefer the securitisation of better quality assets, whilst worse quality assets are retained on the books and funded by deposits. Generally such selective bias would lead financial institutions to champion securitisation transaction with high-risk reference portfolios in spite of increased bankruptcy risk. Hence, GREENBAUM AND THAKOR (1987) demonstrate not only how the between perceived quality of the asset structure comes to matter, but also assess the extent to which certain credit risk management techniques, such as asset securitisation could prove to be a suitable for transforming asset structures.

Moreover, the complexity of securitisation transactions has led researches beyond the confines of scrutinising incentive problems in the association with the asset structure of financial institutions. There exists distinct set of literature that deals with the valuation of asset-backed securities. The common framework of existing models is aligned with the following assumptions. For one, a pool of assets is compiled, which underlies a senioritised collection of different classes of asset-backed securities. Upon simulation of individual asset performance, expected cash flows are aggregated, supporting the coupon and principal payments of the issued obligations. In most cases financial securities generating regular income or other financial commitments (e.g. loans, credit allowances, etc.) sustain these proceeds from the reference portfolio, which serves as collateral for the asset-backed securities. Ultimately, the pricing of the various classes (so-called "tranches") of asset-backed securities is contingent upon prepayments and expected asset default of the underlying collateral portfolio, which impair the allocation of earmarked asset returns. The following studies are representative of the current state of research in the asset pricing asset-backed securities.

CHILDS, OTT AND RIDDIOUGH (1996) employ a structural model for pricing commercial mortgage-backed securities (CMBS) through Monte Carlo simulation based on the correlation structure of individual mortgages in the reference pool. They aggregate the value of each mortgage in order to determine the available amount of asset proceeds supporting each class of CMBS securities. Their proposed valuation methodology courts an optimal design of asset-backed securities and forms an integral part of the established security design literature. RIDDIOUGH (1997) also theoretically solidifies this claim by modelling asymmetric asset value information and non-verifiability of liquidation motives in the context for purposes of optimal security design for asset-backed securities. In the bid for justifying issuers' benefits associated with splitting off a completely risk-free security from the proceeds of the reference portfolio, he plausibly proves the compelling rationale of the so-called first loss provision as credit risk protection (credit enhancement). By retaining the most junior claim on cash flows from the reference portfolio, issuers internalise some or the entire agency cost of asymmetric information, the most critical impediment to fair asset pricing of securitisation transactions. RIDDIOUGH (1997) argues in this context that a junior claim seizes a vital function in the security governance of asset-backed securities due to better asset value information and its role as first loss position. Empirical implications of junior security holders controlling the debt negotiation process with pooled debt structures suggest the validity of this bid for an efficient design of asset-backed securities.

DUFFIE AND GÂRLEANU (2001) follow this common thread of research in their reduced-form credit risk model of asset pricing collateralised debt obligations (CDOs). In line with the diversity score approach devised by rating agencies they calculate the default intensity processes for single debt obligations, which, by definition, are limited to either bonds or loans. In extension to this estimation of individual asset exposure, they obtain the aggregate default intensity of the entire reference portfolio in a similar analytic form. Consequently, the disposable proceeds from the reference portfolio enter an efficient pricing simulation of different security classes (tranches) of a CDO transaction under different priority schemes.

In contrast to DUFFIE AND GÂRLEANU (2001) our study focuses on the default intensities of the *entire* reference portfolio in order to derive efficient pricing of collateralised loan obligations (CLOs), albeit we consider individual asset correlation of loans underlying the translation. In quantifying the default probability and loss severity associated with the cash flow stream from the reference pool for a constant interest rate, we draw on JARROW, AND TURNBULL (1995), who take as given the term structure of credit spreads, i.e. a sufficient number of corporate bond prices, and derive an arbitrage free pseudo-probability of default, as well as JARROW, LANDO AND TURNBULL (1997), who introduce a Markov model for the term structure of credit risk spreads, where rating agencies' default rates and bond prices serve as input, so that investors' risk premium can explicitly estimated for static and variable risk-free interest rates. Additionally, we rely on the breadth of other research in this so-called "yield spread approach" by LITTERMAN AND IBEN (1991), DAS AND TUFANO (1996), ARVANTIS, GREGORY AND LAUREN (1999), ARTZNER AND DELBAEN (1994), NIELSEN AND RONN (1996), and DUFFEE (1996).

We extend the pricing of CLO tranches pursuant to similar considerations suggested by DAS AND TUFANO (1996), who price credit-sensitive debt on the basis of stochastic interest rates, credit ratings and credit spreads, as well as RAMASWAMY AND SUNDARESAN (1986). Hence, we extend JARROW AND TURNBULL (1995) in the spirit of MADAN AND UNAL (1998), who augment the JARROW AND TURNBULL approach by modelling equity depending probability of default and non-constant recovery rate at default. The so-called "adjusted short rate approach" of credit spread modelling proffered by DUFFIE AND SINGLETON (1999), BIELECKI AND RUTKOWSKI (2000), PUGACHEVSKY (1999), BALLAND AND HUGHSTON (2000) comes into play, when we benchmark estimated tranche spreads with defaultable bond yields.

As a more precise formulation for the credit risk involved in the reference pool is sure to produce an improved and more reliable asset pricing technique for asset-backed securities in general and CLOs in particular, our model primarily focuses on the valuation of CLOs. It also gives testimony of a plausible rationale as to why financial institutions securitise assets by providing credit enhancement in the form of first loss coverage. None of the existing models has been able to explicate this issue by means of an inductive approach on the basis of simulated default intensity processes.

Many models assume asymmetric information akin to the "lemons problem" presented by AKERLOF (1970) and explain the gains to be realised by bundling assets and re-packaging them in different tranches prior to issuance in the form of structured finance securities (DEMARZO AND DUFFIE, 1997; RIDDIOUGH, 1997). Generally, the separation of loans from capital requirements levied on the loan book institutions is achieved by means of actual or synthetic asset transfer to non-recourse single-asset entities. Such securitisation conduits, so-called special purpose vehicles (SPVs), are best understood as trust-like structures,[4] which issues two classes of securities upon asset transfer from the sponsor of the securitisation transaction, i.e. the loan-originating financial institutions. Whilst the holders of debt-like notes establish prior claim to the underlying reference portfolio of loans in order of agreed seniority, the issuer retains a residual equity-like class as first loss position. The ratio of debt-like notes and residual equity is commonly referred to as “leverage”, whose variation allows the issuing securitisation conduit to maximise its value for a given set of parameters. Depending on perceived average loan quality in the portfolio, the residual cash flow rights of equity holders act as limited liability for trust claimholders against costly bankruptcy (FROST, 1997) as much as they guarantee senior claimholders a remote probability of suffering losses on their investment. Consequently, the restricted role of equity due to the absence of control rights does, however, upholds bankruptcy protection of senior claimholders, as it serves as an early amortisation trigger for inexpensive prepayment of liquidation value of a deteriorating reference portfolio.

RIDDIOUGH (1997) explains that subordinated security design dominates whole loan sale, since cash flow splitting allows issuers to internalise some or all of the adverse selection risk, which would otherwise apply if they were to consider alternative options of “liquidating” a given pool of assets. Security design meets this objective, however, only if subordination levels increase relative to full information levels in order to limit agency cost for uninformed outside investors. Hence, the combination of early amortisation triggers and credit risk coverage, contractually anchored in CLO security design, also help to limit a likely predisposition of loan securitisation to mispricing due to private information. Packaging strategies, such as pool diversification and loan bundling, add to this rationale, as they soften the asymmetric information dilemma the subordination effect is meant to guard issuers against, and thus, increase “liquidation proceeds” of the reference portfolio (RIDDIOUGH, 1997).

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[4] We assume the securitisation vehicle is registered under the statues governing corporations, and, therefore, pays taxes. However, these taxes are offset by tax credits of debt. Since we do not intend to unveil specific tax

While the appropriation of economic rents from information advantage about asset quality does admittedly come into play as an implicit incentive for the tranching of CLOs in association with the level of credit enhancement, our model concentrates on the optimal pricing of CLOs. Asset pricing loan securitisation could be approached either from the perspective of cash flows generated from the reference portfolio or the expected losses from creditor default. Most models in the literature concentrate on the upside of loan securitisation, i.e. the amount of distributable interest and principal proceeds to be had from the loan pool. We use a slightly different structure in our model.

We analyse the optimal design of loan securitisation transactions from the perspective of credit risk and frictions related to potential default by modelling the loss side through extreme value simulation. By extending concepts and meanings of accepted principles of asset pricing under symmetric information to an asymmetric information context in the risk-neutral valuation of CLOs, expected losses translate into investment risk premium, which entails a certain term structure of credit spreads for the various tranches of a securitisation transaction. In that way, the risk premium commanded by issuers could be compared to empirically observed spreads in loan securitisation, such that CLOs could be accurately priced for the issuing process in a symmetric information setting. Consequently, the suggested model is also a viable alternative asset pricing technique for portfolio credit risk, since its simulation framework rules out stochastic shortcomings frequently encountered in loss distributions of future credit loss on the basis of observed rating transition.

Although we suggest a general approach of pricing contingent claims in the area of portfolio credit risk management, we incorporate considerations, which have emerged in discussions of credit risk modelling in COSSIN (1997), MADAN (1998), MADAM AND UNAL (1994), and DAS (1998). For further information in context of gauging the impact of credit risk on structured finance instruments, we refer readers to HULL AND WHITE (1995) and COOPER AND MARTIN (1995), who make several important observations about credit risk and how it affects the price

advantages of loan securitisation (SULLIVAN, 1998), we consider the tax expense to have the same structure as in the case of the originating company.

of over-the-counter derivatives. Moreover, our analysis makes liberal use of bibliography of key texts presented at the end of this paper.[5]

3 LOSS DISTRIBUTION OF A UNIFORM COLLATERAL PORTFOLIO

Past attempts to simulate credit risk of standard bank loan portfolios has been largely based on the notion that the probability of default of a uniform portfolio is consistent with a normal inverse distribution as the number of loans grows to infinity. According to VASICEK (1987), FINGER (1999) and OVERBECK AND WAGNER (2001) the normal inverse distribution $NID\left(p,\rho\right)$ with default probability $p>0$ as mean and equal pairwise asset correlation $\rho<1$ for a portfolio of h loans with equal exposure $1/h$ for $h\rightarrow+\infty$, the cumulative distribution function with p

$$NID(x,p,\rho)=N\left(\frac{1}{\sqrt{p}}\left(\sqrt{1-\rho}\,N^{-1}(x)-N^{-1}(p)\right)\right) \tag{0.1}$$

denotes the distribution of portfolio losses $0\leq x\leq 1$ by drawing on the assumption on normally distributed asset returns. Its density is represented by

$$f(x,p,\rho)=\frac{1-\rho}{\rho}\times\frac{1}{n\left(N^{-1}(x)\right)}\times n\left(\frac{1}{\sqrt{\rho}}\left(\sqrt{1-\rho}\,N^{-1}(x)-N^{-1}(p)\right)\right), \tag{0.2}$$

with standard deviation of the standard normal distribution function N derived from the bivariate normal distribution $N_2(x,y;\rho)$ with a zero expectation vector,[6] such that

$$\sigma=\sqrt{N_2\left(N^{-1}(p),N^{-1}(p);\rho\right)-p^2}\ . \tag{0.3}$$

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[5] With respect to credit risk hedging, readers might find it worthwhile to consider SORENSEN AND BOLLIER (1994) for a practical explanation of pricing the credit risk in an over-the-counter swap.

[6] The bivariate normal distribution has a symmetric covariance matrix displaying the correlation factor ρ off and covariances on the diagonal.

4 TIME SLICING

Assuming a discrete time grid $t_0 < t_1 < t_j < ... < t_{n-1} < t_n$ to be applied in modelling the development of portfolio losses, which are only accounted for at the end of each time period *j*, the accumulation of absolute estimated losses $\tilde{L}$ over the time horizon *n* can be quantified as

$$\tilde{Y}_n = \sum_{j=1}^{n} \prod_{i=0}^{j-1} (1 - X_i) X_j \equiv \tilde{L} \tag{0.4}$$

with $X_j \sim NID(x, p_j, \rho_j)$ for $j = 1,...,n$, where X_j denotes the relative portfolio loss (in relation to the remaining exposure) at time period *j* on the basis of the normal inverse distribution defined by the portfolio parameters p_j, ρ_j indexed to the time increment $j=1,\ldots, n$, given uniformity of the collateral portfolio and the residues after losses. The estimates losses according to extreme value theory are treated analogously. The collateral balance is assumed to be 1 at time j=0. Once the absolute losses are determined based on the random draft of uniform losses per period as the interim result, aggregating estimated periodic portfolio valuations translates into the generation of absolute losses. This approach is pursuant to the determination of the conditional default rate ("CDR") used by commercial banks in the calculation of loss scenarios on reference portfolios. Periodic default loss is derived by means of combining a constant default rate with projected loan claims loss severity assumptions, i.e. a loss severity percentage that is incurred with respect to aggregate outstanding principal balance of the loan claims at the time of default is multiplied with a certain probability of default (which is reflected by a certain credit risk loss function in this analysis).

Even though the respective density function $f(x, p, \rho)$ could be calculated by product folding, OVERBECK AND WAGNER (2001) state that a closed form display of the results does not seem to be possible and warrants numerical computation. Thus, in absence of reliable historical data pertinent on credit losses, the random variables X_j are generated on the basis of Monte Carlo simulation of a normal inverse distribution of credit losses according to the equation 0.1 above. This approach requires the computation of uniformly distributed random variables $Z \sim U(0, 1)$ and their subsequent transformation to periodic losses for each time step *j* with

$$x = NID^{-1}(z, p, \rho) = N\left(\frac{1}{\sqrt{1-p_j}}\left(N^{-1}(p_j) - \sqrt{\rho_j}N^{-1}(z)\right)\right) \qquad (0.5)$$

by choosing the parameters of the loss function such that the first two moments match the ones obtained from the normal inverse distribution.

Since the occurrence of extreme events takes a pivotal role in risk management considerations as to the accurate approximation of the credit portfolio losses, we need to extend this approach to take account of the extreme tail behaviour of credit events. Hence, the same methodology of generating random variables X_j in equation 0.5 is analogously applied to a Pareto-like distribution of credit losses, which reads in its general form as follows

$$G_{\xi,\beta}(x) = 1 - \left(1 + \frac{\xi x}{\beta}\right)^{-\xi^{-1}} \quad \text{for } \xi \neq 0 \text{ and } x \geq 0. \qquad (0.6)$$

This distribution of collateral losses according to an extreme value distribution as a transformed version of equation 0.6 will be presented as an improvement to the normal inverse distribution of credit losses based on the transformation of uniformly distributed random variables.

5 LOSS CASCADING

The prioritisation of asset claims in the structure of collateralised loan obligations results in portfolio losses $\tilde{L}$ being allocated successively to the constituent m tranches according to their level of seniority, i.e. all investors in tranche k have to bear the aggregate losses up to α_k% of the outstanding notational value of the transaction, investors in the more senior tranche $k+1$ hold for the remaining losses but smaller than α_{k+1}%. Thus, if one tranche k has been fully exhausted in terms of estimated losses as reflected in default tolerance of the structured rating further losses in excess of the scheduled amount for the respective tranche (denoted by the interval $\alpha_k - \alpha_{k-1}$ of default losses the tranche k covers) are allocated to the subsequent, more senior tranche. This bottom-up cascading process perpetuates until all losses are allotted to the relevant tranches in the order of the scheduled default loss for each tranche, i.e. the proportional share $a_k - a_{k-1}$ in total periodic loss on the collateral portfolio, with $0 \leq a_0 < a_1 < ... a_{m-1} < a_m$ set as the boundaries of the respective tranches. These boundaries are time-invariant and lack the notation *j* for time period. The mathematical notation for this allocative routine[7] applies to default losses in the following form

$$L_j^k = \left(x_j - a^{k-1}\right)^+ \wedge \left(a^k - a^{k-1}\right), \tag{0.7}$$

where L_j represents the periodic default loss in time step *j* as the proportional default loss of the collateral portfolio is borne by tranche *k*. The determination of the expected credit loss per tranche in time period *j* and on aggregate over the entire maturity *n*,

$$\tilde{L}^k = \sum_{j=1}^{n} \tilde{L}_j^k = \sum_{j=1}^{n} \int \frac{\left(x_j - a^{k-1}\right)^+ \wedge \left(a^k - a^{k-1}\right)}{a^k - a^{k-1}} df(x_j) \tag{0.8}$$

yields a probability measures $\tilde{L}_j^k$ and $\tilde{L}^k$ of the loss distribution *f(x_j)* and *f(x)* over one time period *j* or on an aggregate basis respectively. The issuing entity tends to retain the lowest, most junior tranche (commonly an equity note) with a credit loss tolerance of $a_0 - a_1$, which absorbs the first loss exposure of the transaction, and, thus, is called the "first loss position" or "first loss piece". This form of credit enhancement displays the extent to which banks are willing to bear part of the losses stemming from the non-performance of the reference portfolio. Such an asset retention establishes the degree of "collateralisation" of the securitisation transaction as issuers pass on asset claims to market investors by means of a special purpose vehicle. The prioritisation of structured claims reduces the default tolerance of the successive tranches, albeit increased interest rate sensitivity, which will be discussed later in this paper. The mezzanine tranches with low and medium investment grade rating are usually sold to capital market investors as notes and commercial paper (in the case of highly rated senior notes). Finally, the most senior tranches are securitised in the form of a credit default

[7] See OVERBECK AND WAGNER (2001) for an abridged representation of this method of loss cascading.

swap with a equally or lower risk-weighted counterparty by means of a credit default swap or some other method of structural provision, such as a bilateral credit guarantee, etc.

6 EXTREME VALUE THEORY AS LOSS FUNCTION

Alternatively to the normal inverse distribution of random variables on a uniform space, one might resort to *extreme value theory* to model the loss density function of credit portfolios. We derive a loss function as a specialised form of a Pareto-like distribution (Fig. A12), which is one-dimensional by definition. Neither the generalised Pareto distribution (GPD) nor the transformed GDP presented in this model are derived from a multi-dimensional distribution with dependent tail events (EMBRECHTS, 2000; EMBRECHTS, MCNEIL AND STRAUMANN, 1999), even though we value contingent claims on a multi-asset portfolio of securitisable loans affected by default losses. This methodology is justified on the grounds of the stochastic characteristics of the reference portfolio. Since the loan pool exhibits equal between-asset correlation, we can do without multi-dimensional distributions by considering the reference portfolio to be one asset, whose credit risk is modelled on aggregate.

Extreme value theory (EVT) propagates a stochastic methodology as part and parcel of a comprehensive risk measure to monitor asset exposure to extremes of random phenomena. EMBRECHTS (2000) describes it as a "canonical theory for the (limit) distribution of normalised maxima of independent, identically distributed random variables", where solving for the right limit results of the equation (0.9) below yields the estimation of the extremal events (EMBRECHTS, KLÜPPELBERG AND MIKOSCH, 1997; EMBRECHTS, RESNICK AND SAMORODNITSKY, 1999; MCNEIL, 1999),

$$M_n = \max\left(X_1,...,X_n\right) \tag{0.9}$$

This is in stark opposition to the theory of averages, where

$$S_n = X_1 + ... + X_n \tag{0.10}$$

describes the general notion of quantiles as multiples of standard deviations, with the Brownian motion as a basic assumption representing what is known to be the most familiar consideration of modelling diffusion processes. Multivariate EVT as an advanced form of estimating the extreme events in a random setting (EMBRECHTS, HAAN and HUANG, 1999), purports to translating the behaviour of such rare events into stochastic processes, evolving dynamically in time and space, by considering issues such as the shape of the distribution density function (skewness and kurtosis) and its variability in stress scenarios. However, the detachment of EVT from the straightjacket of hitherto distributional assumptions on dependent tail behaviour of stochastic processes does come at a certain cost. The methodological elegance of estimating extreme events, be it normalised maxima of i.i.d. events or the behaviour thereof in the context of a stochastic process, admit to restrictions to an unreserved and unqualified adoption in credit risk management. For one, EVT features substantial intrinsic model risk (EMBRECHTS, 2000), for its requires mathematical assumptions about the tail model, whose estimation beyond or at the limit of available data defies reliable verification in practice. The absence of an optimal canonical choice of the threshold above which data is to be used imposes deliberate exogeneity on EVT modelling, which could compound limitations of the model in the presence of non-linearities (RESNIK, 1998). While these qualifications could possibly upset some of the virtues of EVT, a common caveat to EVT, nonetheless, does not hold for the presented model. High dimensional portfolios will not impair the assessment of stochastic properties of extreme events (EMBRECHTS, HAAN AND HUANG, 1999), since we model rare events of default risk in a uniform credit portfolio as a proxy for the valuation of contingent claims on defaultable multi-asset portfolios.

In a nutshell, the use of EVT as a methodology comes to matter as it best describes the stochastic behaviour of extreme events at the cost of strong distributional assumptions, for loss of less presumptive models with equal predictive power. In the context of loan securitisation the modelling of senioritised payout to investors from an underlying reference portfolio of loans over a given period of time, cases of portfolio distress do constitute extreme events in the sense of EVT. Given the objective of the proposed model to explain the effects of loss allocation and the security design provisions governing contingent claims under extreme events, EVT claims methodological attractiveness due to ease of application and flexibility in model calibration. Nevertheless, it certainly falls short of representing the ultimate panacea of risk management due to a multitude of unresolved theoretical issues, such as multiple risk factors and possible computational instability as ML estimated parameters do not necessarily converge (EMBRECHTS, 2000).

In defiance of the standard assumption of an elliptic distribution

$$f(x_j) \sim N(\mu_{x_j}, \sigma_{x_j}), \tag{0.11}$$

since a heavy upper tail of periodic credit losses x_j yields for some positive integer a

$$\int_0^{\infty} x_j^a f(x_j)dx_j \to +\infty, \tag{0.12}$$

the generalised Pareto distribution (GPD) with parameters $\xi \in \mathrm{R}$, $\beta > 0$ is defined by

$$G_{\xi,\beta}(x) = \begin{cases} 1-\left(1+\dfrac{\xi x}{\beta}\right)^{-\xi^{-1}} & \mathit{for}\, \xi \neq 0 \\ 1-\exp\left(-\dfrac{x}{\beta}\right) & \mathit{for}\, \xi = 0 \end{cases}, \tag{0.13}$$

where $x \geq 0$ for $\xi \geq 0$ and $0 \leq x \leq -\dfrac{\beta}{\xi}$ for $\xi < 0$. ξ is a shape parameter of the distribution responsible for the tail behaviour, where the two cases $\xi \geq 0$ and $\xi < 0$ yield heavy tails and light tails respectively.

In order to construct a loss distribution with the same tail behaviour, we improve on the generalised Pareto distribution by following the approach introduced by JUNKER AND SZIMAYER (2001) in allowing for a peak different from zero. Hence, the following loss function *L(x)* can be derived from expanding the support of GPD to **R** by an appropriate transformation,

$$L(x) = 1 - \left(1 + \frac{x \times \left((x - r) + \sqrt{(x - r)^2 + s^2}\right)}{2b \times \left(1 + \exp\left(-\frac{d\,(x - r)}{b}\right)\right)}\right)^{-x^{-1}}, \tag{0.14}$$

for $x > 0$ (heavy tailed), $b > 0$, $s > 0$ and $r \in \mathrm{R}$ (for the treatment of $x \leq 0$ see JUNKER AND SZIMAYER (2001)).

Mapping of the loss function onto a distribution on the uniform interval of random variables in $[0,1]$ is achieved by imposing a upper and lower bound on x such that $x \in [-d; d]$ and

$$L_d(x) = \frac{L(x) - L(-d)}{L(d) - L(-d)}. \tag{0.15}$$

Subsequently, L_U is formed with $u \in [0,1]$ such that

$$L_U(u) = L_d\left(U_d^{-1}(u)\right), \tag{0.16}$$

with U_d being the uniform distribution with $\min = -d$ and $\max = d$. $r_u = U_d^{-1}(r)$ is gained through re-parameterisation, whilst b and s are scale parameters dependent on the level of d, e.g. for d' one obtains $b' = b \times \frac{d'}{d}$. The same holds true for s analogously. The following parameters have been chosen for the simulation: $x = 0.4, b = 26, s = 7.5, p_u = 10^{-4}, d = 10^4$. This parameterisation results in $1 - L(d) = 6 \cdot 10^{-7}$, which has the desired property of leaving the loss tail shape unaffected by the truncation.[8]

[8] Since $L(-d) = 0.05$ the density of L_U does not revert to zero at point $u = 0$, which corresponds to the practical intuition of portfolio losses (reality check of uniform mapping assumption for the distribution of random variables on the uniform interval [0,1]).

7 RESULTS – TERM STRUCTURES OF DEFAULT RATES

Table 2 exhibits the results of a Monte Carlo simulation with a sequence of one million iterations of normal inverse distributed (NID) portfolio losses X_j for $j = 1,...,n$, with the given portfolio parameters p=0.0026 and r=0.17, whereas Table 1 represents the simulations results if we apply extreme value theory (which encompasses the properties of the NID and allows for extreme events to be incorporated in the estimation on the basis of the given shape parameters). The probability of default (commonly abbreviated as PD) is assumed to be constant for each period j at this stage of our analysis, which is in line with the default standard of corporate loans proposed by issuers in their "offering circular" as constant or conditional default rate ("CDR").

The CDR represents an assumed rate of default each month, expressed as a per annum percentage of the aggregate principal balance of the loan claims that are not in default as of such month. It does not purport to be either a historical description of loan default or a predictive measure of the expected rate of default of any pool of loan claims, and, thus, does not impart compliance with possible fluctuations of portfolio default across time. Since banks calculate the CDR for analytical purposes only, it is not an accurate indicator or prediction of actual defaults and losses, as these are likely to differ in timing and amount from the assumed constant end-of-period loss at a constant rate of default. For the purpose of the CDR calculation the outstanding principal balance of the loan claims that are not assumed to be in default is subjected to a periodic default percentage, which complements the monthly decline as a result of scheduled and unscheduled principal payments (prepayments) and expected amortisation. The assumed rate of prepayment and various other CDR assumptions translate into a decline of the outstanding principal balance of loan claims as reference base. The erosion of the aggregate balance is further augmented by any loan claims that have already been in default as a consequence of the application of CDR in the preceding months, irrespective of whether defaulted loan claims are considered liquidated (DEUTSCHE BANK GLOBAL MARKETS, 1998). Thus, a rise in prepayment rates directly affects a change in cumulative defaults.

Consequently, the CDR approach has found entry in the considerations of model assumptions with respect to properties of the reference portfolio of corporate loans. Besides keeping p constant, the reduction of the reference portfolio through prepayments and amortisation is ignored. Amid this simplification of actual accumulation of proceeds and default losses, it recognises the fact that prepayment speed higher than scheduled amortisation might not necessarily reduce aggregate losses, since loan claims with a high default probability are least likely to be prepaid. Furthermore, the timing of defaults is assumed to take place at the end of each period j to ensure consistency in the approximation of relative portfolio losses per period against the background of a declining principal balance. Pursuant to the CDR calculation the accumulation of periodic losses has been modelled in the previous section of this paper (see Eq. 0.4).

The subsequent illustrations (Tabs. 1-4) exhibit the results of applying various distributions to the development of the principle balance of a loan collateral over time, with cumulative losses allocated to tranches according to seniority. The first column denotes the year, the second the respective (forward) default rate p and the third and fourth column list the mean and the standard deviation of the accumulated loss, the estimated expected loss $\tilde{L}_j^k$, and the unexpected loss $s_{\tilde{L}_j^k}$. Since credit risk is measured by the volatility of losses, called unexpected loss, the generated results correspond to the distributional assumptions below (see Fig. 2 below).

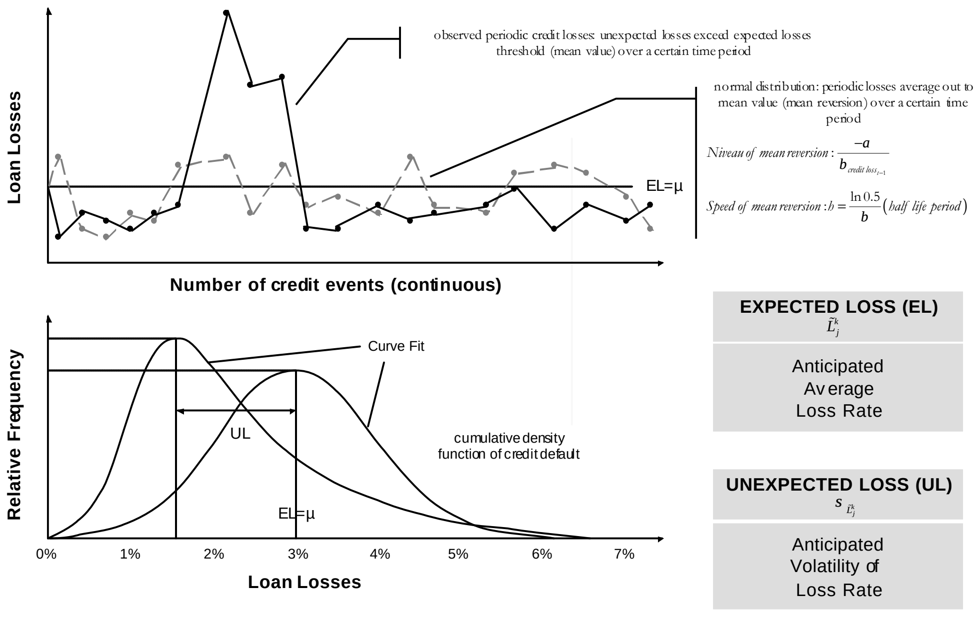

Figure 2. *Volatility of credit losses as a measure of credit risk.*

The remaining columns report on the accumulated expected loss per tranche, whose boundaries have been chosen on the historical basis of most common security designs. In the various tranches all quantities increase monotonously on a cumulative basis, the intriguing insight is depicted in Figures A1-A4 (linear and logarithmic plots).

The term structure of default rates of expected losses has been derived from both the normal inverse distribution in adaptation of the suggested model by VASICEK (1987) and an extreme value theory approach to Pareto distributed credit losses. Moreover, the analysis has been complemented by alternative cumulative distribution functions, a beta distribution and a negative binomial distribution (Tabs. 1-4; complete tables with the simulation results of estimated and unexpected loss as an absolute share of total portfolio loss are included in the appendix as Tabs. 19-26). When all four distribution functions are compared with respect to accurate approximation of total portfolio losses, the estimated expected loss $\tilde{L}_j^k$ and the unexpected loss $s_{\tilde{L}_j^k}$. However, once we move to a tranche-based examination of the results for the various distributions, the expected loss in the distribution tails reflected in the first tranche [0-2.4%] $\tilde{L}_j^1$ showcases material differences across all four approaches. Also in the higher tranches as illustrated in Figure A8, the term structure between the normal inverse distribution and the beta-/negative-binomial distribution as well as the loss distribution according to extreme value theory deviate from each other at an increasing rate, especially in the mezzanine tranche [6.5%-9.0%].

Considering the cumulative incidence of credit losses to be skewed towards the extreme end of the distribution, the normal inverse distribution seems to reflect the "loss reality" most truthfully.[9] According to OVERBECK AND WAGNER (2001) the q-q-plot of quantiles for the beta distribution versus the negative binomial distribution tends to indicate a high degree of similarity on the basis of matched first two moments, with cumulative probabilities reaching levels in the tune of 99.995%. This comparison has been completed *ex post* the adjustment of results obtained from the negative-binomial distribution in dividing the discrete losses *n* by the some large number *s*.[10] Note that the observations tend to fall slightly below the diagonal in the q-q-plot, which is clearly rooted in the cut-off value of *s*. If the same analysis is applied in the context of extreme value theory the q-q-plot of quantiles for the Pareto-based loss function versus the normal inverse distribution (Fig. A12) bears out a clear deviation of EVT quantiles from the normal inverse distribution from 0.05 onwards. Consequently, the extreme value theory might be more attuned to illustrating sporadic credit loss whose statistical impact in expectation models defies the Gaussian assumptions of the normal inverse distribution. Since the degree of coincidence between probability masses is instrumental in predicting of how close the accumulated expected loss per tranche match for both distributions of portfolio loss, the almost identical results of accumulated expected loss per tranche for both the normal inverse distribution and the beta distribution do not come to a great surprise. As the proposed specialised form of a Pareto distribution places a premium on extreme events in line with extreme value theory, the results from the q-q plot hints to a mitigation of default rates for more senior "investor tranches", whilst the first loss position is almost entirely exhausted by estimated default losses.

[9] See also ALTMAN AND SAUNDERS (1998).

[10] In this case s=1000 generated the following parameter input values: a=0.323278 and b=80.4258 (see OVERBECK AND WAGNER (2001)).

7.1 Prime Distribution Functions

7.1.1 Term Structure of Default Rates (Extreme Value Theory)

			Expected and unexpected losses		**$\tilde{L}_j^k$ per tranche (in % of tranche volume)**					
cum./per.	**Yr**	p	$\tilde{L}_j^k$	$s_{\tilde{L}_j^k}$	**0-2.4%**	**2.4-3.9%**	**3.9-6.5%**	**6.5-9%**	**9-10.5%**	**10.5-100%**
cumulative	1	0.0026	**0.002598**	0.004581	0.104285	0.003095	0.000987	0.000375	0.000188	0.000010
periodic			**0.002598**	*0.004581*	*0.104285*	*0.003095*	*0.000987*	*0.000375*	*0.000188*	*0.000010*
cumulative	2	0.0026	**0.005299**	0.006512	0.207327	0.007653	0.002229	0.000797	0.000402	0.000023
periodic			**0.002701**	*0.001931*	*0.103042*	*0.004558*	*0.001242*	*0.000422*	*0.000214*	*0.000013*
cumulative	3	0.0026	**0.007799**	0.007978	0.308168	0.014546	0.003940	0.001332	0.000648	0.000037
periodic			**0.002500**	*0.001466*	*0.100841*	*0.006893*	*0.001711*	*0.000535*	*0.000246*	*0.000014*
cumulative	4	0.0026	**0.010397**	0.009228	0.406098	0.024665	0.006204	0.001994	0.000952	0.000051
periodic			**0.002598**	*0.001250*	*0.097930*	*0.010119*	*0.002264*	*0.000662*	*0.000304*	*0.000014*
cumulative	5	0.0026	**0.012990**	0.010317	0.500079	0.039278	0.009131	0.002777	0.001292	0.000067
periodic			**0.002593**	*0.001089*	*0.093981*	*0.014613*	*0.002927*	*0.000783*	*0.000340*	*0.000016*
cumulative	6	0.0026	**0.015581**	0.011252	0.589083	0.060005	0.012995	0.003645	0.001649	0.000082
periodic			**0.002591**	*0.000935*	*0.089004*	*0.020727*	*0.003864*	*0.000868*	*0.000357*	*0.000015*
cumulative	7	0.0026	**0.018168**	0.012104	0.671323	0.088676	0.018083	0.004711	0.002052	0.000098
periodic			**0.002587**	*0.000852*	*0.082240*	*0.028671*	*0.005088*	*0.001066*	*0.000403*	*0.000016*

			Expected and unexpected losses		**$s_{\tilde{L}_j^k}$ per tranche (in % of tranche volume)**					
cum./per.	**Yr**	p	$\tilde{L}_j^k$	$s_{\tilde{L}_j^k}$	**0-2.4%**	**2.4-3.9%**	**3.9-6.5%**	**6.5-9%**	**9-10.5%**	**10.5-100%**
cumulative	1	0.0026	0.002598	**0.004581**	0.130941	0.050337	0.028247	0.018032	0.013013	0.001526
periodic			*0.002598*	**0.004581**	*0.130941*	*0.050337*	*0.028247*	*0.018032*	*0.013013*	*0.001526*
cumulative	2	0.0026	0.005299	**0.006512**	0.178268	0.078295	0.041886	0.026340	0.019017	0.002261
periodic			*0.002701*	**0.001931**	*0.047327*	*0.027958*	*0.013639*	*0.008308*	*0.006004*	*0.000735*
cumulative	3	0.0026	0.007799	**0.007978**	0.208257	0.106408	0.055182	0.033976	0.024088	0.002737
periodic			*0.002500*	**0.001466**	*0.029989*	*0.028113*	*0.013296*	*0.007636*	*0.005071*	*0.000476*
cumulative	4	0.0026	0.010397	**0.009228**	0.226551	0.136470	0.068699	0.041584	0.029081	0.003191
periodic			*0.002598*	**0.001250**	*0.018294*	*0.030062*	*0.013517*	*0.007608*	*0.004993*	*0.000454*
cumulative	5	0.0026	0.012990	**0.010317**	0.235359	0.169272	0.082556	0.048863	0.033732	0.003616
periodic			*0.002593*	**0.001089**	*0.008808*	*0.032802*	*0.013857*	*0.007279*	*0.004651*	*0.000425*
cumulative	6	0.0026	0.015581	**0.011252**	0.235536	0.205001	0.097288	0.055601	0.038068	0.003977
periodic			*0.002591*	**0.000935**	*0.000177*	*0.035729*	*0.014732*	*0.006738*	*0.004336*	*0.000361*
cumulative	7	0.0026	0.018168	**0.012104**	0.227853	0.243130	0.113521	0.062720	0.042430	0.004301
periodic			*0.002587*	**0.000852**	*-0.007683*	*0.038129*	*0.016233*	*0.007119*	*0.004362*	*0.000324*

Table 1. *Simulation of constant forward probability rates (EVT distribution of portfolio losses) per tranche on a cumulative and periodic basis with results for estimated and unexpected losses as absolute shares of total portfolio losses.*

7.1.2 Term Structure of Default Rates (Normal Inverse Distribution)

			Expected and unexpected losses		$\tilde{L}_j^k$ per tranche (in % of tranche volume)					
cum./per.	**Yr**	p	$\tilde{L}_j^k$	$s_{\tilde{L}_j^k}$	**0-2.4%**	**2.4-3.9%**	**3.9-6.5%**	**6.5-9%**	**9-10.5%**	**10.5-100%**
cumulative	1	0.0026	**0.002593**	0.004588	0.104352	0.004135	0.000830	0.000157	0.000041	0.000001
periodic			**0.002593**	*0.004588*	*0.104352*	*0.004135*	*0.000830*	*0.000157*	*0.000041*	*0.000001*
cumulative	2	0.0026	**0.005186**	0.006491	0.206350	0.010987	0.002129	0.000389	0.000110	0.000001
periodic			**0.002593**	*0.001903*	*0.101998*	*0.006852*	*0.001299*	*0.000232*	*0.000069*	*0.000000*
cumulative	3	0.0026	**0.007771**	0.007921	0.304995	0.021439	0.004068	0.000683	0.000177	0.000002
periodic			**0.002585**	*0.001430*	*0.098645*	*0.010452*	*0.001939*	*0.000294*	*0.000067*	*0.000001*
cumulative	4	0.0026	**0.010356**	0.009126	0.399452	0.036819	0.006922	0.001108	0.000261	0.000003
periodic			**0.002585**	*0.001205*	*0.094457*	*0.015380*	*0.002854*	*0.000425*	*0.000084*	*0.000001*
cumulative	5	0.0026	**0.012934**	0.010181	0.488493	0.058068	0.010884	0.001691	0.000392	0.000004
periodic			**0.002578**	*0.001055*	*0.089041*	*0.021249*	*0.003962*	*0.000583*	*0.000131*	*0.000001*
cumulative	6	0.0026	**0.015500**	0.011127	0.570866	0.086047	0.016401	0.002502	0.000559	0.000006
periodic			**0.002566**	*0.000946*	*0.082373*	*0.027979*	*0.005517*	*0.000811*	*0.000167*	*0.000002*
cumulative	7	0.0026	**0.018059**	0.011991	0.645940	0.121363	0.023847	0.003575	0.000777	0.000008
periodic			**0.002559**	*0.000864*	*0.075074*	*0.035316*	*0.007446*	*0.001073*	*0.000218*	*0.000002*

Table 2. *Simulation of constant forward probability rates (normal inverse distribution of portfolio losses) per tranche on a cumulative and periodic basis.*

8 VARIABLE PORTFOLIO QUALITY - CALCULATION OF THE DEFAULT RATES (EXPECTED/ESTIMATED AND UNEXPECTED) FOR ALL THE TRANCHES

The time-dependent migration of asset quality of collateral during the term of the securitisation transaction is vital in the estimation of the loss cascading mechanism and its implications on the term structure of the individual tranches. Thus, the presented model has also been applied to both a normal inverse distribution and an extreme value distribution of collateral credit loss with varying forward rates of default (constant, increasing and decreasing). In the following section two extreme cases are investigated, i.e. a strictly deteriorating asset portfolio (back loaded) and a strictly improving portfolio (front loaded) with the corresponding effects on the tranching structure at hand in terms of estimated losses. Since a variable portfolio quality can be represented by means of an upward and downward drift of one-year default probabilities, the change in portfolio quality is replicated on the basis of a sequence if increasing and decreasing forward default probabilities p for both prime distributions – extreme value theory loss distribution and normal inverse loss distribution (Tabs. 5-6 and 7-8).

The presented model has been applied for both a normal inverse distribution and an extreme value distribution of collateral credit loss with varying forward rates of default (constant, increasing and decreasing). The typical structure of a CLO transaction reflected in the simulated allocation of losses to the various tranches elicits a peculiar dichotomy of default tolerance of the "first loss position" [0-2.4%] as opposed to the "investor tranches" [2.4-3.9%], [3.9-6.5%], [6.5-9%] and [9-10.5%]. Figs. A5-A7 display the corresponding plots of a deteriorating and improving portfolio. With reference to the former, a comparison with Table 1 and Figure 5 as well as Table 2 and Figure 7 showcases the increase of the first order moment of estimated expected loss $\tilde{L}_j^k$ per tranche (slope of estimated losses) as anticipated, whilst merely the first loss piece flattens from the fifth year onwards as this tranche begins to fill up to the maximum of losses to be absorbed. While the expected loss for the first tranche is a linear function of the time since commencement of the transaction, the expected losses of the other tranches increase in an exponential fashion over the years.

Different forward rates of default losses are limited in altering this stark contrast only in the first periods of the CLO transaction, when an increase in the forward rate affects a smaller first moment of the [2.4-3.9%] tranche than for a decreasing forward rate in the context of a extreme value distribution. The reverse applies for a normal inverse distribution of cumulative default losses, where a change in the forward rate, be it an increase or decrease, directly translates into higher expected losses for investors without the marginal "default baggage" being absorbed by the first loss position. With losses accumulating until the first CLO tranches has been fully exhausted such that subsequent tranches bear excess losses, periodic expected losses for a constant forward rate of an extreme value distribution subside asymptotically; this pattern is reflected in a concave shape of the default term structure of cumulative collateral losses.

			Expected and unexpected losses		$\tilde{L}_j^k$ per tranche (in % of tranche volume)					
cum./per.	**Yr**	r_u	$\tilde{L}_j^k$	$s_{\tilde{L}_j^k}$	**0-2.4%**	**2.4-3.9%**	**3.9-6.5%**	**6.5-9%**	**9-10.5%**	**10.5-100%**
cumulative	1	0.00010	**0.002598**	0.004581	0.104285	0.003095	0.000987	0.000375	0.000188	0.000010
periodic			**0.002598**	*0.004581*	*0.104285*	*0.003095*	*0.000987*	*0.000375*	*0.000188*	*0.000010*
cumulative	2	0.00123	**0.006191**	0.006508	0.248098	0.008190	0.002382	0.000833	0.000413	0.000023
periodic			**0.003593**	*0.004626*	*0.145561*	*0.003188*	*0.001032*	*0.000366*	*0.000197*	*0.000012*
cumulative	3	0.00195	**0.010473**	0.007834	0.416846	0.018131	0.004431	0.001373	0.000653	0.000034
periodic			**0.004282**	*0.004366*	*0.174498*	*0.003318*	*0.000946*	*0.000311*	*0.000158*	*0.000009*
cumulative	4	0.00247	**0.015278**	0.009016	0.598044	0.039707	0.007968	0.002189	0.000976	0.000049
periodic			**0.004805**	*0.004461*	*0.196124*	*0.003444*	*0.000979*	*0.000336*	*0.000171*	*0.000010*
cumulative	5	0.00277	**0.020389**	0.010101	0.770975	0.088717	0.014413	0.003391	0.001416	0.000068
periodic			**0.005111**	*0.004540*	*0.208625*	*0.003570*	*0.001030*	*0.000365*	*0.000192*	*0.000011*
cumulative	6	0.00295	**0.025679**	0.011100	0.908023	0.195662	0.026506	0.005249	0.002007	0.000091
periodic			**0.005289**	*0.004583*	*0.215965*	*0.003606*	*0.001061*	*0.000374*	*0.000194*	*0.000012*
cumulative	7	0.00306	**0.031064**	0.011904	0.981059	0.389878	0.049962	0.008072	0.002774	0.000114
periodic			**0.005386**	*0.004303*	*0.220174*	*0.003614*	*0.001051*	*0.000347*	*0.000173*	*0.000008*

			Expected and unexpected losses		$s_{\tilde{L}_j^k}$ per tranche (in % of tranche volume)					
cum./per.	**Yr**	r_u	$\tilde{L}_j^k$	$s_{\tilde{L}_j^k}$	**0-2.4%**	**2.4-3.9%**	**3.9-6.5%**	**6.5-9%**	**9-10.5%**	**10.5-100%**
cumulative	1	0.00010	0.002598	**0.004581**	0.130941	0.050337	0.028247	0.018032	0.013013	0.001526
periodic			*0.002598*	**0.004581**	*0.130941*	*0.050337*	*0.028247*	*0.018032*	*0.013013*	*0.001526*
cumulative	2	0.00123	0.006191	**0.006508**	0.174874	0.080581	0.043399	0.026735	0.019296	0.002297
periodic			*0.003593*	**0.004626**	*0.128459*	*0.050898*	*0.028846*	*0.017801*	*0.013353*	*0.001674*
cumulative	3	0.00195	0.010473	**0.007834**	0.196178	0.116936	0.058216	0.034156	0.024201	0.002717
periodic			*0.004282*	**0.004366**	*0.127236*	*0.051448*	*0.027190*	*0.016506*	*0.011817*	*0.001324*
cumulative	4	0.00247	0.015278	**0.009016**	0.195799	0.166620	0.076333	0.042914	0.029468	0.003226
periodic			*0.004805*	**0.004461**	*0.126626*	*0.052496*	*0.027589*	*0.017024*	*0.012409*	*0.001539*
cumulative	5	0.00277	0.020389	**0.010101**	0.168668	0.234645	0.100094	0.052945	0.035368	0.003746
periodic			*0.005111*	**0.004540**	*0.126135*	*0.053353*	*0.028524*	*0.017735*	*0.013319*	*0.001588*
cumulative	6	0.00295	0.025679	**0.011100**	0.113185	0.312582	0.131249	0.065124	0.041864	0.004305
periodic			*0.005289*	**0.004583**	*0.125604*	*0.053776*	*0.028916*	*0.018047*	*0.013306*	*0.001701*
cumulative	7	0.00306	0.031064	**0.011904**	0.048280	0.358264	0.172091	0.079550	0.048750	0.004631
periodic			*0.005386*	**0.004303**	*0.125246*	*0.053700*	*0.028669*	*0.017346*	*0.012474*	*0.001050*

Table 3. *Simulation of a deteriorating portfolio (extreme value theory) with separate illustration of estimated and unexpected losses per tranche.*

In the case of rising annual default probabilities (Tab. 3 and Tab. 5), the cumulative expected loss of the first loss position increases less than linear as the upper boundary of the first loss piece does not compare favourably with the exposure of this tranche induced by the high default rate during the initial years. Thus, the deviation of the tranche retained by the issuer and the outstanding, more senior tranches is a reflection of the rapid exhaustion of the loss absorption by the first loss tranche before excess losses are passed onto the higher tranche.

			Expected and unexpected losses		$\tilde{L}_j^k$ per tranche (in % of tranche volume)					
cum./per.	**Yr**	r_u	$\tilde{L}_j^k$	$s_{\tilde{L}_j^k}$	**0-2.4%**	**2.4-3.9%**	**3.9-6.5%**	**6.5-9%**	**9-10.5%**	**10.5-100%**
cumulative	1	0.00367	**0.006005**	0.004523	0.245673	0.003844	0.001091	0.000378	0.000182	0.000009
periodic			**0.006005**	0.004523	0.245673	0.003844	0.001091	0.000378	0.000182	0.000009
cumulative	2	0.00267	**0.011014**	0.006328	0.445650	0.012637	0.003011	0.000892	0.000419	0.000020
periodic			**0.005009**	0.004429	0.204375	0.003659	0.001085	0.000360	0.000178	0.000009
cumulative	3	0.00195	**0.015296**	0.007696	0.608457	0.030299	0.005900	0.001564	0.000693	0.000032
periodic			**0.004282**	0.004366	0.174498	0.003318	0.000946	0.000311	0.000158	0.000009
cumulative	4	0.00154	**0.019189**	0.008889	0.742734	0.063996	0.010507	0.002531	0.001046	0.000047
periodic			**0.003894**	0.004440	0.158171	0.003325	0.001020	0.000345	0.000169	0.000009
cumulative	5	0.00123	**0.022782**	0.010016	0.846424	0.121583	0.017450	0.003799	0.001493	0.000066
periodic			**0.003593**	0.004626	0.145561	0.003188	0.001032	0.000366	0.000197	0.000012
cumulative	6	0.00094	**0.026099**	0.011032	0.917378	0.206659	0.027686	0.005460	0.002017	0.000086
periodic			**0.003317**	0.004623	0.134103	0.003262	0.001042	0.000364	0.000176	0.000010
cumulative	7	0.00083	**0.029309**	0.011953	0.961671	0.318082	0.042558	0.007584	0.002645	0.000107
periodic			**0.003209**	0.004607	0.129703	0.003157	0.000987	0.000365	0.000195	0.000011

			Expected and unexpected losses		$s_{\tilde{L}_j^k}$ per tranche (in % of tranche volume)					
cum./per.	**Yr**	r_u	$\tilde{L}_j^k$	$s_{\tilde{L}_j^k}$	**0-2.4%**	**2.4-3.9%**	**3.9-6.5%**	**6.5-9%**	**9-10.5%**	**10.5-100%**
cumulative	1	0.00367	0.006005	**0.004523**	0.124453	0.055172	0.029266	0.018088	0.012631	0.001715
periodic			0.006005	**0.004523**	0.124453	0.055172	0.029266	0.018088	0.012631	0.001715
cumulative	2	0.00267	0.011014	**0.006328**	0.159532	0.097505	0.047893	0.027566	0.019229	0.002198
periodic			0.005009	**0.004429**	0.126442	0.054316	0.029122	0.017752	0.012535	0.001253
cumulative	3	0.00195	0.015296	**0.007696**	0.170597	0.145666	0.065797	0.036069	0.024743	0.002656
periodic			0.004282	**0.004366**	0.127236	0.051448	0.027190	0.016506	0.011817	0.001324
cumulative	4	0.00154	0.019189	**0.008889**	0.162596	0.202429	0.086183	0.045515	0.030438	0.003059
periodic			0.003894	**0.004440**	0.128220	0.051941	0.028494	0.017093	0.012322	0.001283
cumulative	5	0.00123	0.022782	**0.010016**	0.138867	0.262821	0.108775	0.055492	0.036301	0.003628
periodic			0.003593	**0.004626**	0.128459	0.050898	0.028846	0.017801	0.013353	0.001674
cumulative	6	0.00094	0.026099	**0.011032**	0.106689	0.317039	0.133944	0.066194	0.041936	0.004178
periodic			0.003317	**0.004623**	0.129227	0.051637	0.028862	0.017647	0.012598	0.001760
cumulative	7	0.00083	0.029309	**0.011953**	0.072741	0.354321	0.161875	0.077491	0.047751	0.004688
periodic			0.003209	**0.004607**	0.128773	0.050739	0.028080	0.017886	0.013405	0.001735

Table 4. *Simulation of improving portfolio (extreme value theory) with separate illustration of estimated and unexpected losses per tranche.*

			Expected and unexpected losses		$\tilde{L}_j^k$ per tranche (in % of tranche volume)					
cum./per.	**Yr**	r_u	$\tilde{L}_j^k$	$s_{\tilde{L}_j^k}$	**0-2.4%**	**2.4-3.9%**	**3.9-6.5%**	**6.5-9%**	**9-10.5%**	**10.5-100%**
cumulative	1	0.0026	**0.002593**	0.004579	0.104369	0.004101	0.000831	0.000147	0.000039	0.000001
periodic			**0.002593**	0.004579	0.104369	0.004101	0.000831	0.000147	0.000039	0.000001
cumulative	2	0.0036	**0.006178**	0.007552	0.241895	0.016947	0.003589	0.000685	0.000200	0.000004
periodic			**0.003585**	0.002973	0.137526	0.012846	0.002758	0.000538	0.000161	0.000003
cumulative	3	0.0043	**0.010451**	0.010198	0.393470	0.045190	0.010055	0.001928	0.000516	0.000008
periodic			**0.004273**	0.002646	0.151575	0.028243	0.006466	0.001243	0.000316	0.000004
cumulative	4	0.0048	**0.015207**	0.012650	0.542116	0.096050	0.023047	0.004522	0.001182	0.000016
periodic			**0.004756**	0.002452	0.148646	0.050860	0.012992	0.002594	0.000666	0.000008
cumulative	5	0.0051	**0.020238**	0.014826	0.673930	0.172279	0.044946	0.009036	0.002325	0.000030
periodic			**0.005031**	0.002176	0.131814	0.076229	0.021899	0.004514	0.001143	0.000014
cumulative	6	0.0053	**0.025426**	0.016816	0.780726	0.271508	0.078663	0.016656	0.004250	0.000054
periodic			**0.005188**	0.001990	0.106796	0.099229	0.033717	0.007620	0.001925	0.000024
cumulative	7	0.0054	**0.030681**	0.018618	0.860469	0.386342	0.125166	0.028539	0.007366	0.000094
periodic			**0.005255**	0.001802	0.079743	0.114834	0.046503	0.011883	0.003116	0.000040

Table 5. *Simulation of deteriorating portfolio (normal inverse distribution) with separate illustration of estimated losses per tranche.*

			Expected and unexpected losses		$\tilde{L}_j^k$ per tranche (in % of tranche volume)					
cum./per.	**Yr**	$r\,u$	$\tilde{L}_j^k$	$s_{\tilde{L}_j^k}$	**0-2.4%**	**2.4-3.9%**	**3.9-6.5%**	**6.5-9%**	**9-10.5%**	**10.5-100%**
cumulative	1	0.0060	**0.005992**	0.006269	0.241622	0.010208	0.001379	0.000144	0.000023	0.000000
periodic			**0.005992**	*0.006269*	*0.241622*	*0.010208*	*0.001379*	*0.000144*	*0.000023*	*0.000000*
cumulative	2	0.0050	**0.010961**	0.008220	0.431596	0.032250	0.004144	0.000389	0.000061	0.000000
periodic			**0.004969**	*0.001951*	*0.189974*	*0.022042*	*0.002765*	*0.000245*	*0.000037*	*0.000000*
cumulative	3	0.0043	**0.015214**	0.009431	0.580569	0.068669	0.008830	0.000719	0.000097	0.000001
periodic			**0.004253**	*0.001211*	*0.148973*	*0.036419*	*0.004686*	*0.000330*	*0.000037*	*0.000000*
cumulative	4	0.0039	**0.019060**	0.010333	0.698890	0.121829	0.016286	0.001245	0.000147	0.000001
periodic			**0.003846**	*0.000902*	*0.118321*	*0.053160*	*0.007456*	*0.000526*	*0.000050*	*0.000000*
cumulative	5	0.0036	**0.022595**	0.011046	0.790304	0.190679	0.027340	0.002020	0.000225	0.000001
periodic			**0.003535**	*0.000713*	*0.091414*	*0.068850*	*0.011054*	*0.000775*	*0.000078*	*0.000000*
cumulative	6	0.0033	**0.025819**	0.011616	0.857477	0.270135	0.042336	0.003105	0.000323	0.000001
periodic			**0.003224**	*0.000570*	*0.067173*	*0.079456*	*0.014996*	*0.001085*	*0.000098*	*0.000000*
cumulative	7	0.0032	**0.028936**	0.012125	0.907148	0.359962	0.062859	0.004687	0.000454	0.000002
periodic			**0.003117**	*0.000509*	*0.049671*	*0.089827*	*0.020523*	*0.001582*	*0.000131*	*0.000000*

Table 6. *Simulation of improving portfolio (normal inverse distribution) with separate illustration of estimated losses per tranche.*

9 CALCULATION OF THE DEFAULT RATES (EXPECTED/ESTIMATED AND UNEXPECTED) FOR ALL THE TRANCHES UNDER LOSS RECOVERY

The development of portfolio losses can also be considered in the context of loss recovery, which mitigates periodic default loss. We update each tranche *k* for each time period *j*, given the simulation results in section 7. The random variable X denotes the total amount of loan default loss of the underlying reference portfolio during a given year with random default. Hence, X can be written as $X = I \times B$, where I is a discriminate random variable indicator, which denotes the uniform probability *p* of default loss to occur in

$$I = \begin{cases} 1 & E\left(X \middle| I = 1\right) = m_{X|I=1} = m \\ 0 & E\left(X \middle| I = 0\right) = m_{X|I=1} = 0 \end{cases} \tag{0.17}$$

and B represents the amount of bad debt actually written off in the case of default, indicated by $I = 1$, upon loan recovery of m of the individual loan value. Hence, adjusting for loan recovery yields the density function of default loss below:

$$f_X(x) = \begin{cases} x > 0 & 1 - p \\ x = 0 & p \times f_B(x) \end{cases}. \tag{0.18}$$

In other words, B is commensurate to the capacity of loan originators to allay loss severity by appropriating the maximum residual loan value (i.e. the pledged collateral) included in the securitised reference portfolio. The mean and the standard deviation of B is denoted as $\mu_B = m$ and σ_B, so that $f_B(x) \sim N(\mu_B, \sigma_B)$. The probability p of loan default could either be *static* or *variable* (see section 8) if we were to consider dynamic credit performance in a multi-period setting.

So for each tranche k at any point in time j, we can refine the loss function $L_j^k(X)$ of total default loss X_j^k by adjusting for some loss recovery factor B, which mitigates default loss allocated to each "senioritised" tranche by the factor $1-m$. Hence, the new loss function

$$L_j^k(X) \sim f_X(x) = \begin{cases} x > 0 & 1-p \\ x = 0 & p \times f_B(x) \end{cases} \tag{0.19}$$

does not betray the acknowledgement – though admittedly simplistic – of recovering bad debt in loan servicing as a pivotal consideration in determining the value of the securitised reference portfolio as an underlying asset in pricing contingent claims. Successful debtor monitoring and workout proceedings ultimately lower the discount investors demand from issuers in the form of spreads over some benchmark market interest rate.

Thus, we need to derive the formulae of the expected and unexpected loss (mean and standard deviation) of the new loss function $\hat{L}_j^k(X)$, which is a function of the capacity of bad debt recovery on the existing loss function $\hat{L}_j^k(X) = f\left(L_j^k(X)\right)$.

$$\mu_X = E\left(\mu_{X|I=1}\right) = p \times \mu_{X|I=1} + \underbrace{(1-p)\mu_{X|I=0}}_{0} = p \times \mu_{X|I=1} = p \times m \tag{0.20}$$

and

$$\sigma_X^2 = \mu_{\sigma^2_{X|I}} + \sigma^2_{\mu_{X|I}}, \tag{0.21}$$

where

$$m_{s^2_{X|I}} = p \times s^2_{X|I=1} + (1-p) \times s^2_{X|I=0} \text{ with } \begin{cases} s^2_{X|I=1} = s^2_B \\ s^2_{X|I=0} = 0 \end{cases} \quad (0.22)$$

so that

$$m_{s^2_{X|I}} = p \times s^2_B + (1-p) \times 0 = p \times s^2_B, \quad (0.23)$$

and

$$\begin{aligned} & s^2_{m_{X|I}} = E\left(m^2_{X|I}\right) - E\left(m_{X|I}\right)^2 = \\ & \left(p \times m^2_{X|I=1} + \underbrace{(1-p) \times m^2_{X|I=0}}_{0} \right) - m^2_X = \\ & \left(p \times m^2_{X|I=1} \right) - (p \times m)^2 = \\ & m^2 p (1-p). \end{aligned} \quad (0.24)$$

Hence,

$$s^2_X = m_{s^2_{X|I}} + s^2_{m_{X|I}} = p \times s^2_B + m^2 p(1-p) = p\left[s^2_B + m^2 (1-p) \right] \quad (0.25)$$

Since we conditioned the mean and variance of X on the case of default (I=1), we need to incorporate both the mean value of losses m^k_j and the volatility of underlying asset loss $s^2_{m^k_j}$ for each period and tranche in order to do justice to the definition $\hat{L}^k_j(X) = f\left(L^k_j(X)\right)$. First, m itself is a proportion of the nominal expected loss, where $p = m^k_j$ for an initial portfolio value of 1 in period $j = 0$, such that eq. 24 yields

$$m_X = m_j \times m. \quad (0.26)$$

Second, given $\operatorname{cov}\left(s^2_B, s^2_{m^k_j}\right) = 0$ we transpose eqs. 27 and 28 to

$$\mu_{\sigma^2_{X|I}} = p\times\left(\sigma^2_B + \sigma^2_{\mu^k_j}\right) \tag{0.27}$$

and

$$\sigma^2_{\mu_{X|I}} = m^2 p\left(1-p\right) + \sigma^2_{\mu^k_j} \,. \tag{0.28}$$

Hence,

$$\sigma^2_{X^k_j} = p\left(\sigma^2_B + m^2\left(1-p\right)\right) + \sigma^2_{\mu^k_j}\left(1+p\right). \tag{0.29}$$

In keeping with the notation used in this paper, we write

$$\hat{L}^k_j(X) \sim f\left(\mu_{X^k_j}, \sigma^2_{X^k_j}\right), \tag{0.30}$$

where

$$\mu_X = \mu^k_j \times m \,, \tag{0.31}$$

$$\sigma^2_{X^k_j} = p\left(\sigma^2_B + m^2\left(1-p\right)\right) + \sigma^2_{\mu^k_j}\left(1+p\right), \tag{0.32}$$

Based on the above formulae we recalculate Tables 9-11 for a mean recovery rate of 45% for both the total periodic losses of the entire portfolio $\hat{L}_j(X)$ and each tranche after loss cascading $\hat{L}^k_j(X)$ for both constant and variable default rates (analogous to section 8 above). By imposing the additional condition of bad debt recovery on our simulation results in section 7 we privilege equally the pivotal role of both the probability of debtor default and the ability of loan servicers of the underlying reference portfolio to affect the loss cascading mechanism in pricing loan securitisation. We specify the above expressions (equations 35 and 36) to

$$\mu_{X^k_j} = p\times m \tag{0.33}$$

and

$$\sigma^2_{X_j^k} = p\left(\sigma_B^2 + \mu^2\left(1-p\right)\right) + \sigma^2_{\mu_j^k}\left(1+p\right), \tag{0.34}$$

where $\mu = 0.45$ and $\sigma_B = 0.1$.[11] We also obtain new measures for the time-dependent relationship between unexpected and expected loss after recovery (see Appendix, Tabs. 31-34). For matters of conciseness we only provide results for extreme value theory in this regard.

9.1 Term Structure of Default Rates (Extreme Value Theory) – Constant Default Rate and Recovery

			Expected and unexpected losses		**$\tilde{L}_j^k$ per tranche (in % of tranche volume)**					
cum./per.	**Yr**	p	$\tilde{L}_j^k$	$\sigma_{\tilde{L}_j^k}$	**0-2.4%**	**2.4-3.9%**	**3.9-6.5%**	**6.5-9%**	**9-10.5%**	**10.5-100%**
cumulative	1	0.0026	**0.001169**	0.005144	0.046928	0.001393	0.000444	0.000169	0.000085	0.000005
periodic			**0.001169**	*0.005144*	*0.046928*	*0.001393*	*0.000444*	*0.000169*	*0.000085*	*0.000005*
cumulative	2	0.0026	**0.002385**	0.007652	0.093297	0.003444	0.001003	0.000359	0.000181	0.000010
periodic			**0.001215**	*0.002509*	*0.046369*	*0.002051*	*0.000559*	*0.000190*	*0.000096*	*0.000006*
cumulative	3	0.0026	**0.003510**	0.009652	0.138676	0.006546	0.001773	0.000599	0.000292	0.000017
periodic			**0.001125**	*0.002000*	*0.045378*	*0.003102*	*0.000770*	*0.000241*	*0.000111*	*0.000006*
cumulative	4	0.0026	**0.004679**	0.011456	0.182744	0.011099	0.002792	0.000897	0.000428	0.000023
periodic			**0.001169**	*0.001804*	*0.044069*	*0.004554*	*0.001019*	*0.000298*	*0.000137*	*0.000006*
cumulative	5	0.0026	**0.005846**	0.013097	0.225036	0.017675	0.004109	0.001250	0.000581	0.000030
periodic			**0.001167**	*0.001641*	*0.042291*	*0.006576*	*0.001317*	*0.000352*	*0.000153*	*0.000007*
cumulative	6	0.0026	**0.007011**	0.014584	0.265087	0.027002	0.005848	0.001640	0.000742	0.000037
periodic			**0.001166**	*0.001487*	*0.040052*	*0.009327*	*0.001739*	*0.000391*	*0.000161*	*0.000007*
cumulative	7	0.0026	**0.008176**	0.015987	0.302095	0.039904	0.008137	0.002120	0.000923	0.000044
periodic			**0.001164**	*0.001403*	*0.037008*	*0.012902*	*0.002290*	*0.000480*	*0.000181*	*0.000007*

			Expected and unexpected losses		**$\sigma_{\tilde{L}_j^k}$ per tranche (in % of tranche volume)**					
cum./per.	**Yr**	p	$\tilde{L}_j^k$	$\sigma_{\tilde{L}_j^k}$	**0-2.4%**	**2.4-3.9%**	**3.9-6.5%**	**6.5-9%**	**9-10.5%**	**10.5-100%**
cumulative	1	0.0026	0.001169	**0.005144**	0.163522	0.051118	0.028475	0.018115	0.013054	0.001528
periodic			*0.001169*	**0.005144**	*0.163522*	*0.051118*	*0.028475*	*0.018115*	*0.013054*	*0.001528*
cumulative	2	0.0026	0.002385	**0.007652**	0.234452	0.080123	0.042382	0.026512	0.019102	0.002266
periodic			*0.001215*	**0.002509**	*0.070930*	*0.029005*	*0.013907*	*0.008397*	*0.006049*	*0.000738*
cumulative	3	0.0026	0.003510	**0.009652**	0.285836	0.109816	0.056047	0.034260	0.024224	0.002745
periodic			*0.001125*	**0.002000**	*0.051384*	*0.029694*	*0.013665*	*0.007748*	*0.005122*	*0.000479*
cumulative	4	0.0026	0.004679	**0.011456**	0.323820	0.142212	0.070052	0.042007	0.029280	0.003201
periodic			*0.001169*	**0.001804**	*0.037984*	*0.032396*	*0.014005*	*0.007747*	*0.005056*	*0.000457*
cumulative	5	0.0026	0.005846	**0.013097**	0.350708	0.178411	0.084541	0.049450	0.034002	0.003630
periodic			*0.001167*	**0.001641**	*0.026888*	*0.036199*	*0.014489*	*0.007443*	*0.004721*	*0.000428*
cumulative	6	0.0026	0.007011	**0.014584**	0.367329	0.218992	0.100110	0.056370	0.038412	0.003994
periodic			*0.001166*	**0.001487**	*0.016621*	*0.040582*	*0.015569*	*0.006920*	*0.004410*	*0.000364*
cumulative	7	0.0026	0.008176	**0.015987**	0.374306	0.263857	0.117451	0.063712	0.042857	0.004321
periodic			*0.001164*	**0.001403**	*0.006977*	*0.044864*	*0.017341*	*0.007342*	*0.004445*	*0.000327*

[11] We assume the recovery rate to take the value 45 percent in accordance with the industry standard and the specifications set forth by the Basle Committee (1999).

Table 7. *Simulation of constant forward probability rates (EVT distribution of portfolio losses) per tranche on a cumulative and periodic basis with results for estimated and unexpected losses as absolute shares of total portfolio losses under loss recovery.*

9.2 Term Structure of Default Rates (Extreme Value Theory) – Variable Default Rate and Recovery

			Expected and unexpected losses		$\tilde{L}_j^k$ per tranche (in % of tranche volume)					
cum./per.	**Yr**	r_u	$\tilde{L}_j^k$	$s_{\tilde{L}_j^k}$	**0-2.4%**	**2.4-3.9%**	**3.9-6.5%**	**6.5-9%**	**9-10.5%**	**10.5-100%**
cumulative	1	0.00010	**0.001169**	0.005144	0.046928	0.001393	0.000444	0.000169	0.000085	0.000005
periodic			**0.001169**	*0.005144*	*0.046928*	*0.001393*	*0.000444*	*0.000169*	*0.000085*	*0.000005*
cumulative	2	0.00123	**0.002786**	0.010547	0.111644	0.003686	0.001072	0.000375	0.000186	0.000010
periodic			**0.001617**	*0.005404*	*0.065502*	*0.001435*	*0.000464*	*0.000165*	*0.000089*	*0.000005*
cumulative	3	0.00195	**0.004713**	0.015838	0.187581	0.008159	0.001994	0.000618	0.000294	0.000015
periodic			**0.001927**	*0.005291*	*0.078524*	*0.001493*	*0.000426*	*0.000140*	*0.000071*	*0.000004*
cumulative	4	0.00247	**0.006875**	0.021337	0.269120	0.017868	0.003586	0.000985	0.000439	0.000022
periodic			**0.002162**	*0.005499*	*0.088256*	*0.001550*	*0.000441*	*0.000151*	*0.000077*	*0.000005*
cumulative	5	0.00277	**0.009175**	0.026981	0.346939	0.039923	0.006486	0.001526	0.000637	0.000031
periodic			**0.002300**	*0.005644*	*0.093881*	*0.001607*	*0.000464*	*0.000164*	*0.000086*	*0.000005*
cumulative	6	0.00295	**0.011555**	0.032706	0.408610	0.088048	0.011928	0.002362	0.000903	0.000041
periodic			**0.002380**	*0.005725*	*0.097184*	*0.001623*	*0.000477*	*0.000168*	*0.000087*	*0.000005*
cumulative	7	0.00306	**0.013979**	0.038171	0.441477	0.175445	0.022483	0.003632	0.001248	0.000051
periodic			**0.002424**	*0.005465*	*0.099078*	*0.001626*	*0.000473*	*0.000156*	*0.000078*	*0.000004*

			Expected and unexpected losses		$s_{\tilde{L}_j^k}$ per tranche (in % of tranche volume)					
cum./per.	**Yr**	r_u	$\tilde{L}_j^k$	$s_{\tilde{L}_j^k}$	**0-2.4%**	**2.4-3.9%**	**3.9-6.5%**	**6.5-9%**	**9-10.5%**	**10.5-100%**
cumulative	1	0.00010	0.001169	**0.005144**	0.163522	0.051118	0.028475	0.018115	0.013054	0.001528
periodic			*0.001169*	**0.005144**	*0.163522*	*0.051118*	*0.028475*	*0.018115*	*0.013054*	*0.001528*
cumulative	2	0.00123	0.002786	**0.010547**	0.335880	0.102822	0.057559	0.035996	0.026449	0.003204
periodic			*0.001617*	**0.005404**	*0.172358*	*0.051704*	*0.029085*	*0.017882*	*0.013396*	*0.001676*
cumulative	3	0.00195	0.004713	**0.015838**	0.514505	0.155111	0.084966	0.052570	0.038300	0.004530
periodic			*0.001927*	**0.005291**	*0.178626*	*0.052289*	*0.027407*	*0.016574*	*0.011851*	*0.001326*
cumulative	4	0.00247	0.006875	**0.021337**	0.697911	0.208483	0.112781	0.069668	0.050746	0.006071
periodic			*0.002162*	**0.005499**	*0.183406*	*0.053372*	*0.027814*	*0.017098*	*0.012446*	*0.001541*
cumulative	5	0.00277	0.009175	**0.026981**	0.883815	0.262747	0.141542	0.087484	0.064106	0.007662
periodic			*0.002300*	**0.005644**	*0.185904*	*0.054264*	*0.028762*	*0.017815*	*0.013360*	*0.001590*
cumulative	6	0.00295	0.011555	**0.032706**	0.070855	0.317445	0.170704	0.105613	0.077454	0.009365
periodic			*0.002380*	**0.005725**	*0.187040*	*0.054698*	*0.029161*	*0.018129*	*0.013348*	*0.001703*
cumulative	7	0.00306	0.013979	**0.038171**	0.258468	0.372068	0.199616	0.123035	0.089965	0.010417
periodic			*0.002424*	**0.005465**	*0.187613*	*0.054624*	*0.028912*	*0.017422*	*0.012511*	*0.001052*

Table 8. *Simulation of a deteriorating portfolio (extreme value theory) with separate illustration of estimated and unexpected losses per tranche under loss recovery.*

Under consideration of loss recovery during the life of the transaction for both deteriorating and improving quality of the underying portfolio Table 8 above and Table 9 below) the cumulative expected loss of the first loss position substantiates similar properties as simulation results presented in section 7 for extreme value theory. Again, we detect a sizeable dichotomy between the first loss position and the more senior tranches due to the loss absorbing effect of the first loss tranche. At the same time the degree of unexpected loss in relation to expected default loss increases in the seniority of issued tranches, with first loss position exhibiting a more accurate prediction of its default exposure.

			Expected and unexpected losses		$\tilde{L}_j^k$ per tranche (in % of tranche volume)					
cum./per.	**Yr**	*r u*	$\tilde{L}_j^k$	$s_{\tilde{L}_j^k}$	**0-2.4%**	**2.4-3.9%**	**3.9-6.5%**	**6.5-9%**	**9-10.5%**	**10.5-100%**
cumulative	1	0.00367	**0.002702**	0.005819	0.110553	0.001730	0.000491	0.000170	0.000082	0.000004
periodic			**0.002702**	*0.005819*	*0.110553*	*0.001730*	*0.000491*	*0.000170*	*0.000082*	*0.000004*
cumulative	2	0.00267	**0.004956**	0.011329	0.200543	0.005687	0.001355	0.000401	0.000189	0.000009
periodic			**0.002254**	*0.005511*	*0.091969*	*0.001647*	*0.000488*	*0.000162*	*0.000080*	*0.000004*
cumulative	3	0.00195	**0.006883**	0.016620	0.273806	0.013635	0.002655	0.000704	0.000312	0.000014
periodic			**0.001927**	*0.005291*	*0.078524*	*0.001493*	*0.000426*	*0.000140*	*0.000071*	*0.000004*
cumulative	4	0.00154	**0.008636**	0.021902	0.334230	0.028798	0.004728	0.001139	0.000471	0.000021
periodic			**0.001752**	*0.005282*	*0.071177*	*0.001496*	*0.000459*	*0.000155*	*0.000076*	*0.000004*
cumulative	5	0.00123	**0.010252**	0.027306	0.380891	0.054712	0.007853	0.001710	0.000672	0.000030
periodic			**0.001617**	*0.005404*	*0.065502*	*0.001435*	*0.000464*	*0.000165*	*0.000089*	*0.000005*
cumulative	6	0.00094	**0.011745**	0.032647	0.412820	0.092997	0.012459	0.002457	0.000908	0.000039
periodic			**0.001493**	*0.005341*	*0.060346*	*0.001468*	*0.000469*	*0.000164*	*0.000079*	*0.000005*
cumulative	7	0.00083	**0.013189**	0.037948	0.432752	0.143137	0.019151	0.003413	0.001190	0.000048
periodic			**0.001444**	*0.005302*	*0.058366*	*0.001421*	*0.000444*	*0.000164*	*0.000088*	*0.000005*

			Expected and unexpected losses		$s_{\tilde{L}_j^k}$ per tranche (in % of tranche volume)					
cum./per.	**Yr**	*r u*	$\tilde{L}_j^k$	$s_{\tilde{L}_j^k}$	**0-2.4%**	**2.4-3.9%**	**3.9-6.5%**	**6.5-9%**	**9-10.5%**	**10.5-100%**
cumulative	1	0.00367	0.002702	**0.005819**	0.192579	0.056160	0.029519	0.018171	0.012670	0.001717
periodic			*0.002702*	**0.005819**	*0.192579*	*0.056160*	*0.029519*	*0.018171*	*0.012670*	*0.001717*
cumulative	2	0.00267	0.004956	**0.011329**	0.377811	0.111413	0.058892	0.036003	0.025243	0.002972
periodic			*0.002254*	**0.005511**	*0.185232*	*0.055253*	*0.029373*	*0.017831*	*0.012573*	*0.001255*
cumulative	3	0.00195	0.006883	**0.016620**	0.556437	0.163702	0.086299	0.052577	0.037094	0.004298
periodic			*0.001927*	**0.005291**	*0.178626*	*0.052289*	*0.027407*	*0.016574*	*0.011851*	*0.001326*
cumulative	4	0.00154	0.008636	**0.021902**	0.731917	0.216487	0.115029	0.069746	0.049453	0.005582
periodic			*0.001752*	**0.005282**	*0.175480*	*0.052785*	*0.028730*	*0.017169*	*0.012358*	*0.001285*
cumulative	5	0.00123	0.010252	**0.027306**	0.904274	0.268191	0.144113	0.087627	0.062848	0.007259
periodic			*0.001617*	**0.005404**	*0.172358*	*0.051704*	*0.029085*	*0.017882*	*0.013396*	*0.001676*
cumulative	6	0.00094	0.011745	**0.032647**	0.074359	0.320655	0.173216	0.105354	0.075484	0.009021
periodic			*0.001493*	**0.005341**	*0.170084*	*0.052464*	*0.029103*	*0.017727*	*0.012636*	*0.001762*
cumulative	7	0.00083	0.013189	**0.037948**	0.242705	0.372192	0.201524	0.123321	0.088931	0.010758
periodic			*0.001444*	**0.005302**	*0.168346*	*0.051537*	*0.028307*	*0.017966*	*0.013447*	*0.001737*

Table 9. *Simulation of improving portfolio (extreme value theory) with separate illustration of estimated and unexpected losses per tranche under loss recovery.*

10 RISK-NEUTRAL PRICING OF CLO TRANCHES

The proportion of collateral losses allocated to each tranche of a CLO transaction according to contractual prioritisation constitutes a default rate, which serves as a basis for the calculation of its risk-neutral price. In the risk-neutral valuation of CLOs we do is nothing else but extend concepts and meanings of accepted principles of asset pricing under symmetric information to an asymmetric information context, where expected losses are the risk premium, as unexpected losses cannot be determined in a straightforward fashion without incorporating some simplifying assumptions. The proposed risk-neutral pricing of CLO tranches heavily draws on risky bond analysis and the analysis of the term structure of credit spreads (LELAND AND TOFT, 1996). Hence, the definition of the threshold, which determines the accepted credit exposure of noteholders in an ordinary fixed income transaction, is adapted to fit the expected term structure of credit losses (loss distribution) of CLO transactions for purposes of risk-based profit management of loan portfolios. Further, we consider the spreads of investors of each tranche (above the risk-free rate, be it fixed or stochastic) equal to the risk premium associated with expected losses. We subsequently compare the required risk-neutral rate of return of each tranche to the pricing of bonds of amenable quality, matched first moments and maturity.

In absence of arbitrage in efficient markets risk-neutrality corresponds to a "good deal" proposition in the security design of CLOs, such that the tranche spreads reflect a certain expectation of future losses. Hence, the value of each CLO tranche ought to be driven by the valuation of interest proceeds generated from the securitised asset portfolio, subject to a certain degree of attrition, such as debtor default and prepayments. Given the estimated default rates per tranche of the transaction, we are now in the position to compute the rates of return for each individual tranche by calculating the net present value of expected cash flows according to

$$\begin{aligned}&\sum_{m=1}^{n-1}\frac{\left(1-\sum_{j=1}^{m}\tilde{L}_j^k\right)r_k}{\prod_{l=1}^{m}\left(1+r_{f_l}\right)}+\frac{\left(1-\sum_{j=1}^{n}\tilde{L}_j^k\right)\left(1+r_k\right)}{\prod_{l=1}^{n}\left(1+r_{f_l}\right)}=\\&=\sum_{m=1}^{n}\frac{\left(1-\sum_{j=1}^{m}\tilde{L}_j^k\right)r_k}{\prod_{l=1}^{n}\left(1+r_{f_l}\right)}+\frac{\left(1-\sum_{j=1}^{n}\tilde{L}_j^k\right)}{\prod_{l=1}^{n}\left(1+r_{f_l}\right)}=1,\end{aligned} \tag{0.35}$$

where $\sum_{j=1}^{m}\tilde{L}_j^k$ denotes the accumulated expected loss in the tranche k up to year j and r_f represents the risk-free forward rate during the applicable time period. By solving the equation above for r_k over j=7 for each tranche, the following results are obtained for a collateral portfolio with a constant, a deteriorating and an improving forward rate of default at a constant risk-free rate $r_{f_l}=r_f=5.0\%$ for each time period (Tab. 10).

In absence of an appropriate analytical approach to capture the complexity of loan securitisation, investors are inclined to deal with such structure finance products in an undifferentiated way as especially senior tranches of CLOs are regarded as virtually risk-free (BURGHARDT, 2001). Given that this shortcoming results in notorious mispricing as risks are underestimated, the presented method of deriving the returns of CLO tranches based on the term structure of periodic defaults might provide guidance as to the proper mean-variance consistent pricing of CLO transactions.

Distribution and Collateral Performance		**Returns per Tranche**					
Constant	$\tilde{L}_j^k$ *cum. (EVT/const.)*	20.34747%	6.28610%	5.25952%	5.06849%	5.03000%	5.00140%
	$\tilde{L}_j^k$ *cum. (NID/const.)*	20.56017%	6.79431%	5.33870%	5.05067%	5.01100%	5.00010%
Deteriorating	$\tilde{L}_j^k$ *cum. (EVT/deter.)*	37.49613%	10.81930%	5.69241%	5.11364%	5.03970%	5.00160%
	$\tilde{L}_j^k$ *cum. (NID/deter.)*	30.43862%	10.19510%	6.78903%	5.39333%	5.10090%	5.00120%
Improving	$\tilde{L}_j^k$ *cum. (EVT/improv.)*	49.12123%	9.87893%	5.60218%	5.10820%	5.03820%	5.00150%
	$\tilde{L}_j^k$ *cum. (NID/improv.)*	42.75610%	10.94410%	6.00440%	5.06577%	5.00640%	5.00000%

Table 10. *Investor spreads in the various tranches under two different default distributions of the collateral portfolio (extreme value theory (EVT) and normal inverse distribution (NID)) on a cumulative basis for constant, increasing and decreasing forward rates of loan default.*

The above spreads are the risk-neutral returns of the relevant tranches based on a normal inverse distribution and an extreme value distribution of collateral default. The first loss position [0-2.4%] retained by sponsor of the transaction as credit enhancement absorbs most of the periodic losses throughout the lifetime of the transaction (that is, seven years in this model), and, thus, the expected spread of risk-neutral investors amounts to a cumulative average annual return of 20.35 % and 20.56% for either case of loss distribution, i.e. normal inverse distribution (NID) and Pareto-like distribution according to extreme value theory (EVT) of credit events in the reference portfolio. Successive tranches claim lower spreads/returns as their default tolerance decreases from a maximum accumulated credit loss of 6.29% (EVT)) and 6.79% (NID) of notational value for the [2.4-3.9%] tranche to almost the risk-free rate of return for the [10.5-100%] senior tranche.

Generally, the prediction of expected investor returns significantly varies with the methodology applied in modelling credit risk of the underlying reference portfolio. In all tranches but the [3.9-6.5%] mezzanine "investor" tranche default losses under the EVT model protrude the projection of credit loss under the NID approach for a constant forward rate of default. The pattern of a higher degree of estimated default under the extreme value approach is consistent for the first loss position only if we extend the exposition of expected risk-neutral returns per tranche to the case of a deteriorating and improving collateral quality.

The change of the periodic forward rate of credit default entails higher returns for all tranches, irrespective of whether the first moment of the terms structure is positive or negative. In this context an intriguing paradox of expected returns for each tranche emerges. An increasing forward rate of default (deteriorating portfolio quality) is actually more favourable for the first loss piece compared to a decline in the periodic default rate. For mezzanine and senior tranches spreads are lower under an improving rather than a deteriorating rate of default. The credit enhancement commands risk-neutral spreads well beyond 30% per period (for a deteriorating portfolio) and 40% per period (for an improving portfolio) and reduces the chances of collateral default for mezzanine and senior tranches accordingly. These diametrically opposite expectations as to risk-neutral returns for the issuer (who generally retains the first loss position) and investors (who hold the mezzanine and senior tranches) is robust even if the second moment of the forward rate is changed for both deterioration and improvement of portfolio quality. Thus, an increase in the default rate of the collateral portfolio over time (deteriorating portfolio quality) requires a lower default tolerance and correspondingly lower risk-neutral spreads per period from holders of the most junior tranche (credit enhancement), as the second moment of expected losses of an improving collateral portfolio is smaller than it is for a deteriorating portfolio, provided that first moments are matched. Conversely, spreads of the first loss position are less sensitive to a reduction in the collateral default rate as long as the first period defaults are set at a level such that the risk horizon of seven years is insufficient to offset higher risk-neutral returns (due to portfolio deterioration) to the level of a constant forward rate. The application of EVT in the determination of total cumulative portfolio default loss pronounces this effect, as the flat tail behaviour of this Pareto-style distribution puts more weight on the first tranches as credit enhancement.

11 REALITY CHECK

11.1 Ratio of estimated and unexpected losses

The spreads actually offered to investors (returns minus the assumed risk-free rate), however, price significantly wider than the derived risk-neutral returns and defy the above assumption of risk-neutral expectations in deriving the term structure of CLO tranches. Since this observation is not all too surprising in the light of the complexity of securitisation structures, the spread concession solicits complementary inferences drawn from investment choices that largely cause generic asset backed structures to be usually cheaper than plain vanilla corporate bonds. Investors might just command higher returns for CLO tranches on the theory that not only liquidity or other additional risks, but also the degree of unexpected loss matters in the mean-variance trade-off of financial investment.[12] The relationship between unexpected and expected loss (ratio of unexpected and expected loss $s_{\tilde{L}_j^k} \big/ \tilde{L}_j^k$) epitomises a margin of error in the estimation of a loss function to describe the distribution of credit losses. If applied to the entire portfolio in general and all tranches in particular for cumulative as well as periodic losses (in the case of both NID and EVT credit risk loss functions) it could substantiate this conjecture as a reality check as to the viability of risk neutral spreads.

Whilst all ratios decrease over time at a rate similar to a normal inverse distribution setting (Figs. A4 and A5), the EVT loss function suggest a lower starting value during the first year of the securitisation transaction especially for the first loss position retained by the issuer and the senior piece [10.5-100%]. Most importantly, the application of an EVT in the disclosure of unexpected losses $s_{\tilde{L}_j^k}$ per tranche complements this observation in a most insightful way, since apparently asymptotic development of the variance in portfolio losses allocated to the first loss position of the issuer [0-2.4%] augments the favourable property of a linear increase of the $s_{\tilde{L}_j^k} \big/ \tilde{L}_j^k$ ratio for the first loss piece. At the same time the required spread for the first

[12] Synthetic bank CLOs feature even higher spreads than traditional CLOs. This pricing disparity is frequently attributed to the fact that lower secondary liquidity, a less receptive investor base for credit derivative based products and additional risk arising from the increased leverage of the senior tranches in partially funded structures are prime characteristics bearing additional exposure for investors in synthetic CLOs (BATCHVAROV ET AL. 2000).

tranche does not attract additional compensation for excessive variations in unexpected losses $s_{\tilde{L}_j^k}$. As EVT estimates reduce the margin of error compared to other distributions, a verification of risk-neutral spreads on CLO tranches seems to be more realistic. Thus, Tables 1, 2 and 13 (where extreme value theory is applied to portfolio losses) are a consequence of a "canonical theory for the (limit) distribution of normalised maxima of independent, identically distributed random variables" (EMBRECHTS, 2000), which effects more weight being attached to credit losses of extreme events absorbed by the first loss position and the alleviation of default risk imposed on more senior "investor tranches".

			Unexpected divided by exp. losses	$s_{\tilde{L}_j^k} \Big/ \tilde{L}_j^k$ per tranche					
cum./per.	**Yr**	p	$s_{\tilde{L}_j^k} \Big/ \tilde{L}_j^k$	**0-2.4%**	**2.4-3.9%**	**3.9-6.5%**	**6.5-9%**	**9-10.5%**	**10.5-100%**
cumulative	1	0.0026	0.763279	0.255607	16.263974	28.619048	48.085333	69.218085	152.600000
periodic			*0.763279*	*0.255607*	*16.263974*	*28.619048*	*48.085333*	*69.218085*	*152.600000*
cumulative	2	0.0026	0.228911	0.859840	10.230629	18.791386	33.048934	47.305970	98.304348
periodic			*0.714920*	*0.459298*	*6.133831*	*10.981481*	*19.687204*	*28.056075*	*56.538462*
cumulative	3	0.0026	0.022952	0.675790	7.315276	14.005584	25.507508	37.172840	73.972973
periodic			*0.586400*	*0.297389*	*4.078485*	*7.770894*	*14.272897*	*20.613821*	*34.000000*
cumulative	4	0.0026	0.887564	0.557873	5.532941	10.073340	20.854564	30.547269	62.568627
periodic			*0.481139*	*0.186807*	*2.970847*	*5.970406*	*10.492447*	*16.424342*	*32.428571*
cumulative	5	0.0026	0.794226	0.470644	4.309588	9.041288	17.595607	26.108359	53.970149
periodic			*0.419977*	*0.093721*	*2.244714*	*4.734199*	*9.296296*	*13.679412*	*26.562500*
cumulative	6	0.0026	0.722162	0.399835	3.416399	7.486572	15.254047	23.085506	48.500000
periodic			*0.360865*	*0.001989*	*0.723790*	*3.812629*	*7.762673*	*12.145658*	*24.066667*
cumulative	7	0.0026	0.666226	0.339409	2.741779	6.277775	13.313522	20.677388	43.887755
periodic			*0.329339*	*-0.093422*	*0.329880*	*3.190448*	*6.678236*	*10.823821*	*20.250000*

Table 11. $s_{\tilde{L}_j^k} \Big/ \tilde{L}_j^k$ *ratio for each tranche based on a simulation of constant forward probability rates (EVT distribution of portfolio losses).*

			Unexpect. divided by exp. losses	$s_{\tilde{L}_j^k} \Big/ \tilde{L}_j^k$ per tranche					
cum./per.	**Yr**	r_u	$s_{\tilde{L}_j^k} \Big/ \tilde{L}_j^k$	**0-2.4%**	**2.4-3.9%**	**3.9-6.5%**	**6.5-9%**	**9-10.5%**	**10.5-100%**
cumulative	1	0.000100	0.763279	0.255607	16.263974	28.619048	48.085333	69.218085	152.600000
periodic			*0.763279*	*0.255607*	*16.263974*	*28.619048*	*48.085333*	*69.218085*	*152.600000*
cumulative	2	0.001231	0.051203	0.704859	9.838950	18.219563	32.094838	46.721550	99.869565
periodic			*0.287503*	*0.882510*	*15.965496*	*27.951550*	*48.636612*	*67.781726*	*139.500000*
cumulative	3	0.001945	0.748019	0.470625	6.449506	13.138343	24.876912	37.061256	79.911765
periodic			*0.019617*	*0.729154*	*15.505726*	*28.742072*	*53.073955*	*74.791139*	*147.111111*
cumulative	4	0.002469	0.590130	0.327399	4.196237	9.579945	19.604386	30.192623	65.836735
periodic			*0.928408*	*0.645643*	*15.242741*	*28.180797*	*50.666667*	*72.567251*	*153.900000*
cumulative	5	0.002771	0.495414	0.218772	2.644871	6.944703	15.613388	24.977401	55.088235
periodic			*0.888280*	*0.604602*	*14.944818*	*27.693204*	*48.589041*	*69.369792*	*144.363636*
cumulative	6	0.002954	0.432260	0.124650	0.597561	4.951671	12.406935	20.858994	47.307692
periodic			*0.866515*	*0.581594*	*14.912923*	*27.253534*	*48.254011*	*68.587629*	*140.750000*
cumulative	7	0.003055	0.383209	0.049212	0.918913	3.444438	9.855055	17.573901	40.622807
periodic			*0.798923*	*0.568850*	*14.858882*	*27.277831*	*49.988473*	*72.104046*	*130.250000*

Table 12. $s_{\tilde{L}_j^k} \Big/ \tilde{L}_j^k$ *ratio for each tranche based on a simulation of increasing forward probability rates of a deteriorating portfolio (EVT distribution of portfolio losses).*

			Unexpect. divided by exp. losses	$s_{\tilde{L}_j^k} / \tilde{L}_j^k$ per tranche					
cum./per.	**Yr**	r_u	$s_{\tilde{L}_j^k} / \tilde{L}_j^k$	**0-2.4%**	**2.4-3.9%**	**3.9-6.5%**	**6.5-9%**	**9-10.5%**	**10.5-100%**
cumulative	1	0.003673	0.753206	0.506580	14.352758	26.824931	47.851852	69.401099	190.555556
periodic			0.753206	*0.506580*	*14.352758*	*26.824931*	*47.851852*	*69.401099*	*190.555556*
cumulative	2	0.002669	0.574541	0.357976	7.715834	15.906011	30.903587	45.892601	109.900000
periodic			*0.884208*	*0.618676*	*14.844493*	*26.840553*	*49.311111*	*70.421348*	*139.222222*
cumulative	3	0.001945	0.503138	0.280376	4.807617	10.152034	23.062020	35.704185	83.000000
periodic			*0.019617*	*0.729154*	*15.505726*	*28.742072*	*53.073955*	*74.791139*	*147.111111*
cumulative	4	0.001538	0.463234	0.218916	3.163151	8.202436	17.983011	29.099426	65.085106
periodic			*0.140216*	*0.810642*	*15.621353*	*27.935294*	*49.544928*	*72.911243*	*142.555556*
cumulative	5	0.001231	0.439645	0.164063	2.161659	6.233524	14.607002	24.314133	54.969697
periodic			*0.287503*	*0.882510*	*15.965496*	*27.951550*	*48.636612*	*67.781726*	*139.500000*
cumulative	6	0.000940	0.422698	0.116298	0.534117	4.837969	12.123443	20.791274	48.581395
periodic			*0.393729*	*0.963640*	*15.829859*	*27.698656*	*48.480769*	*70.579545*	*176.000000*
cumulative	7	0.000834	0.407827	0.075640	0.113930	3.803633	10.217695	18.053308	43.813084
periodic			*0.435650*	*0.992830*	*16.071904*	*28.449848*	*49.002740*	*68.743590*	*157.727273*

Table 13. $s_{\tilde{L}_j^k} / \tilde{L}_j^k$ *ratio for each tranche based on a simulation of decreasing forward probability rates of an improving portfolio (EVT distribution of portfolio losses).*

			Unexpected divided by exp. losses
cum./per.	**Yr**	p	$s_{\tilde{L}_j^k} / \tilde{L}_j^k$
cumulative	1	0.0026	0.769379
periodic			*0.769379*
cumulative	2	0.0026	0.251639
periodic			*0.733899*
cumulative	3	0.0026	0.019303
periodic			*0.553191*
cumulative	4	0.0026	0.881228
periodic			*0.466151*
cumulative	5	0.0026	0.787150
periodic			*0.409232*
cumulative	6	0.0026	0.717871
periodic			*0.368667*
cumulative	7	0.0026	0.663990
periodic			*0.337632*

Table 14. $s_{\tilde{L}_j^k} / \tilde{L}_j^k$ *ratio for total amount of tranches based on a simulation of constant forward probability rates (normal inverse distribution of portfolio losses).*

			Unexpected divided by exp. losses
cum./per.	**Yr**	r_u	$s_{\tilde{L}_j^k} / \tilde{L}_j^k$
cumulative	1	0.0026	0.765908
periodic			*0.765908*
cumulative	2	0.0036	0.222402
periodic			*0.829289*
cumulative	3	0.0043	0.975792
periodic			*0.619237*
cumulative	4	0.0048	0.831854
periodic			*0.515559*
cumulative	5	0.0051	0.732582
periodic			*0.432518*
cumulative	6	0.0053	0.661370
periodic			*0.383577*
cumulative	7	0.0054	0.606825
periodic			*0.342912*

Table 15. $s_{\tilde{L}_j^k} / \tilde{L}_j^k$ *ratio for total amount of tranches based on a simulation of increasing forward probability rates of a deteriorating portfolio (normal inverse distribution of portfolio losses).*

			Unexpected divided by exp. losses
cum./per.	**Yr**	r_u	$s_{\tilde{L}_j^k} / \tilde{L}_j^k$
cumulative	1	0.0060	0.046224
periodic			*0.046224*
cumulative	2	0.0050	0.749932
periodic			*0.392634*
cumulative	3	0.0043	0.619890
periodic			*0.284740*
cumulative	4	0.0039	0.542130
periodic			*0.234529*
cumulative	5	0.0036	0.488869
periodic			*0.201697*
cumulative	6	0.0033	0.449901
periodic			*0.176799*
cumulative	7	0.0032	0.419028
periodic			*0.163298*

Table 16. $s_{\tilde{L}_j^k} / \tilde{L}_j^k$ *ratio for total amount of tranches based on a simulation of decreasing forward probability rates of an improving portfolio (normal inverse distribution of portfolio losses).*

			Unexpected divided by exp. losses
cum./per.	**Yr**	*p*	$s_{\tilde{L}_j^k} \Big/ \tilde{L}_j^k$
cumulative	1	0.0026	0.756325
periodic			*0.756325*
cumulative	2	0.0026	0.246136
periodic			*0.743383*
cumulative	3	0.0026	0.017332
periodic			*0.564103*
cumulative	4	0.0026	0.880955
periodic			*0.472713*
cumulative	5	0.0026	0.786955
periodic			*0.406006*
cumulative	6	0.0026	0.717086
periodic			*0.362818*
cumulative	7	0.0026	0.663182
periodic			*0.337617*

Table 17. $s_{\tilde{L}_j^k} \Big/ \tilde{L}_j^k$ *ratio for total amount of tranches based on a simulation of constant forward probability rates (beta distribution of portfolio losses).*

			Unexpected divided by exp. losses
cum./per.	**Yr**	*p*	$s_{\tilde{L}_j^k} \Big/ \tilde{L}_j^k$
cumulative	1	0.0026	0.770777
periodic			*0.770777*
cumulative	2	0.0026	0.246104
periodic			*0.726054*
cumulative	3	0.0026	0.019931
periodic			*0.564341*
cumulative	4	0.0026	0.880166
periodic			*0.458705*
cumulative	5	0.0026	0.787173
periodic			*0.412451*
cumulative	6	0.0026	0.716911
periodic			*0.365571*
cumulative	7	0.0026	0.662943
periodic			*0.337481*

Table 18. $s_{\tilde{L}_j^k} \Big/ \tilde{L}_j^k$ *ratio for total amount of tranches based on a simulation of constant forward probability rates (negative binomial distribution of portfolio losses).*

As shown in Figures A8 and A9, all ratios decrease in time; but the more interesting result is that they differ considerably in orders of magnitude. In contrast to the whole portfolio and the first loss position [0-2.4%], which yield a ratio of $s_{\tilde{L}_j^k} \Big/ \tilde{L}_j^k$ in the order of one on the basis of cumulative losses, the second tranche [2.4-3.9%] exhibits a $s_{\tilde{L}_j^k} \Big/ \tilde{L}_j^k$ ratio of order 10 and the remaining, more senior ratios increase roughly by a factor of two on the basis of this reference level. The relationship of periodic expected and unexpected default rates exhibits the second moment of a declining impact of $s_{\tilde{L}_j^k}$ as the CLO transaction matures. Consequently, it can be concluded that the development of expected losses over time are more favourable in the case of the first loss piece, but also the variation of unexpected losses around the expected value of the most junior tranche retained by the issuer is much lower than for the investor tranches, and thus necessitates additional risk premium. Note that by incorporating a normally distributed recovery rate (see Appendix, Tables 31-34) the ratio of ratio of $s_{\tilde{L}_j^k} \Big/ \tilde{L}_j^k$ maintains a consistenly higher level than in the previous case without loss recovery. This result indicates that close debtor monitoring (proxied by higher loss recovery) could mitigate moral hazards effects between the fmost junor tranche and mezzanine and senior tranches.

In general, the obtained results of the term structure of expected and unexpected losses, $\tilde{L}_j^k$ and $s_{\tilde{L}_j^k}$, for a CLO transaction have critical implications on how the security design of securitisation transactions has to be viewed in the light of loan default and its variability over time. Our results back out the fact that sponsors of CLO transactions, who usually retain the most junior tranche as a first loss position in the transaction, find themselves disposed to a constant first moment of expected losses (linear increase), while investors holding mezzanine (and senior) claims on the collateral portfolio might be discomforted in anticipation of exponentially increasing losses.

11.2 Comparison with zero-bonds

The development of variance in unexpected losses in relation to scheduled defaults of the collateral portfolio ($s_{\tilde{L}_j^k} \Big/ \tilde{L}_j^k$ *(unexpected to expected losses)* ratio) per period might be indicative of the abnormal spreads on CLO tranches, if one was to take risk-neutral returns as the reference base. However, as the difference between the observed spreads in the CLO market and the calculated risk-neutral spreads depends on the assumptions entering the loss function of credit risk, the default rates per tranche and the corresponding risk-neutral spreads should be subjected to empirical examination. Since the calculation of expected spreads on CLO tranches rekindles the derivation of the yield-to-maturity of zero-coupon bonds, the term structure of periodic default probabilities of selected tranches could be directly benchmarked with comparable zero-bonds, whose internal rate of return is calibrated based on the default rates for rating classes published in the rating reports of Moody's Investor Services. This is accomplished by matching the first moments of either the "one-year" default probability ("lower boundary") or the accumulated "seven-year" default probability ("upper boundary") assigned by Moody's to a suitable corporate bond[13] to the expected loss of the respective CLO tranche according to the following scaling:

The bond default rate per period (matched to the one-year-default rate) as lower boundary

$$\frac{\tilde{L}_1^k}{\tilde{L}_1^{Bond_{high}}} \times \tilde{L}_j^{Bond} \tag{0.36}$$

and the bond default rate per period (matched to the seven-year-default rate) as upper boundary

$$\frac{\tilde{L}_7^k}{\tilde{L}_7^{Bond_{low}}} \times \tilde{L}_j^{Bond}, \tag{0.37}$$

where the exponential growth of default losses allocated to CLO tranches suggest to use the high expected loss of a lower rated bond as a matching first moment for the seven-year-default rate and the low default rate of a higher rated bond as a matching first moment for the one-year-default rate. The approximation of default rate patterns of zero-bonds and CLO tranches establishes an orientation as to the lower and upper boundaries of the CLO term structure if it had the same expected loss properties as zero-bonds.

Thus, the following steps have been completed:

(i) comparison of default rates of varying rating classes of zero-bonds (Moody's) and the estimated expected default based on NID and EVT distributions *(and consideration of deteriorating and improving collateral quality)*

[13] See also Wilson and Fabozzi (1995).

(ii) comparison of calculated spreads (term structure) of zero-bonds and the expected return for the different CLO tranches, purely based on risk-neutral returns *(and consideration of deteriorating and improving collateral quality)*

Figures A10 and A11 exhibit matched first moments of the expected losses (default rate term structure) of both various rating classes of zero-bonds and the CLO tranches, with Table 19 illustrating matched first moments of upper and lower boundaries of risk-neutral spreads on selected zero-coupon bonds (which have been chosen as close matches in the analysis of the default rate term structure in Figures A1-A7). Figures A10 and A11 illustrate the term structure of default rates for the first loss position taken by the sponsor [0-2.4%] and the "investor tranches" [2.4-3.9%], [3.9-6.5%] and [6.5-9%] for both credit risk loss functions (NID and EVT) for a constant forward default rate across seven periods on a cumulative basis. The same methodology has been extended for an increasing and decreasing forward default rate of an EVT loss function.

Against the background of a particular probability distribution the term structure of a CLO tranche is edged by the default term structures of two zero-coupon bonds as upper and lower boundary of collateral default. In contrast to zero-bonds, whose periodic default rate increases linearly over time, structured default tolerance rises exponentially over time for all tranches but the most junior tranche (first loss position) [0-2.4%]. The "investor" tranches display a similar degree of convexity for both probability distributions of collateral default, which contrasts sharply with the linear (and in some cases concave) increase of cumulative expected losses of the first loss provision held by the sponsor of the CLO transaction.

In as far as the previous comparison of default term structures allows for the identification of benchmark zero-coupon bonds, the observed difference of risk-neutral returns on CLO tranches and suitable zero-coupon bonds (whose spreads have been derived on the basis of the same risk-neutral return calculation) reflects the exponential nature of expected losses associated with the peculiar loss cascading effect of CLOs as a *pars pro toto* of structured transactions with subordination as credit enhancement. Table 20 shows the risk-neutral returns of the CLO tranches, whose matching upper and lower boundaries of selected benchmark zero-coupon bonds are consistent across different default probability distributions of varying forward rates of default. In the case of a constant forward rate simulated risk-neutral returns display the smallest degree of deviation from return expectations for linearly increasing cumulative defaults of zero-coupon bonds, where the EVT distribution makes a strong bid for attention as it commands higher spreads than the NID default probability distribution for the first loss position [0-2.4%] and the most senior "investor tranche" [6.5-9%]. It has to be noted that the bond spreads do not attract reference as a first moment match with the CLO tranches as opposed to the "matched comparison" the term structure of default rates between zero-coupon bonds and CLO tranches.

Moody's rating	**Zero bond returns per rating class** *(for a constant risk-free interest rate of 5%)*	**CLO benchmark**
Aaa	5.00074%	
Aa1	5.00771%	
Aa2	5.01596%	
Aa3	**5.03278%**	**upper bound Tranche 4**
A1	**5.05892%**	**upper bound Tranche 4**
A2	5.10357%	
A3	**5.16282%**	**upper bound Tranche 3**
Baa1	5.24663%	
Baa2	**5.35818%**	**lower bound Tranche 3 & Tranche 4**
Baa3	5.65911%	
Ba3	**6.10601%**	**upper bound Tranche 2**
B1	**6.73539%**	**lower bound Tranche 2**
B2	7.47054%	
B3	8.41114%	
Caa	9.52890%	

Table 19. *Risk-neutral returns on zero-coupon bonds given their default term structure (Moody's Investor Services), matched with the CLO tranches in first moment in either the first or seventh period.*

Distribution and Collateral Performance		**Returns per Tranche**					
Constant	$\tilde{L}_j^k$ *cum. (EVT/const.)*	20.34747%	6.28610%	5.25952%	5.06849%	5.03000%	5.00140%
	$\tilde{L}_j^k$ *cum. (NID/const.)*	20.56017%	6.79431%	5.33870%	5.05067%	5.01100%	5.00010%
Deteriorating	$\tilde{L}_j^k$ *cum. (EVT/deter.)*	37.49613%	10.81930%	5.69241%	5.11364%	5.03970%	5.00160%
	$\tilde{L}_j^k$ *cum. (NID/deter.)*	30.43862%	10.19510%	6.78903%	5.39333%	5.10090%	5.00120%
Improving	$\tilde{L}_j^k$ *cum. (EVT/improv.)*	49.12123%	9.87893%	5.60218%	5.10820%	5.03820%	5.00150%
	$\tilde{L}_j^k$ *cum. (NID/improv.)*	42.75610%	10.94410%	6.00440%	5.06577%	5.00640%	5.00000%
Benchmark Zero Bonds	*upper bound*	20.15625% (Caa)	6.73539% (Ba2)	5.35818% (Baa2)	5.05892% (A1)		
	lower bound	10.33656% (B3)	6.10601% (Ba1)	5.16282% (A3)	5.03278% (Aa3)		

Table 20. *Comparison of risk-neutral returns of CLO tranches for different distributions of credit loss on a cumulative basis with a constant, increasing and decreasing forward rate of default.*

12 INTRODUCTION OF STOCHASTIC RISK-FREE INTEREST RATES

In simulation of the interest rate r_{f_t}, we need to distinguish between two cases: (i) a variable (stochastic) risk-free interest rate based on the fitted distribution of observed LIBOR rates and (ii) a constant risk-free rate, which is the average value of the stochastic interest rate across time. In this section we allow for a varying risk-free interest rate per period. Due to the egression of the United Kingdom from the European exchange rate system (European Monetary System (EMS)) on 16.09.1992 (JORION, 2001),[14] we restrict the database of interest rates to 12-month LIBOR rates quoted at the daily market's closing from 04.00.1993 to 02.10.2001 in order to avoid a "change point" in the time series of observed daily interest rates.[15] The observed data points do not display significant historical bias ("momentum effect") and heteroscedasticity is low, such that they can be safely regarded independent and identically distributed. Since only the first 1,000 observations contain 460 zero returns, the simulation of stochastic interest rates for the given investment horizon in the presented model requires the transformation of daily LIBOR rates to end-of-the week quotes. This methodology does not harm the statistical validity of extrapolating future interest rates, as the intra-week rates do not fluctuate, so that a particular end-of-week effect of daily 12-month LIBOR rates can be confidently ruled out. After this conversion of daily rates will are still left with 447 observations to substantiate the simulation.

12.1 Interest Rate Model

Generally speaking, interest rate models focus on extrapolating the development of returns on fixed income securities in relation to their maturity. The most common approach would assume one or more factors to explain the interest rate term structure. After the time-varying dynamics of a single- or multi-factor model have been specified the imposition of certain

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14 One could also argue in favour of using observations only after the December 1995 Madrid Summit, mainly because it was then that a concrete timetable for the introduction of the euro was agreed upon and much of the detailed preparatory work was set in motion. At this point in time, some have argued, the convergence process of European monetary policy commences, as the implications of the 1992 ERM crisis gradually began to be offset by visible evidence of practical advances in the introducing the euro.

15 This starting date of the time series was chosen insofar as some time is needed for the event (Great Britain left the European Monetary System (EMS) on 16th September 1992) to manifest itself in the new model.

expectation hypotheses yields an explicit result for future interest rates. Thus, we employ the interest rate model proposed by Hull and White (HULL, 1993):

$$d \ln r = [q(t) - a \ln r] dt + s\, dz \qquad (0.38)$$

Since the model considers logarithmic interest rates $d \ln r$ instead of nominal r, we guard against negative interest rates that might arise in the course of the analysis. Moreover, mean reversion is permitted for $a > 0$. We substitute the constant m for the term structure parameter $q(t)$, given that the objective of this exercise is predicated on the simulation of the 12-month interest rate at a certain point rather than an entire yield curve. The conversion of this discretisation yields a AR(1) process with $a > 0$:

$$\begin{aligned} \ln r_{t+1} - \ln r_t &= q(t) - a \ln r_t + s e_t = m - a \ln r_t + s e_t \\ \ln r_{t+1} &= m - a \ln r_t + s e_t + \ln r_t = m + (1-a) \ln r_t + s e_t \\ \ln r_{t+1} &= m + a \ln r_t + s e_t, \end{aligned} \qquad (0.39)$$

where we abstain from imposing normality on the independent, uniformly distributed values of e_t, because no option prices are determined in the course of this analysis.

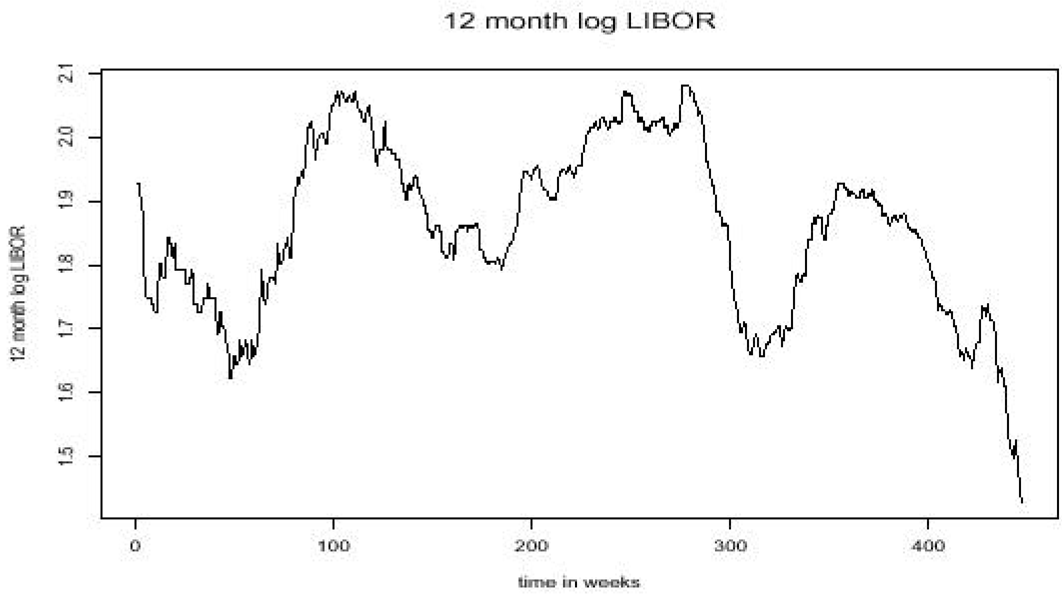

Figure 3. *The time series of logarithmic end-of-week LIBOR interest rates*

The plot of logarithmic end-of-week LIBOR interest rates exhibits only a weak form of mean reversion, since it takes about two years on average until the level of LIBOR rates has returned to the original base level. Consequently, the time series of the returns of 12-month LIBOR rates

$$R_t = \frac{z_t - z_{t-1}}{z_{t-1}} \text{ in } \hat{z}_t = \left(1 + \hat{m} + \hat{s}e\right) z_{t-1} \tag{0.40}$$

is at the brink of non-stationarity, given the weak mean reversion of the LIBOR time series on almost nine years of historic data results in an a value close to 0. We use maximum likelihood estimation to determine the parameters of the probability distribution of AR(1) residuals of logarithmic interest rates.

Due to the weak mean reversion of the given LIBOR rates the ML estimation proves to be inconclusive even with the YULE WALKER starting estimator. We have left the matter at the robust YULE WALKER estimator $\hat{a}$ for a . The results are illustrated in Table 21 below.

parameter	$\hat{a}$	$\hat{a}$
estimator	0.977686	0.022314
std. error	0.027836	0.027836
t-value	35.12	0.80

Table 21. *Yule Walker estimation results for mean reversion.*

The residuals $R_t = \ln r_t - \hat{a} \ln r_{t-1}$ are clearly heavy tailed, as the deviation of a line signifying the empirical quantiles against standard normal quantiles in the Q-Q plot (Fig. 4) below demonstrates.

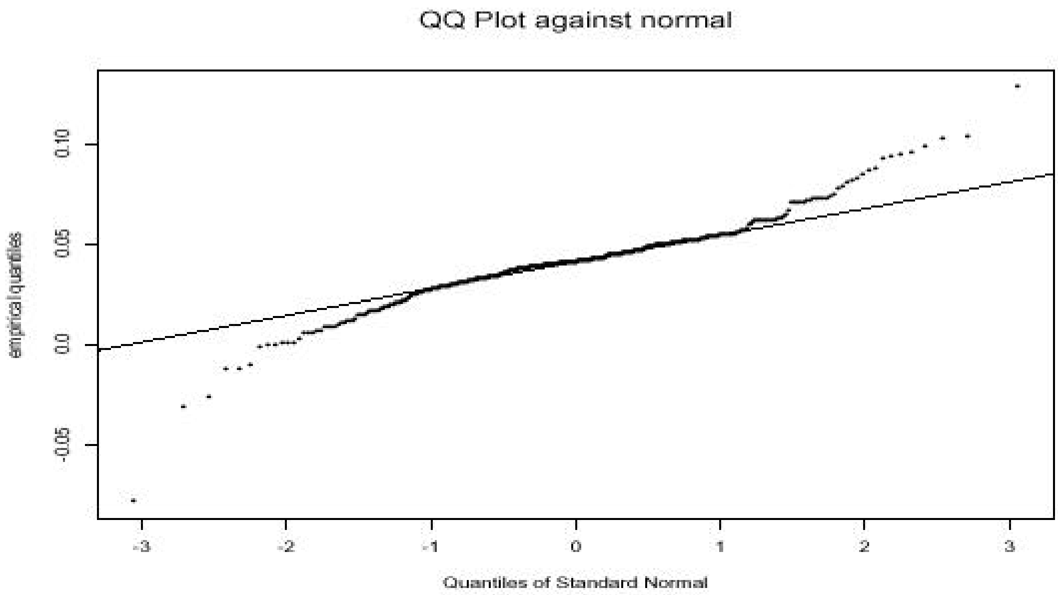

Figure 4. *Q-Q plot of observed distribution of LIBOR rates and a standard normal distribution.*

Nonetheless, the tails, i.e. the positive and negative deviation of the observed distribution of LIBOR rates from a standard normal distribution, appear to be symmetric, we have fitted a t-distribution on the mean-adjusted and scaled residuals of the observed data points[16], such that

$$R_t = m + se_t \quad \text{for } e_t \sim t_n . \tag{0.41}$$

At this point one could certainly consider more advanced interest rate models, such as the Cox-Ingersoll-Ross (CIR) model (1985). In the CIR model[17] we would assume R_t to be the

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[16] See also CAMPELL AND MACKINLAY (1997); SANDMANN (1999).

[17] The CIR model assumes $P_T(t)$ to be the price of a zero bond with a nominal value set to 1, i.e. a non-dividend paying security, which pays exactly 1 at maturity *T*. The so-called "yield-to-maturity (YTM)" $Y_T(t)$ denotes the logarithmic return of the zero bond at continuous compounding of interest, such that the relationship between the bond price and its investment return is $P_T(t) = \exp\{-Y_T(t)t\}$ for the remaining maturity $t = T - t$. The short rate is defined as $r(t) = \lim_{T \to t} Y_T(t)$ and is considered the most important estimator value of the interest rate term structure. Since $r(t)$ represents the sport rate at the limit for $T \to t$, the expected price of the zero bond at time *t* is $P_T(t) = E\left[\exp\left(-\int_t^T r(s)\,ds \middle| \Omega_t\right)\right]$ for a variable but deterministic interest rate and past prices Ω_t. Thus, $P_T(t)$ represents the present value of a zero bond over its lifetime with a constant return rate of $Y_T(t)$ or repeated investment in zero bonds in future periods $t-1,\ldots,T$ at a risky short-term rate. The CIR model includes the short-term interest rate as a factor (as most one-factor models), whose dynamics is specified as a time-continuous stochastic process $dr(t) = a(b - r(t))dt + s\sqrt{r(t)}dW_t$, with a Wiener process W_t and constant parameters *a,b* and s .

price of a zero bond $P_T(t)$ with a nominal value set to 1, i.e. a non-dividend paying security, which pays exactly 1 at maturity *T*. We include the short-term interest rate as a factor (as most one-factor models), whose dynamics is specified as a time-continuous stochastic process

$$dr(t) = a\big(b - r(t)\big)dt + s\sqrt{r(t)}dW_t, \tag{0.42}$$

with a Wiener process W_t and constant parameters *a,b* and s. The volatility $s\sqrt{r(t)}$ increases in the interest rate level, and $r(t)$ is specified as a Markov process, such that $P_T(t)$ becomes a function of the current short-term rate[18],

$$P_T(t) = V\big(r(t), t\big). \tag{0.43}$$

Thus, the complete CIR model reads as

$$P_T(t) = V\big(r(t), t\big) = \exp\big\{A(T - t) + B(T - T)r(t)\big\}, \tag{0.44}$$

where

$$y = \sqrt{a^2 + 2s^2} \tag{0.45}$$

and

$$g(t) = 2y + (a + y)\{\exp(yt) - 1\}, \tag{0.46}$$

such that

$$A(t) = \frac{2ab}{s^2}\ln\frac{2y\exp\left\{(a+y)\frac{t}{2}\right\}}{g(t)} \tag{0.47}$$

and

The volatility $s\sqrt{r(t)}$ increases in the interest rate level, and $r(t)$ is specified as a Markov process, such that $P_T(t)$ becomes a function of the current short-term rate, $P_T(t) = V\big(r(t), t\big)$. We consider the Wiener process mean reverting as a positive value for *a* offsets any deviation from the stationary mean value *b*. By using Itos Lemma we obtain the differential equation $a(b-r)\frac{\partial V(r,t)}{\partial r} + \frac{1}{2}s^2 r\frac{\partial^2 V(r,t)}{\partial r^2} + \frac{\partial V(r,t)}{\partial t} - rV(r,t) = 0$ with the boundary condition $V(r,T) = P_T(T) = 1$. This set of equations yields the above CIR model (eq. 0.37).

18 See also DUFFIE (1997); KARATZAS AND SHREVE (1991); WILMOTT, HOWISON AND DEWYNNE (1995); BENNINGA (1997).

$$B(t) = \frac{2\{1 - \exp(y t)\}}{g(t)}.^{[19]} \quad (0.48)$$

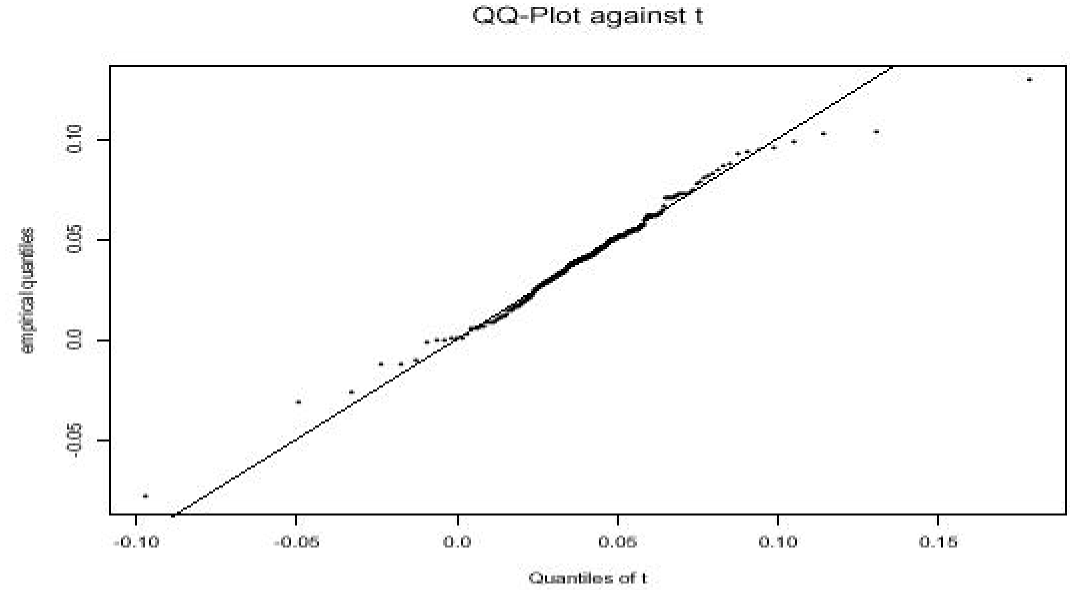

Figure 5. *Q-Q plot of observed residuals and t-distribution*

The estimated parameters for the 12-month LIBOR interest rate are as follows:

	$\hat{m}$	$\hat{s}$	$\hat{n}$
estimator	0.040622	0.011517	2643781
std. error	0.000674	0.000791	0.419292
t-value	60.26	14.55	3.92

Table 22. *Estimation of residuals for a fitted t-distribution on 12-month LIBOR*

The estimated chi-square statistic c_{19}^{2} is 24.52, which equates to a p-value of 17.69%.

[19] The interest rate term structure, i.e. the yield-to-maturity, converges to $Y_{\lim} = \frac{2ab}{y + a}$ as the remaining maturity $T - t$ becomes larger. If the short-term interest rate $r(t) > b$, the first moment of the interest rate term structure is negative, whereas $r(t) < Y_{\lim}$ indicates rising future interest rates. If the short-term interest rate is $b < r(t) < Y_{\lim}$, the yield curve could increase before a negative second moment induces declining interest rate in the long-term.

12.2 Simulation

We simulate one million paths of estimated LIBOR interest rates over seven years, i.e. 350 time increments, based on the presented model. Figure 6 is an exemplary exposition of a few paths.

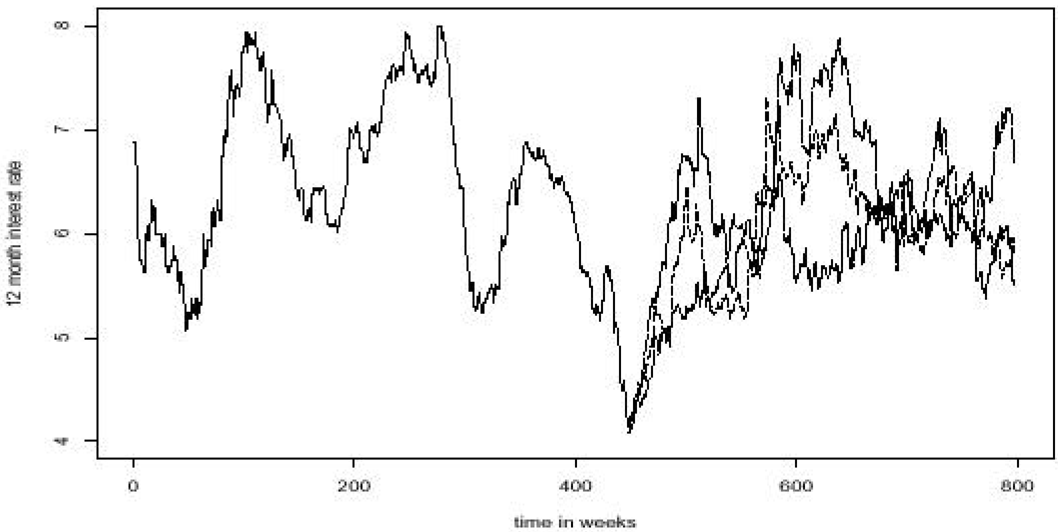

Figure 6. *Simulation of LIBOR rates following* HULL AND WHITE *(1993)*

Table 23 below shows the estimated spreads per tranche investors ought to demand, if stochastic risk-free interest rates (12-month LIBOR) averaged over one million iterations of the suggested interest rate model are applied to the risk-neutral pricing of CLOs (eq. 0.19). Analogous to the pervious section, when a constant risk-free rate of 5.0% is used, spreads are calculated for both models of credit risk (EVT and NID) and three possibilities of portfolio quality (constant, deteriorating, improving). For illustrative purposes investor spreads per tranche have also been calculated with the average stochastic interest-rate (Tab. 24) as the constant risk-free rate

$$\exp\left(\frac{m}{a}\right) = 6.174769\% \tag{0.49}$$

In the case of a fixed risk-free rate (see Section 8), we have maximised the discount factor $\prod_{l=1}^{m}\left(1+r_{f_l}\right)$ with the summation of risk-free interest rates r_{f_l} per period held constant, such that $r_{f_l} = r_{f_{l+1}}$. In this section stochastic interest rates are introduced. Even though the summation of stochastic interest rates per period *j* equates to the summation of constant risk-free interest rates $\sum_{l=1}^{m}\left(1+r_{f_l}\right)$ due to mean reversion for $t \rightarrow \infty$ in general and $m \rightarrow \infty$ in our model, stochastic interest rates vary over time, such that $r_{f_l} \neq r_{f_{l+1}}$. Thus, if we substitute stochastic periodic interest rates for the constant risk-free interest rates r_{f_l}, $\prod_{l=1}^{m}\left(1+r_{f_l}\right)$ is generally smaller for stochastic interest rates than for constant interest rates. If we recapitulate the term structure of default rates for each tranche of a CLO transaction (eq. 0.19), the risk-neutral spreads need to be lower for stochastic interest rates to offset a smaller value in the denominator.

Distribution and Collateral Performance		Returns per Tranche (at avg. stochastic risk-free rate)					
Constant	$\tilde{L}_j^k$ *cum. (EVT/const.)*	22.39463%	7.30974%	6.29268%	6.10283%	6.06451%	6.03589%
	$\tilde{L}_j^k$ *cum. (NID/const.)*	20.62188%	7.81517%	6.37059%	6.08477%	6.04547%	6.03454%
Deteriorating	$\tilde{L}_j^k$ *cum. (EVT/deter.)*	38.26656%	10.74418%	6.71721%	6.14718%	6.07401%	6.03609%
	$\tilde{L}_j^k$ *cum. (NID/deter.)*	30.38123%	12.15254%	7.79993%	6.42232%	6.13398%	6.03570%
Improving	$\tilde{L}_j^k$ *cum. (EVT/improv.)*	50.15165%	10.84695%	6.63082%	6.14206%	6.07255%	6.03599%
	$\tilde{L}_j^k$ *cum. (NID/improv.)*	43.89018%	10.92903%	6.92431%	6.09964%	6.04086%	6.03446%

Table 23. *Expected spreads per tranche based on the average variable (stochastic) risk-free rate as constant discount rate*

Distribution and Collateral Performance		Returns per Tranche (at variable risk-free rate)					
Constant	$\tilde{L}_j^k$ *cum. (EVT/const.)*	22.62031%	7.45478%	6.43406%	6.24347%	6.20499%	6.17623%
	$\tilde{L}_j^k$ *cum. (NID/const.)*	20.84458%	7.96229%	6.51219%	6.22531%	6.18585%	6.17488%
Deteriorating	$\tilde{L}_j^k$ *cum. (EVT/deter.)*	38.56835%	10.90002%	6.85962%	6.28793%	6.21451%	6.17644%
	$\tilde{L}_j^k$ *cum. (NID/deter.)*	30.65209%	12.31435%	7.94590%	6.56384%	6.27462%	6.17605%
Improving	$\tilde{L}_j^k$ *cum. (EVT/improv.)*	50.54813%	10.00315%	6.77326%	6.28283%	6.21305%	6.17634%
	$\tilde{L}_j^k$ *cum. (NID/improv.)*	44.25293%	12.09255%	7.06783%	6.24022%	6.18122%	6.17480%

Table 24. *Expected spreads per tranche based on the periodic stochastic/variable risk-free rate*

However, the effect of variable (stochastic) interest rates is not only confined to a lower level of spreads investors will demand in return for periodic defaults in each tranche. The implications of a lower periodic discount rate $\prod_{l=1}^{m}\left(1+r_{f_l}\right)$ will be more pronounced the more time has elapsed in a CLO transaction. This has a significant bearing on the different term structures of default rates of the first loss position (constant expected losses per period) and the "investor tranches" (non-linear increase of expected losses per period). The interaction of term structures and stochastic interest rates means that spreads for the first loss position will decline to a higher extent than for mezzanine and senior tranches as we permit the periodic discount rate $\prod_{l=1}^{m}\left(1+r_{f_l}\right)$ to vary across time by a introducing stochastic risk-free interest rate.

12.3 Economic explanation

Since we benchmark the term structure of default for each tranche of the CLO transaction to comparable zero bonds, we might resort to the pricing behaviour of bonds to explain the difference of spreads between constant and stochastic risk-free interest rates. As the intrinsic value of volatility in time-varying interest rates induce a higher bond price than constant interest rates, investment in bonds is safer under a constant discount rate per period. Consequently, a constant interest rate leads to a marginal increase of the periodic discount rate $\sum_{l=1}^{m}\left(1+r_{f_l}\right)$, which requires higher investment spreads according to equation 0.19. Thus, the calculated spreads of each tranche ought to be lower for stochastic interest rates than under constant interest rates.

13 CONCLUSION – MODELLING THE LOSS FUNCTION AND LOSS CASCADING

The presented analysis purposted to a comprehensive illustration of loan securitisation from the perspective of the default term structure and the pricing of CLO tranches. We base our methodology on divergent loss accumulation of tranches of debt securities issued on loan portfolios due to the priorisation of cumulative loan loss most commonly found in subordinated structures of securitisations. This so-called loss cascading mechanism was modelled by transforming simulating uniformly distributed periodic (annual) random portfolio losses into two prime cumulative distributions, an (i) extreme value theory loss function and a (ii) normal inverse distribution function. Furthermore both a beta distribution as well as a negative binomial distribution of loan default augment this determination of default term structures in comparative terms. The simulation was augmented by also taking into consideration normally distributed loss recovery. By virtue of a reality check on the tranche pricing for the simulations results of portfolio loss assumed accurate we privilege equally concerns about contingencies of estimation risk and state variable uncertainty, on the one hand, and the economic virtues of viable and realistic model specification, on the other hand.

We identified a dichotomy of estimated losses per tranche between the first loss position and the "investors tranches". Whereas the term structure of cumulative loan loss increase linearly with respect to the first tranche, the remaining tranches are subjected to an exponentially increasing loss burden and follow a convex term structure. The latter aspect translates into excess spreads for tranches issued to investors, as the pricing of the transaction on the grounds of risk-neutral default-probabilities corresponds to a more convex term structure compared to corporate bonds with the same average life and similar rating. This observation is underpinned by the relevance of the variance of default losses (cf. $s_{\tilde{L}_j^k} \big/ \tilde{L}_j^k$ ratio), whose impact on estimated investor returns increases exponentially for tranches beyond the first loss position. Considering the structural features of securitisation transactions this result bears critical relevance to any model of securitisation in the context of asymmetric information as the issuer is compelled to retain a portion of the low risk loan category for the market to remain in equilibrium ex ceteris paribus. Consequently, the retention of the lowest tranche eventuates not only enhanced calibration of measures to manage credit risk exposure but also the transfer of additional risks and higher variance of loss severity to investors. Given the high level of cumulative probability (stochastic weight) associated with extreme events in the case of the extreme value theory loss function, its approximation of extreme type behaviour of credit losses is superior to the tail behaviour of the normal inverse distribution as well as the beta- and negative binomial distribution, which shows almost perfect coincidence of probability masses. It warrants noting that extreme value theory improves on the modelling of default risk by means of the normal inverse distribution as the variance of unexpected loss in relation to scheduled defaults in the collateral portfolio is significantly reduced, while most losses accrue to the first loss credit default cover of the most junior tranche. Nonetheless, even if approximating portfolio performance across time depends on the chosen loss function, the effect of loss cascading remains to be consistently most pronounced in the first loss position across all distributions. As opposed to the beta- and negative-binomial distribution, which yield a significantly higher estimated risk exposure of the CLO tranches after the respective parameters have been calibrated by matching the first two moments, the extreme value theory loss function concentrates an escalation of expected cumulative losses on the most junior tranche (especially if the forward rate is flexible as to reflect time-varying collateral portfolio quality), whilst marginal periodic losses display asymptotic properties. The implications of variable portfolio quality on the cumulative and periodic loss burden per tranche reinforce the conclusion that the partial retention of assets of the collateral pool does not only reflect adverse effects of information asymmetries in the securitisation market. Bank issuers also have the ability to subdivide and redirect cash flows from underlying assets among a range of sold and retained interests in order to reduce their total risk exposure in linear future default and to enhance tranche spreads allocated to investors. Given this structural discretion in security design on part of issuers, the management of unexpected risk represents the decisive element in the selection of the securitised loans, i.e. the credit risk of the collateral portfolio.

Since 60 to 95 % (depending on the modelled loss function of the loan portfolio) of the default loss is concentrated in the first loss position (whose level is subject to the willingness of the issuer to provide credit cover), the aforementioned incentives for securitisation persist, recent financial innovation in the area of loan securitisation has drawn increased attention to the possibility of subparticipation.[20]

[20] The mechanism of interest subparticipation has been devised by issuers to reduce the illiquidity of the first loss piece of securitisation transactions in order to ameliorate the marketability of the credit enhancement held by the sponsor of the transaction as an equity tranche. Payments out of available interest generated from the overall reference portfolio are partially used to offset first losses of noteholders of the first loss position. By doing so, the principal amount of the outstanding first loss piece is reduced through the amount of interest subparticipation, in an amount equal to the allocated realised losses. Even though the claim of first loss noteholders to the interest subparticipation is an unsecured claim against the issuer, the economic rationale behind this concept is regulatory capital relief, as no capital has to be held against interest income under the current regulatory standards. Since the first loss piece achieves the rating of the issuer, the placement of credit enhancement under interest subparticipation is cost efficient. However, the capital efficiency derived from such an arrangement is associated with substantial institutional risk in view of potential future changes in the regulatory framework, which has hitherto not given clear guidance on the capital treatment of the concept of

Despite of regulatory change ("internal ratings-based approach"), which renders obsolete efforts of institutional arbitrage between minimum capital requirements and economic capital associated with a certain risk category of loans, the continuous presence of exogenous non-systematic risk warrants limiting the impact of unexpected risk. Once linear expected losses allocated to the credit enhancement are accounted for by means of the security design, the benefit of the securitisation process derives from the removal of unexpected losses from the bank loan book, provided that possible efficiency loss due to adverse selection in the securitisation of illiquid loans matches the remedial effect of the credit enhancement. The lower the error margin of modelled periodic credit losses the more accurate risk-neutral calculation of tranche spreads will be. With the gap between risk-neutral spreads and observed prices of CLO tranches gradually narrowing as we estimate a reduction of unexpected loss in proportion to unexpected loss of the securitisation transaction, the model predictions militate towards CLO pricing in efficient markets. Nonetheless, the significant decline of marginal unexpected losses over time in the first loss position held by the issuer/sponsor of the transaction allows issuers more predictable investment risk than capital market investors, who hold the mezzanine and senior tranches (investor tranches). In view of inherent intransparency of actual asset quality in current CLO markets (especially in bank-based financial systems), an extension of this approach to unexpected loss is instrumental in estimating the term structure of CLOs in a sufficiently liquidity market for bank loans. Therefore, a consideration of integrating sophisticated derivative structures and comprehensive asset pricing methods in the security design of CLOs will further common understanding of the term structure and the pricing of CLO transactions.

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interest subparticipation in the provision of credit enhancement. The new proposal for a revision of the Basel Accord indicates the possibility that the fist loss position will most likely be subjected to a full deduction from capital in this thinly regulated area of structured finance. Given present regulatory uncertainty as to the future capital treatment of structural provisions, such credit enhancement and the interest subparticipation, it is worthwhile incorporating a regulatory call of the first loss piece, which allows for the possible restructuring and subsequent sale of the most junior tranche to capital market investors.

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15 APPENDIX

			Expected and unexpected losses		$\tilde{L}_j$ per tranche (in % of tranche volume)						$\tilde{L}_j^k$ per tranche (abs. share of total exp. losses per period)					
cum./per.	**Yr**	p	$\tilde{L}_j^k$	$s_{\tilde{L}_j^k}$	**0-2.4%**	**2.4-3.9%**	**3.9-6.5%**	**6.5-9%**	**9-10.5%**	**10.5-100%**	**0-2.4%**	**2.4-3.9%**	**3.9-6.5%**	**6.5-9%**	**9-10.5%**	**10.5-100%**
cumulative	1	0.0026	**0.002598**	0.004581	0.104285	0.003095	0.000987	0.000375	0.000188	0.000010	0.002503	0.000046	0.000026	0.000009	0.000005	0.000009
periodic			**0.002598**	*0.004581*	*0.104285*	*0.003095*	*0.000987*	*0.000375*	*0.000188*	*0.000010*	*0.002503*	*0.000046*	*0.000026*	*0.000009*	*0.000005*	*0.000009*
cumulative	2	0.0026	**0.005299**	0.006512	0.207327	0.007653	0.002229	0.000797	0.000402	0.000023	0.004976	0.000115	0.000058	0.000020	0.000010	0.000020
periodic			**0.002701**	*0.001931*	*0.103042*	*0.004558*	*0.001242*	*0.000422*	*0.000214*	*0.000013*	*0.002473*	*0.000068*	*0.000032*	*0.000011*	*0.000005*	*0.000012*
cumulative	3	0.0026	**0.007799**	0.007978	0.308168	0.014546	0.003940	0.001332	0.000648	0.000037	0.007396	0.000218	0.000102	0.000033	0.000016	0.000033
periodic			**0.002500**	*0.001466*	*0.100841*	*0.006893*	*0.001711*	*0.000535*	*0.000246*	*0.000014*	*0.002420*	*0.000103*	*0.000044*	*0.000013*	*0.000006*	*0.000012*
cumulative	4	0.0026	**0.010397**	0.009228	0.406098	0.024665	0.006204	0.001994	0.000952	0.000051	0.009746	0.000370	0.000161	0.000050	0.000024	0.000045
periodic			**0.002598**	*0.001250*	*0.097930*	*0.010119*	*0.002264*	*0.000662*	*0.000304*	*0.000014*	*0.002350*	*0.000152*	*0.000059*	*0.000017*	*0.000008*	*0.000012*
cumulative	5	0.0026	**0.012990**	0.010317	0.500079	0.039278	0.009131	0.002777	0.001292	0.000067	0.012002	0.000589	0.000237	0.000069	0.000032	0.000059
periodic			**0.002593**	*0.001089*	*0.093981*	*0.014613*	*0.002927*	*0.000783*	*0.000340*	*0.000016*	*0.002256*	*0.000219*	*0.000076*	*0.000020*	*0.000009*	*0.000014*
cumulative	6	0.0026	**0.015581**	0.011252	0.589083	0.060005	0.012995	0.003645	0.001649	0.000082	0.014138	0.000900	0.000338	0.000091	0.000041	0.000073
periodic			**0.002591**	*0.000935*	*0.089004*	*0.020727*	*0.003864*	*0.000868*	*0.000357*	*0.000015*	*0.002136*	*0.000311*	*0.000100*	*0.000022*	*0.000009*	*0.000013*
cumulative	7	0.0026	**0.018168**	0.012104	0.671323	0.088676	0.018083	0.004711	0.002052	0.000098	0.016112	0.001330	0.000470	0.000118	0.000051	0.000087
periodic			**0.002587**	*0.000852*	*0.082240*	*0.028671*	*0.005088*	*0.001066*	*0.000403*	*0.000016*	*0.001974*	*0.000430*	*0.000132*	*0.000027*	*0.000010*	*0.000014*

			Expected and unexpected losses		$s_{\tilde{L}_j^k}$ per tranche (in % of tranche volume)						$s_{\tilde{L}_j^k}$ per tranche (abs. share of total exp. losses per period)					
cum./per.	**Yr**	p	$\tilde{L}_j^k$	$s_{\tilde{L}_j^k}$	**0-2.4%**	**2.4-3.9%**	**3.9-6.5%**	**6.5-9%**	**9-10.5%**	**10.5-100%**	**0-2.4%**	**2.4-3.9%**	**3.9-6.5%**	**6.5-9%**	**9-10.5%**	**10.5-100%**
cumulative	1	0.0026	0.002598	**0.004581**	0.130941	0.050337	0.028247	0.018032	0.013013	0.001526	0.003143	0.000755	0.000734	0.000451	0.000325	0.001351
periodic			*0.002598*	**0.004581**	*0.130941*	*0.050337*	*0.028247*	*0.018032*	*0.013013*	*0.001526*	*0.003143*	*0.000755*	*0.000734*	*0.000451*	*0.000325*	*0.001351*
cumulative	2	0.0026	0.005299	**0.006512**	0.178268	0.078295	0.041886	0.026340	0.019017	0.002261	0.004278	0.001174	0.001089	0.000659	0.000475	0.002001
periodic			*0.002701*	**0.001931**	*0.047327*	*0.027958*	*0.013639*	*0.008308*	*0.006004*	*0.000735*	*0.001136*	*0.000419*	*0.000355*	*0.000208*	*0.000150*	*0.000650*
cumulative	3	0.0026	0.007799	**0.007978**	0.208257	0.106408	0.055182	0.033976	0.024088	0.002737	0.004998	0.001596	0.001435	0.000849	0.000602	0.002422
periodic			*0.002500*	**0.001466**	*0.029989*	*0.028113*	*0.013296*	*0.007636*	*0.005071*	*0.000476*	*0.000720*	*0.000422*	*0.000346*	*0.000191*	*0.000127*	*0.000421*
cumulative	4	0.0026	0.010397	**0.009228**	0.226551	0.136470	0.068699	0.041584	0.029081	0.003191	0.005437	0.002047	0.001786	0.001040	0.000727	0.002824
periodic			*0.002598*	**0.001250**	*0.018294*	*0.030062*	*0.013517*	*0.007608*	*0.004993*	*0.000454*	*0.000439*	*0.000451*	*0.000351*	*0.000190*	*0.000125*	*0.000402*
cumulative	5	0.0026	0.012990	**0.010317**	0.235359	0.169272	0.082556	0.048863	0.033732	0.003616	0.005649	0.002539	0.002146	0.001222	0.000843	0.003200
periodic			*0.002593*	**0.001089**	*0.008808*	*0.032802*	*0.013857*	*0.007279*	*0.004651*	*0.000425*	*0.000211*	*0.000492*	*0.000360*	*0.000182*	*0.000116*	*0.000376*
cumulative	6	0.0026	0.015581	**0.011252**	0.235536	0.205001	0.097288	0.055601	0.038068	0.003977	0.005653	0.003075	0.002529	0.001390	0.000952	0.003520
periodic			*0.002591*	**0.000935**	*0.000177*	*0.035729*	*0.014732*	*0.006738*	*0.004336*	*0.000361*	*0.000004*	*0.000536*	*0.000383*	*0.000168*	*0.000108*	*0.000319*
cumulative	7	0.0026	0.018168	**0.012104**	0.227853	0.243130	0.113521	0.062720	0.042430	0.004301	0.005468	0.003647	0.002952	0.001568	0.001061	0.003806
periodic			*0.002587*	**0.000852**	*-0.007683*	*0.038129*	*0.016233*	*0.007119*	*0.004362*	*0.000324*	*-0.000184*	*0.000572*	*0.000422*	*0.000178*	*0.000109*	*0.000287*

Table 25. *Simulation of constant forward probability rates (extreme value theory loss function as distribution of portfolio losses) of default losses on a cumulative and periodic basis – losses per tranche with either the tranche % or the absolute value of losses per period as reference base.*

			Expected and unexpected losses		$\tilde{L}_j^k$ per tranche (in % of tranche volume)						$\tilde{L}_j^k$ per tranche (abs. share of total exp. losses per period)					
cum./per.	**Yr**	$\hat{p}$	$\tilde{L}_j^k$	$s_{\tilde{L}_j^k}$	**0-2.4%**	**2.4-3.9%**	**3.9-6.5%**	**6.5-9%**	**9-10.5%**	**10.5-100%**	**0-2.4%**	**2.4-3.9%**	**3.9-6.5%**	**6.5-9%**	**9-10.5%**	**10.5-100%**
cumulative	1	0.00010	**0.002598**	0.004581	0.104285	0.003095	0.000987	0.000375	0.000188	0.000010	0.002503	0.000046	0.000026	0.000009	0.000005	0.000009
periodic			**0.002598**	*0.004581*	*0.104285*	*0.003095*	*0.000987*	*0.000375*	*0.000188*	*0.000010*	*0.002503*	*0.000046*	*0.000026*	*0.000009*	*0.000005*	*0.000009*
cumulative	2	0.00123	**0.006191**	0.006508	0.248098	0.008190	0.002382	0.000833	0.000413	0.000023	0.005954	0.000123	0.000062	0.000021	0.000010	0.000020
periodic			**0.003593**	*0.004626*	*0.145561*	*0.003188*	*0.001032*	*0.000366*	*0.000197*	*0.000012*	*0.003452*	*0.000076*	*0.000036*	*0.000011*	*0.000006*	*0.000012*
cumulative	3	0.00195	**0.010473**	0.007834	0.416846	0.018131	0.004431	0.001373	0.000653	0.000034	0.010004	0.000272	0.000115	0.000034	0.000016	0.000030
periodic			**0.004282**	*0.004366*	*0.174498*	*0.003318*	*0.000946*	*0.000311*	*0.000158*	*0.000009*	*0.004050*	*0.000149*	*0.000053*	*0.000014*	*0.000006*	*0.000010*
cumulative	4	0.00247	**0.015278**	0.009016	0.598044	0.039707	0.007968	0.002189	0.000976	0.000049	0.014353	0.000596	0.000207	0.000055	0.000024	0.000043
periodic			**0.004805**	*0.004461*	*0.196124*	*0.003444*	*0.000979*	*0.000336*	*0.000171*	*0.000010*	*0.004349*	*0.000324*	*0.000092*	*0.000020*	*0.000008*	*0.000013*
cumulative	5	0.00277	**0.020389**	0.010101	0.770975	0.088717	0.014413	0.003391	0.001416	0.000068	0.018503	0.001331	0.000375	0.000085	0.000035	0.000060
periodic			**0.005111**	*0.004540*	*0.208625*	*0.003570*	*0.001030*	*0.000365*	*0.000192*	*0.000011*	*0.004150*	*0.000735*	*0.000168*	*0.000030*	*0.000011*	*0.000017*
cumulative	6	0.00295	**0.025679**	0.011100	0.908023	0.195662	0.026506	0.005249	0.002007	0.000091	0.021793	0.002935	0.000689	0.000131	0.000050	0.000081
periodic			**0.005289**	*0.004583*	*0.215965*	*0.003606*	*0.001061*	*0.000374*	*0.000194*	*0.000012*	*0.003289*	*0.001604*	*0.000314*	*0.000046*	*0.000015*	*0.000020*
cumulative	7	0.00306	**0.031064**	0.011904	0.981059	0.389878	0.049962	0.008072	0.002774	0.000114	0.023545	0.005848	0.001299	0.000202	0.000069	0.000101
periodic			**0.005386**	*0.004303*	*0.220174*	*0.003614*	*0.001051*	*0.000347*	*0.000173*	*0.000008*	*0.001753*	*0.002913*	*0.000610*	*0.000071*	*0.000019*	*0.000020*

			Expected and unexpected losses		$s_{\tilde{L}_j^k}$ per tranche (in % of tranche volume)						$s_{\tilde{L}_j^k}$ per tranche (abs. share of total exp. losses per period)					
cum./per.	**Yr**	$\hat{p}$	$\tilde{L}_j^k$	$s_{\tilde{L}_j^k}$	**0-2.4%**	**2.4-3.9%**	**3.9-6.5%**	**6.5-9%**	**9-10.5%**	**10.5-100%**	**0-2.4%**	**2.4-3.9%**	**3.9-6.5%**	**6.5-9%**	**9-10.5%**	**10.5-100%**
cumulative	1	0.00010	0.002598	**0.004581**	0.130941	0.050337	0.028247	0.018032	0.013013	0.001526	0.003143	0.000755	0.000734	0.000451	0.000325	0.001351
periodic			*0.002598*	**0.004581**	*0.130941*	*0.050337*	*0.028247*	*0.018032*	*0.013013*	*0.001526*	*0.003143*	*0.000755*	*0.000734*	*0.000451*	*0.000325*	*0.001351*
cumulative	2	0.00123	0.006191	**0.006508**	0.174874	0.080581	0.043399	0.026735	0.019296	0.002297	0.004197	0.001209	0.001128	0.000668	0.000482	0.002033
periodic			*0.003593*	**0.004626**	*0.128459*	*0.050898*	*0.028846*	*0.017801*	*0.013353*	*0.001674*	*0.001054*	*0.000454*	*0.000394*	*0.000218*	*0.000157*	*0.000682*
cumulative	3	0.00195	0.010473	**0.007834**	0.196178	0.116936	0.058216	0.034156	0.024201	0.002717	0.004708	0.001754	0.001514	0.000854	0.000605	0.002405
periodic			*0.004282*	**0.004366**	*0.127236*	*0.051448*	*0.027190*	*0.016506*	*0.011817*	*0.001324*	*0.000511*	*0.000545*	*0.000385*	*0.000186*	*0.000123*	*0.000372*
cumulative	4	0.00247	0.015278	**0.009016**	0.195799	0.166620	0.076333	0.042914	0.029468	0.003226	0.004699	0.002499	0.001985	0.001073	0.000737	0.002855
periodic			*0.004805*	**0.004461**	*0.126626*	*0.052496*	*0.027589*	*0.017024*	*0.012409*	*0.001539*	*-0.000009*	*0.000745*	*0.000471*	*0.000219*	*0.000132*	*0.000450*
cumulative	5	0.00277	0.020389	**0.010101**	0.168668	0.234645	0.100094	0.052945	0.035368	0.003746	0.004048	0.003520	0.002602	0.001324	0.000884	0.003315
periodic			*0.005111*	**0.004540**	*0.126135*	*0.053353*	*0.028524*	*0.017735*	*0.013319*	*0.001588*	*-0.000651*	*0.001020*	*0.000618*	*0.000251*	*0.000148*	*0.000460*
cumulative	6	0.00295	0.025679	**0.011100**	0.113185	0.312582	0.131249	0.065124	0.041864	0.004305	0.002716	0.004689	0.003412	0.001628	0.001047	0.003810
periodic			*0.005289*	**0.004583**	*0.125604*	*0.053776*	*0.028916*	*0.018047*	*0.013306*	*0.001701*	*-0.001332*	*0.001169*	*0.000810*	*0.000304*	*0.000162*	*0.000495*
cumulative	7	0.00306	0.031064	**0.011904**	0.048280	0.358264	0.172091	0.079550	0.048750	0.004631	0.001159	0.005374	0.004474	0.001989	0.001219	0.004098
periodic			*0.005386*	**0.004303**	*0.125246*	*0.053700*	*0.028669*	*0.017346*	*0.012474*	*0.001050*	*-0.001558*	*0.000685*	*0.001062*	*0.000361*	*0.000172*	*0.000289*

Table 26. *Simulation of increasing forward probability rates (extreme value theory loss function as distribution of portfolio losses) of default losses on a cumulative and periodic basis – losses per tranche with either the tranche % or the absolute value of losses per period as reference base.*

			Expected and unexpected losses		$\tilde{L}_j^k$ per tranche (in % of tranche volume)						$\tilde{L}_j^k$ per tranche (abs. share of total exp. losses per period)					
cum./per.	Yr	p	$\tilde{L}_j^k$	$s\ \tilde{L}_j^k$	0-2.4%	2.4-3.9%	3.9-6.5%	6.5-9%	9-10.5%	10.5-100%	0-2.4%	2.4-3.9%	3.9-6.5%	6.5-9%	9-10.5%	10.5-100%
cumulative	1	0.00367	**0.006005**	0.004523	0.245673	0.003844	0.001091	0.000378	0.000182	0.000009	0.005896	0.000058	0.000028	0.000009	0.000005	0.000008
periodic			**0.006005**	*0.004523*	*0.245673*	*0.003844*	*0.001091*	*0.000378*	*0.000182*	*0.000009*	*0.005896*	*0.000058*	*0.000028*	*0.000009*	*0.000005*	*0.000008*
cumulative	2	0.00267	**0.011014**	0.006328	0.445650	0.012637	0.003011	0.000892	0.000419	0.000020	0.010696	0.000190	0.000078	0.000022	0.000010	0.000018
periodic			**0.005009**	*0.004429*	*0.204375*	*0.003659*	*0.001085*	*0.000360*	*0.000178*	*0.000009*	*0.004799*	*0.000132*	*0.000050*	*0.000013*	*0.000006*	*0.000010*
cumulative	3	0.00195	**0.015296**	0.007696	0.608457	0.030299	0.005900	0.001564	0.000693	0.000032	0.014603	0.000454	0.000153	0.000039	0.000017	0.000028
periodic			**0.004282**	*0.004366*	*0.174498*	*0.003318*	*0.000946*	*0.000311*	*0.000158*	*0.000009*	*0.003907*	*0.000265*	*0.000075*	*0.000017*	*0.000007*	*0.000011*
cumulative	4	0.00154	**0.019189**	0.008889	0.742734	0.063996	0.010507	0.002531	0.001046	0.000047	0.017826	0.000960	0.000273	0.000063	0.000026	0.000042
periodic			**0.003894**	*0.004440*	*0.158171*	*0.003325*	*0.001020*	*0.000345*	*0.000169*	*0.000009*	*0.003223*	*0.000505*	*0.000120*	*0.000024*	*0.000009*	*0.000013*
cumulative	5	0.00123	**0.022782**	0.010016	0.846424	0.121583	0.017450	0.003799	0.001493	0.000066	0.020314	0.001824	0.000454	0.000095	0.000037	0.000058
periodic			**0.003593**	*0.004626*	*0.145561*	*0.003188*	*0.001032*	*0.000366*	*0.000197*	*0.000012*	*0.002489*	*0.000864*	*0.000181*	*0.000032*	*0.000011*	*0.000017*
cumulative	6	0.00094	**0.026099**	0.011032	0.917378	0.206659	0.027686	0.005460	0.002017	0.000086	0.022017	0.003100	0.000720	0.000137	0.000050	0.000076
periodic			**0.003317**	*0.004623*	*0.134103*	*0.003262*	*0.001042*	*0.000364*	*0.000176*	*0.000010*	*0.001703*	*0.001276*	*0.000266*	*0.000042*	*0.000013*	*0.000018*
cumulative	7	0.00083	**0.029309**	0.011953	0.961671	0.318082	0.042558	0.007584	0.002645	0.000107	0.023080	0.004771	0.001107	0.000190	0.000066	0.000095
periodic			**0.003209**	*0.004607*	*0.129703*	*0.003157*	*0.000987*	*0.000365*	*0.000195*	*0.000011*	*0.001063*	*0.001671*	*0.000387*	*0.000053*	*0.000016*	*0.000019*

			Expected and unexpected losses		$s\ \tilde{L}_j^k$ per tranche (in % of tranche volume)						$s\ \tilde{L}_j^k$ per tranche (abs. share of total exp. losses per period)					
cum./per.	Yr	p	$\tilde{L}_j^k$	$s\ \tilde{L}_j^k$	0-2.4%	2.4-3.9%	3.9-6.5%	6.5-9%	9-10.5%	10.5-100%	0-2.4%	2.4-3.9%	3.9-6.5%	6.5-9%	9-10.5%	10.5-100%
cumulative	1	0.00367	0.006005	**0.004523**	0.124453	0.055172	0.029266	0.018088	0.012631	0.001715	0.002987	0.000828	0.000761	0.000452	0.000316	0.001518
periodic			*0.006005*	**0.004523**	*0.124453*	*0.055172*	*0.029266*	*0.018088*	*0.012631*	*0.001715*	*0.002987*	*0.000828*	*0.000761*	*0.000452*	*0.000316*	*0.001518*
cumulative	2	0.00267	0.011014	**0.006328**	0.159532	0.097505	0.047893	0.027566	0.019229	0.002198	0.003829	0.001463	0.001245	0.000689	0.000481	0.001945
periodic			*0.005009*	**0.004429**	*0.126442*	*0.054316*	*0.029122*	*0.017752*	*0.012535*	*0.001253*	*0.000842*	*0.000635*	*0.000484*	*0.000237*	*0.000165*	*0.000427*
cumulative	3	0.00195	0.015296	**0.007696**	0.170597	0.145666	0.065797	0.036069	0.024743	0.002656	0.004094	0.002185	0.001711	0.000902	0.000619	0.002351
periodic			*0.004282*	**0.004366**	*0.127236*	*0.051448*	*0.027190*	*0.016506*	*0.011817*	*0.001324*	*0.000266*	*0.000722*	*0.000466*	*0.000213*	*0.000138*	*0.000405*
cumulative	4	0.00154	0.019189	**0.008889**	0.162596	0.202429	0.086183	0.045515	0.030438	0.003059	0.003902	0.003036	0.002241	0.001138	0.000761	0.002707
periodic			*0.003894*	**0.004440**	*0.128220*	*0.051941*	*0.028494*	*0.017093*	*0.012322*	*0.001283*	*-0.000192*	*0.000851*	*0.000530*	*0.000236*	*0.000142*	*0.000357*
cumulative	5	0.00123	0.022782	**0.010016**	0.138867	0.262821	0.108775	0.055492	0.036301	0.003628	0.003333	0.003942	0.002828	0.001387	0.000908	0.003211
periodic			*0.003593*	**0.004626**	*0.128459*	*0.050898*	*0.028846*	*0.017801*	*0.013353*	*0.001674*	*-0.000569*	*0.000906*	*0.000587*	*0.000249*	*0.000147*	*0.000504*
cumulative	6	0.00094	0.026099	**0.011032**	0.106689	0.317039	0.133944	0.066194	0.041936	0.004178	0.002561	0.004756	0.003483	0.001655	0.001048	0.003698
periodic			*0.003317*	**0.004623**	*0.129227*	*0.051637*	*0.028862*	*0.017647*	*0.012598*	*0.001760*	*-0.000772*	*0.000813*	*0.000654*	*0.000268*	*0.000141*	*0.000487*
cumulative	7	0.00083	0.029309	**0.011953**	0.072741	0.354321	0.161875	0.077491	0.047751	0.004688	0.001746	0.005315	0.004209	0.001937	0.001194	0.004149
periodic			*0.003209*	**0.004607**	*0.128773*	*0.050739*	*0.028080*	*0.017886*	*0.013405*	*0.001735*	*-0.000815*	*0.000559*	*0.000726*	*0.000282*	*0.000145*	*0.000451*

Table 27. *Simulation of decreasing forward probability rates (extreme value theory loss function as distribution of portfolio losses) of default losses on a cumulative and periodic basis – losses per tranche with either the tranche % or the absolute value of losses per period as reference base.*

			Expected and unexpected losses		$\tilde{L}_j^k$ per tranche (in % of tranche volume)						$\tilde{L}_j^k$ per tranche (abs. share of total exp. losses per period)					
cum./per.	**Yr**	p	$\tilde{L}_j^k$	$s\ \tilde{L}_j^k$	**0-2.4%**	**2.4-3.9%**	**3.9-6.5%**	**6.5-9%**	**9-10.5%**	**10.5-100%**	**0-2.4%**	**2.4-3.9%**	**3.9-6.5%**	**6.5-9%**	**9-10.5%**	**10.5-100%**
cumulative	1	0.0026	**0.002593**	0.004588	0.104352	0.004135	0.000830	0.000157	0.000041	0.000001	0.002504	0.000062	0.000022	0.000004	0.000001	0.000001
periodic			**0.002593**	*0.004588*	*0.104352*	*0.004135*	*0.000830*	*0.000157*	*0.000041*	*0.000001*	*0.002504*	*0.000062*	*0.000022*	*0.000004*	*0.000001*	*0.000001*
cumulative	2	0.0026	**0.005186**	0.006491	0.206350	0.010987	0.002129	0.000389	0.000110	0.000001	0.004952	0.000165	0.000055	0.000010	0.000003	0.000001
periodic			**0.002593**	*0.001903*	*0.101998*	*0.006852*	*0.001299*	*0.000232*	*0.000069*	*0.000000*	*0.002448*	*0.000103*	*0.000034*	*0.000006*	*0.000002*	*0.000000*
cumulative	3	0.0026	**0.007771**	0.007921	0.304995	0.021439	0.004068	0.000683	0.000177	0.000002	0.007320	0.000322	0.000106	0.000017	0.000004	0.000002
periodic			**0.002585**	*0.001430*	*0.098645*	*0.010452*	*0.001939*	*0.000294*	*0.000067*	*0.000001*	*0.002367*	*0.000157*	*0.000050*	*0.000007*	*0.000002*	*0.000001*
cumulative	4	0.0026	**0.010356**	0.009126	0.399452	0.036819	0.006922	0.001108	0.000261	0.000003	0.009587	0.000552	0.000180	0.000028	0.000007	0.000003
periodic			**0.002585**	*0.001205*	*0.094457*	*0.015380*	*0.002854*	*0.000425*	*0.000084*	*0.000001*	*0.002267*	*0.000231*	*0.000074*	*0.000011*	*0.000002*	*0.000001*
cumulative	5	0.0026	**0.012934**	0.010181	0.488493	0.058068	0.010884	0.001691	0.000392	0.000004	0.011724	0.000871	0.000283	0.000042	0.000010	0.000004
periodic			**0.002578**	*0.001055*	*0.089041*	*0.021249*	*0.003962*	*0.000583*	*0.000131*	*0.000001*	*0.002137*	*0.000319*	*0.000103*	*0.000015*	*0.000003*	*0.000001*
cumulative	6	0.0026	**0.015500**	0.011127	0.570866	0.086047	0.016401	0.002502	0.000559	0.000006	0.013701	0.001291	0.000426	0.000063	0.000014	0.000005
periodic			**0.002566**	*0.000946*	*0.082373*	*0.027979*	*0.005517*	*0.000811*	*0.000167*	*0.000002*	*0.001977*	*0.000420*	*0.000143*	*0.000020*	*0.000004*	*0.000002*
cumulative	7	0.0026	**0.018059**	0.011991	0.645940	0.121363	0.023847	0.003575	0.000777	0.000008	0.015503	0.001820	0.000620	0.000089	0.000019	0.000007
periodic			**0.002559**	*0.000864*	*0.075074*	*0.035316*	*0.007446*	*0.001073*	*0.000218*	*0.000002*	*0.001802*	*0.000530*	*0.000194*	*0.000027*	*0.000005*	*0.000002*

Table 28. *Simulation of constant forward probability rates (normal inverse distribution of portfolio losses) of default losses on a cumulative and periodic basis – losses per tranche with either the tranche % or the absolute value of losses per period as reference base.*

			Expected and unexpected losses		$\tilde{L}_j^k$ per tranche (in % of tranche volume)						$\tilde{L}_j^k$ per tranche (abs. share of total exp. losses per period)					
cum./per.	**Yr**	$\hat{p}$	$\tilde{L}_j^k$	$s\ \tilde{L}_j^k$	**0-2.4%**	**2.4-3.9%**	**3.9-6.5%**	**6.5-9%**	**9-10.5%**	**10.5-100%**	**0-2.4%**	**2.4-3.9%**	**3.9-6.5%**	**6.5-9%**	**9-10.5%**	**10.5-100%**
cumulative	1	0.0026	**0.002593**	0.004579	0.104369	0.004101	0.000831	0.000147	0.000039	0.000001	0.002505	0.000062	0.000022	0.000004	0.000001	0.000001
periodic			**0.002593**	*0.004579*	*0.104369*	*0.004101*	*0.000831*	*0.000147*	*0.000039*	*0.000001*	*0.002505*	*0.000062*	*0.000022*	*0.000004*	*0.000001*	*0.000001*
cumulative	2	0.0026	**0.006178**	0.007552	0.241895	0.016947	0.003589	0.000685	0.000200	0.000004	0.005805	0.000254	0.000093	0.000017	0.000005	0.000004
periodic			**0.003585**	*0.002973*	*0.137526*	*0.012846*	*0.002758*	*0.000538*	*0.000161*	*0.000003*	*0.003301*	*0.000193*	*0.000072*	*0.000013*	*0.000004*	*0.000003*
cumulative	3	0.0026	**0.010451**	0.010198	0.393470	0.045190	0.010055	0.001928	0.000516	0.000008	0.009443	0.000678	0.000261	0.000048	0.000013	0.000007
periodic			**0.004273**	*0.002646*	*0.151575*	*0.028243*	*0.006466*	*0.001243*	*0.000316*	*0.000004*	*0.003638*	*0.000424*	*0.000168*	*0.000031*	*0.000008*	*0.000004*
cumulative	4	0.0026	**0.015207**	0.012650	0.542116	0.096050	0.023047	0.004522	0.001182	0.000016	0.013011	0.001441	0.000599	0.000113	0.000030	0.000014
periodic			**0.004756**	*0.002452*	*0.148646*	*0.050860*	*0.012992*	*0.002594*	*0.000666*	*0.000008*	*0.003568*	*0.000763*	*0.000338*	*0.000065*	*0.000017*	*0.000007*
cumulative	5	0.0026	**0.020238**	0.014826	0.673930	0.172279	0.044946	0.009036	0.002325	0.000030	0.016174	0.002584	0.001169	0.000226	0.000058	0.000027
periodic			**0.005031**	*0.002176*	*0.131814*	*0.076229*	*0.021899*	*0.004514*	*0.001143*	*0.000014*	*0.003164*	*0.001143*	*0.000569*	*0.000113*	*0.000029*	*0.000012*
cumulative	6	0.0026	**0.025426**	0.016816	0.780726	0.271508	0.078663	0.016656	0.004250	0.000054	0.018737	0.004073	0.002045	0.000416	0.000106	0.000048
periodic			**0.005188**	*0.001990*	*0.106796*	*0.099229*	*0.033717*	*0.007620*	*0.001925*	*0.000024*	*0.002563*	*0.001488*	*0.000877*	*0.000191*	*0.000048*	*0.000021*
cumulative	7	0.0026	**0.030681**	0.018618	0.860469	0.386342	0.125166	0.028539	0.007366	0.000094	0.020651	0.005795	0.003254	0.000713	0.000184	0.000083
periodic			**0.005255**	*0.001802*	*0.079743*	*0.114834*	*0.046503*	*0.011883*	*0.003116*	*0.000040*	*0.001914*	*0.001723*	*0.001209*	*0.000297*	*0.000078*	*0.000035*

Table 29. *Simulation of increasing forward probability rates (normal inverse distribution of portfolio losses) of default losses on a cumulative and periodic basis – losses per tranche with either the tranche % or the absolute value of losses per period as reference base.*

			Expected and unexpected losses		$\tilde{L}_j^k$ per tranche (in % of tranche volume)						$\tilde{L}_j^k$ per tranche (abs. share of total exp. losses per period)					
cum./per.	**Yr**	p	$\tilde{L}_j^k$	$s_{\tilde{L}_j^k}$	**0-2.4%**	**2.4-3.9%**	**3.9-6.5%**	**6.5-9%**	**9-10.5%**	**10.5-100%**	**0-2.4%**	**2.4-3.9%**	**3.9-6.5%**	**6.5-9%**	**9-10.5%**	**10.5-100%**
cumulative	1	0.0026	**0.005992**	0.006269	0.241622	0.010208	0.001379	0.000144	0.000023	0.000000	0.005799	0.000153	0.000036	0.000004	0.000001	0.000000
periodic			**0.005992**	*0.006269*	*0.241622*	*0.010208*	*0.001379*	*0.000144*	*0.000023*	*0.000000*	*0.005799*	*0.000153*	*0.000036*	*0.000004*	*0.000001*	*0.000000*
cumulative	2	0.0026	**0.010961**	0.008220	0.431596	0.032250	0.004144	0.000389	0.000061	0.000000	0.010358	0.000484	0.000108	0.000010	0.000002	0.000000
periodic			**0.004969**	*0.001951*	*0.189974*	*0.022042*	*0.002765*	*0.000245*	*0.000037*	*0.000000*	*0.004559*	*0.000331*	*0.000072*	*0.000006*	*0.000001*	*0.000000*
cumulative	3	0.0026	**0.015214**	0.009431	0.580569	0.068669	0.008830	0.000719	0.000097	0.000001	0.013934	0.001030	0.000230	0.000018	0.000002	0.000000
periodic			**0.004253**	*0.001211*	*0.148973*	*0.036419*	*0.004686*	*0.000330*	*0.000037*	*0.000000*	*0.003575*	*0.000546*	*0.000122*	*0.000008*	*0.000001*	*0.000000*
cumulative	4	0.0026	**0.019060**	0.010333	0.698890	0.121829	0.016286	0.001245	0.000147	0.000001	0.016773	0.001827	0.000423	0.000031	0.000004	0.000001
periodic			**0.003846**	*0.000902*	*0.118321*	*0.053160*	*0.007456*	*0.000526*	*0.000050*	*0.000000*	*0.002840*	*0.000797*	*0.000194*	*0.000013*	*0.000001*	*0.000000*
cumulative	5	0.0026	**0.022595**	0.011046	0.790304	0.190679	0.027340	0.002020	0.000225	0.000001	0.018967	0.002860	0.000711	0.000051	0.000006	0.000001
periodic			**0.003535**	*0.000713*	*0.091414*	*0.068850*	*0.011054*	*0.000775*	*0.000078*	*0.000000*	*0.002194*	*0.001033*	*0.000287*	*0.000019*	*0.000002*	*0.000000*
cumulative	6	0.0026	**0.025819**	0.011616	0.857477	0.270135	0.042336	0.003105	0.000323	0.000001	0.020579	0.004052	0.001101	0.000078	0.000008	0.000001
periodic			**0.003224**	*0.000570*	*0.067173*	*0.079456*	*0.014996*	*0.001085*	*0.000098*	*0.000000*	*0.001612*	*0.001192*	*0.000390*	*0.000027*	*0.000002*	*0.000000*
cumulative	7	0.0026	**0.028936**	0.012125	0.907148	0.359962	0.062859	0.004687	0.000454	0.000002	0.021772	0.005399	0.001634	0.000117	0.000011	0.000002
periodic			**0.003117**	*0.000509*	*0.049671*	*0.089827*	*0.020523*	*0.001582*	*0.000131*	*0.000000*	*0.001192*	*0.001347*	*0.000534*	*0.000040*	*0.000003*	*0.000000*

Table 30. *Simulation of decreasing forward probability rates (normal inverse distribution of portfolio losses) of default losses on a cumulative and periodic basis – losses per tranche with either the tranche % or the absolute value of losses per period as reference base.*

			Expected and unexpected losses		$\tilde{L}_j^k$ per tranche (in % of tranche volume)						$\tilde{L}_j^k$ per tranche (abs. share of total exp. losses per period)					
cum./per.	**Yr**	p	$\tilde{L}_j^k$	$s\ \tilde{L}_j^k$	**0-2.4%**	**2.4-3.9%**	**3.9-6.5%**	**6.5-9%**	**9-10.5%**	**10.5-100%**	**0-2.4%**	**2.4-3.9%**	**3.9-6.5%**	**6.5-9%**	**9-10.5%**	**10.5-100%**
cumulative	1	0.0026	**0.002569**	0.004512	0.002851	0.000209	0.000004	0.000000	0.000000	0.000000	0.000068	0.000003	0.000000	0.000000	0.000000	0.000000
periodic			**0.002569**	*0.004512*	*0.002851*	*0.000209*	*0.000004*	*0.000000*	*0.000000*	*0.000000*	*0.000068*	*0.000003*	*0.000000*	*0.000000*	*0.000000*	*0.000000*
cumulative	2	0.0026	**0.005176**	0.006450	0.009512	0.000850	0.000021	0.000000	0.000000	0.000000	0.000228	0.000013	0.000001	0.000000	0.000000	0.000000
periodic			**0.002607**	*0.001938*	*0.006661*	*0.000641*	*0.000017*	*0.000000*	*0.000000*	*0.000000*	*0.000160*	*0.000010*	*0.000000*	*0.000000*	*0.000000*	*0.000000*
cumulative	3	0.0026	**0.007789**	0.007924	0.021589	0.002135	0.000061	0.000000	0.000000	0.000000	0.000518	0.000032	0.000002	0.000000	0.000000	0.000000
periodic			**0.002613**	*0.001474*	*0.012077*	*0.001285*	*0.000039*	*0.000000*	*0.000000*	*0.000000*	*0.000290*	*0.000019*	*0.000001*	*0.000000*	*0.000000*	*0.000000*
cumulative	4	0.0026	**0.010391**	0.009154	0.039937	0.004462	0.000185	0.000002	0.000000	0.000000	0.000958	0.000067	0.000005	0.000000	0.000000	0.000000
periodic			**0.002602**	*0.001230*	*0.018348*	*0.002327*	*0.000124*	*0.000002*	*0.000000*	*0.000000*	*0.000440*	*0.000035*	*0.000003*	*0.000000*	*0.000000*	*0.000000*
cumulative	5	0.0026	**0.012955**	0.010195	0.065128	0.008176	0.000377	0.000003	0.000000	0.000000	0.001563	0.000123	0.000010	0.000000	0.000000	0.000000
periodic			**0.002564**	*0.001041*	*0.025191*	*0.003714*	*0.000192*	*0.000001*	*0.000000*	*0.000000*	*0.000605*	*0.000056*	*0.000005*	*0.000000*	*0.000000*	*0.000000*
cumulative	6	0.0026	**0.015510**	0.011122	0.098164	0.013608	0.000714	0.000009	0.000000	0.000000	0.002356	0.000204	0.000019	0.000000	0.000000	0.000000
periodic			**0.002555**	*0.000927*	*0.033036*	*0.005432*	*0.000337*	*0.000006*	*0.000000*	*0.000000*	*0.000793*	*0.000081*	*0.000009*	*0.000000*	*0.000000*	*0.000000*
cumulative	7	0.0026	**0.018078**	0.011989	0.138913	0.021659	0.001298	0.000030	0.000000	0.000000	0.003334	0.000325	0.000034	0.000001	0.000000	0.000000
periodic			**0.002568**	*0.000867*	*0.040749*	*0.008051*	*0.000584*	*0.000021*	*0.000000*	*0.000000*	*0.000978*	*0.000121*	*0.000015*	*0.000001*	*0.000000*	*0.000000*

Table 31. *Simulation of constant forward probability rates (beta distribution of portfolio losses) of default losses on a cumulative and periodic basis – losses per tranche with either the tranche % or the absolute value of losses per period as reference base.*

			Expected and unexpected losses		$\tilde{L}_j^k$ per tranche (in % of tranche volume)						$\tilde{L}_j^k$ per tranche (abs. share of total exp. losses per period)					
cum./per.	**Yr**	p	$\tilde{L}_j^k$	$s_{\tilde{L}_j^k}$	**0-2.4%**	**2.4-3.9%**	**3.9-6.5%**	**6.5-9%**	**9-10.5%**	**10.5-100%**	**0-2.4%**	**2.4-3.9%**	**3.9-6.5%**	**6.5-9%**	**9-10.5%**	**10.5-100%**
cumulative	1	0.0026	**0.002587**	0.004581	0.105659	0.003039	0.000205	0.000010	0.000000	0.000000	0.002536	0.000046	0.000005	0.000000	0.000000	0.000000
periodic			**0.002587**	*0.004581*	*0.105659*	*0.003039*	*0.000205*	*0.000010*	*0.000000*	*0.000000*	*0.002536*	*0.000046*	*0.000005*	*0.000000*	*0.000000*	*0.000000*
cumulative	2	0.0026	**0.005197**	0.006476	0.209482	0.009744	0.000864	0.000017	0.000000	0.000000	0.005028	0.000146	0.000022	0.000000	0.000000	0.000000
periodic			**0.002610**	*0.001895*	*0.103823*	*0.006705*	*0.000659*	*0.000007*	*0.000000*	*0.000000*	*0.002492*	*0.000101*	*0.000017*	*0.000000*	*0.000000*	*0.000000*
cumulative	3	0.0026	**0.007777**	0.007932	0.308187	0.021358	0.002205	0.000091	0.000003	0.000000	0.007396	0.000320	0.000057	0.000002	0.000000	0.000000
periodic			**0.002580**	*0.001456*	*0.098705*	*0.011614*	*0.001341*	*0.000074*	*0.000003*	*0.000000*	*0.002369*	*0.000174*	*0.000035*	*0.000002*	*0.000000*	*0.000000*
cumulative	4	0.0026	**0.010356**	0.009115	0.402161	0.038983	0.004387	0.000225	0.000004	0.000000	0.009652	0.000585	0.000114	0.000006	0.000000	0.000000
periodic			**0.002579**	*0.001183*	*0.093974*	*0.017625*	*0.002182*	*0.000134*	*0.000001*	*0.000000*	*0.002255*	*0.000264*	*0.000057*	*0.000003*	*0.000000*	*0.000000*
cumulative	5	0.0026	**0.012926**	0.010175	0.489310	0.063951	0.008158	0.000425	0.000028	0.000000	0.011743	0.000959	0.000212	0.000011	0.000001	0.000000
periodic			**0.002570**	*0.001060*	*0.087149*	*0.024968*	*0.003771*	*0.000200*	*0.000023*	*0.000000*	*0.002092*	*0.000375*	*0.000098*	*0.000005*	*0.000001*	*0.000000*
cumulative	6	0.0026	**0.015511**	0.011120	0.569918	0.096979	0.013770	0.000770	0.000050	0.000000	0.013678	0.001455	0.000358	0.000019	0.000001	0.000000
periodic			**0.002585**	*0.000945*	*0.080608*	*0.033028*	*0.005612*	*0.000345*	*0.000022*	*0.000000*	*0.001935*	*0.000495*	*0.000146*	*0.000009*	*0.000001*	*0.000000*
cumulative	7	0.0026	**0.018083**	0.011988	0.642122	0.138027	0.021825	0.001309	0.000076	0.000000	0.015411	0.002070	0.000567	0.000033	0.000002	0.000000
periodic			**0.002572**	*0.000868*	*0.072204*	*0.041048*	*0.008055*	*0.000539*	*0.000026*	*0.000000*	*0.001733*	*0.000616*	*0.000209*	*0.000013*	*0.000001*	*0.000000*

Table 32. *Simulation of constant forward probability rates (negative binomial distribution of portfolio losses) of default losses on a cumulative and periodic basis – losses per tranche with either the tranche % or the absolute value of losses per period as reference base.*

			Unexpected divided by exp. losses	$s_{\tilde{L}_j^k} / \tilde{L}_j^k$ per tranche					
cum./per.	**Yr**	p	$s_{\tilde{L}_j^k} / \tilde{L}_j^k$	**0-2.4%**	**2.4-3.9%**	**3.9-6.5%**	**6.5-9%**	**9-10.5%**	**10.5-100%**
cumulative	1	0.0026	4.399632	3.484512	36.702854	64.110433	107.346421	154.297022	339.564720
periodic			*4.399632*	*3.484512*	*36.702854*	*64.110433*	*107.346421*	*154.297022*	*339.564720*
cumulative	2	0.0026	3.785829	2.512960	23.265408	42.253033	73.920746	105.595136	218.906721
periodic			*3.342004*	*0.529687*	*14.141035*	*24.883264*	*44.217836*	*62.810301*	*126.092875*
cumulative	3	0.0026	3.360607	2.061186	16.776855	30.611230	57.157314	83.073508	164.836492
periodic			*0.777466*	*0.132350*	*9.572895*	*17.747653*	*32.184500*	*46.269872*	*76.006829*
cumulative	4	0.0026	2.448552	0.771988	12.812742	25.092041	46.815071	68.438096	139.493259
periodic			*0.543029*	*0.861934*	*7.114355*	*13.746811*	*26.005602*	*36.959719*	*72.514717*
cumulative	5	0.0026	2.240594	0.558457	10.093890	20.574793	39.571379	58.482570	120.385362
periodic			*0.406757*	*0.635773*	*5.504792*	*10.000140*	*20.124482*	*30.859098*	*59.478937*
cumulative	6	0.0026	2.080032	0.385690	8.110153	17.119327	34.366668	50.764189	108.229350
periodic			*0.275055*	*0.414983*	*4.350938*	*8.953731*	*17.715189*	*27.450049*	*53.932499*
cumulative	7	0.0026	0.955407	0.239034	6.612258	14.433535	30.053660	46.412161	97.979813
periodic			*0.204816*	*0.188537*	*3.477341*	*7.573891*	*15.306088*	*24.512671*	*45.450935*

Table 33. $s_{\tilde{L}_j^k} / \tilde{L}_j^k$ *ratio for each tranche based on a simulation of constant forward probability rates (EVT distribution of portfolio losses) under loss recovery.*

			Unexpect. divided by exp. losses	$s_{\tilde{L}_j^k} / \tilde{L}_j^k$ per tranche					
cum./per.	**Yr**	$r u$	$s_{\tilde{L}_j^k} / \tilde{L}_j^k$	**0-2.4%**	**2.4-3.9%**	**3.9-6.5%**	**6.5-9%**	**9-10.5%**	**10.5-100%**
cumulative	1	0.000100	4.399632	3.484512	36.702854	64.110433	107.346421	154.297022	339.564720
periodic			*4.399632*	*3.484512*	*36.702854*	*64.110433*	*107.346421*	*154.297022*	*339.564720*
cumulative	2	0.001231	3.785829	3.008487	27.899062	53.698372	96.028673	142.314033	309.612802
periodic			*3.342004*	*2.631317*	*36.040773*	*62.628416*	*108.570975*	*150.105864*	*310.453937*
cumulative	3	0.001945	3.360607	2.742849	19.011110	42.612144	85.086138	130.338390	296.099856
periodic			*2.745313*	*2.274788*	*35.020228*	*64.381490*	*118.428884*	*166.678943*	*327.366741*
cumulative	4	0.002469	3.103498	2.593312	10.667847	30.453772	70.725589	115.541203	275.345540
periodic			*2.543102*	*2.078119*	*34.438088*	*63.135083*	*113.080495*	*160.738280*	*342.453638*
cumulative	5	0.002771	2.940677	2.547467	6.581402	20.823275	57.330605	100.606003	250.379582
periodic			*2.453967*	*0.980200*	*33.777884*	*62.053598*	*108.465116*	*154.634826*	*320.261827*
cumulative	6	0.002954	2.830469	2.620724	3.605366	14.311552	44.712487	85.759867	228.695160
periodic			*2.405616*	*0.924590*	*33.707930*	*60.077412*	*107.721293*	*152.896657*	*315.453997*
cumulative	7	0.003055	2.730647	2.850588	2.120712	8.878558	33.871663	72.070196	203.054503
periodic			*2.254745*	*0.893580*	*33.587668*	*60.130860*	*110.574107*	*160.709078*	*292.119219*

Table 34. $s_{\tilde{L}_j^k} / \tilde{L}_j^k$ *ratio for each tranche based on a simulation of increasing forward probability rates of a deteriorating portfolio (EVT distribution of portfolio losses) under loss recovery.*

			Unexpect. divided by exp. losses	$s_{\tilde{L}_j^k} / \tilde{L}_j^k$ per tranche					
cum./per.	Yr	r_u	$s_{\tilde{L}_j^k} / \tilde{L}_j^k$	0-2.4%	2.4-3.9%	3.9-6.5%	6.5-9%	9-10.5%	10.5-100%
cumulative	1	0.003673	2.153361	0.741965	32.466114	60.125725	106.827696	154.702873	423.910819
periodic			*2.153361*	*0.741965*	*32.466114*	*60.125725*	*106.827696*	*154.702873*	*423.910819*
cumulative	2	0.002669	2.285866	0.883944	19.592067	43.464264	89.692804	133.882010	330.185942
periodic			2.444718	2.014072	33.557040	60.160123	110.069756	156.969883	309.835719
cumulative	3	0.001945	2.414625	2.032232	12.006405	32.504371	74.704197	118.949257	298.438110
periodic			*2.745313*	*2.274788*	*35.020228*	*64.381490*	*118.428884*	*166.678943*	*327.366741*
cumulative	4	0.001538	2.536279	2.189857	7.517381	24.328460	60.236739	105.061912	263.940601
periodic			*3.014149*	*2.465404*	*35.278269*	*62.591515*	*110.587890*	*162.502512*	*217.243193*
cumulative	5	0.001231	2.663346	2.374104	4.901839	18.352531	50.257479	93.544953	244.403871
periodic			*3.342004*	*2.631317*	*36.040773*	*62.628416*	*108.570975*	*150.105864*	*310.453937*
cumulative	6	0.000940	2.779611	2.602486	3.448034	13.903233	42.879270	83.164265	233.096603
periodic			*3.578179*	*2.818469*	*35.740968*	*62.066461*	*108.224317*	*159.543795*	*390.565240*
cumulative	7	0.000834	2.877246	2.871633	2.600252	10.522826	36.134792	74.716369	223.428582
periodic			3.671349	2.884306	36.276896	63.734063	109.384782	153.243456	350.959123

Table 35. $s_{\tilde{L}_j^k} / \tilde{L}_j^k$ *ratio for each tranche based on a simulation of decreasing forward probability rates of an improving portfolio (EVT distribution of portfolio losses) under loss recovery.*

16 APPENDIX II: FIGURES

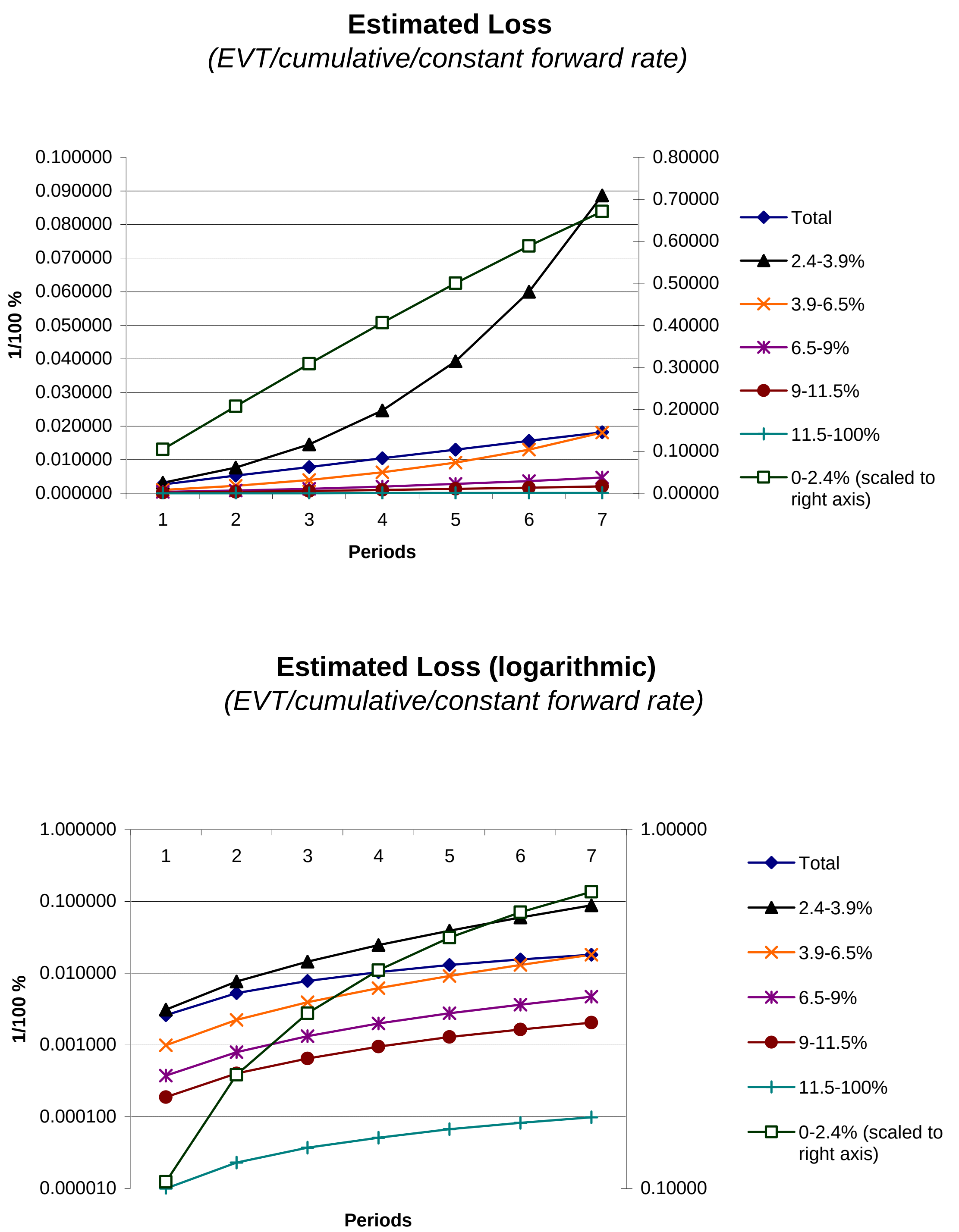
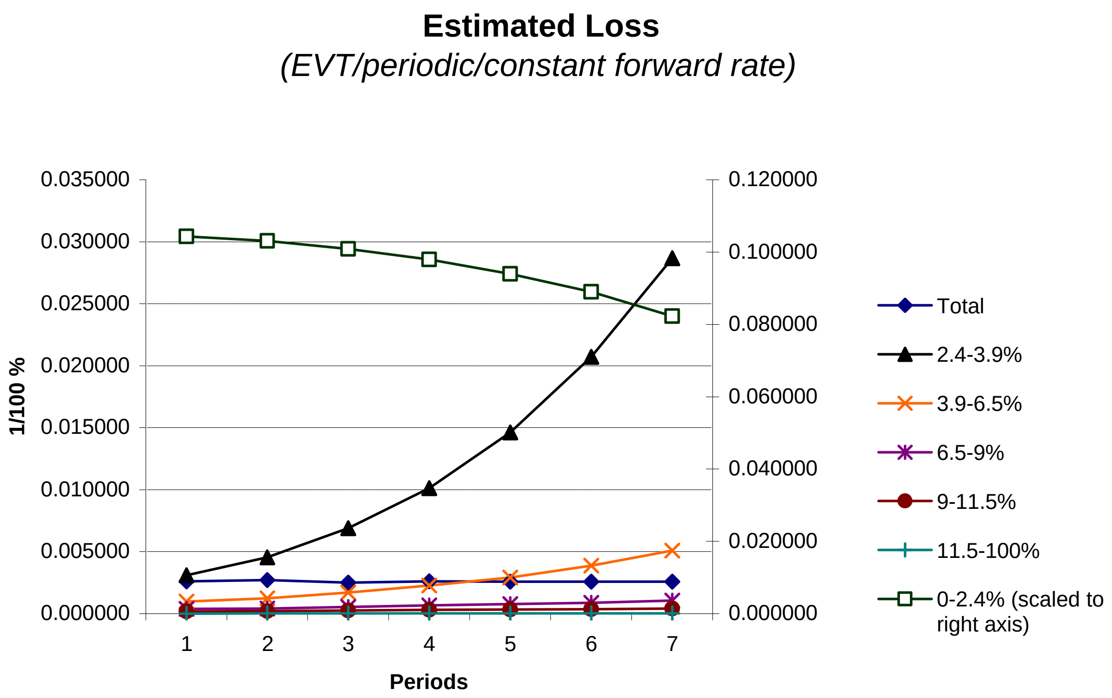


Figure A1. *Term structure of cumulative expected losses in various tranches of a collateral portfolio (extreme value simulation), depicted on a linear and a logarithmic scale. In the linear version of the loss cascades the first tranche [0-2.4%] scales with the right axis.*



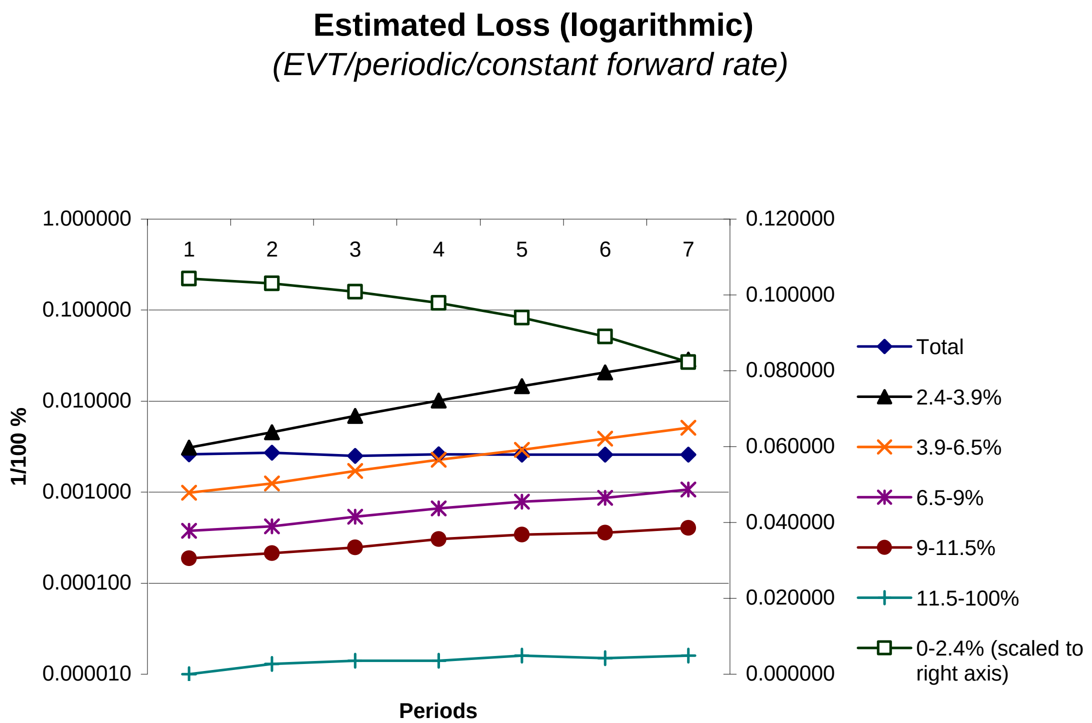
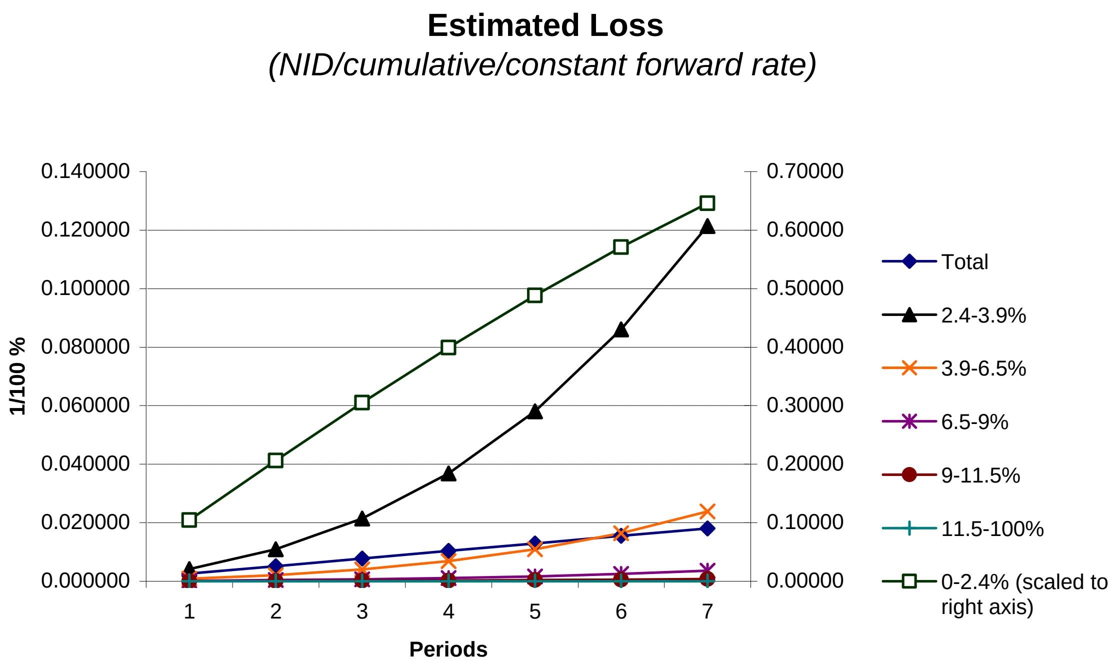


Figure A2. *Term structure of periodic expected losses in various tranches of a collateral portfolio (extreme value simulation), depicted on a linear and a logarithmic scale. In the linear version of the loss cascades the first tranche [0-2.4%] scales with the right axis.*



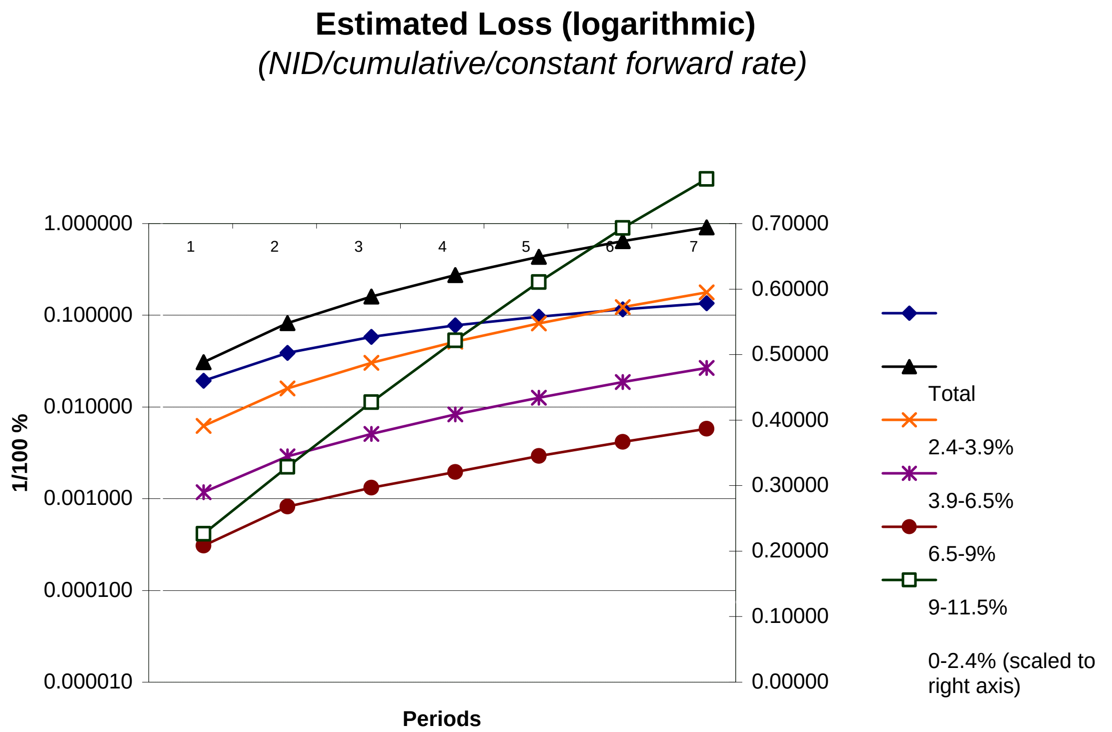
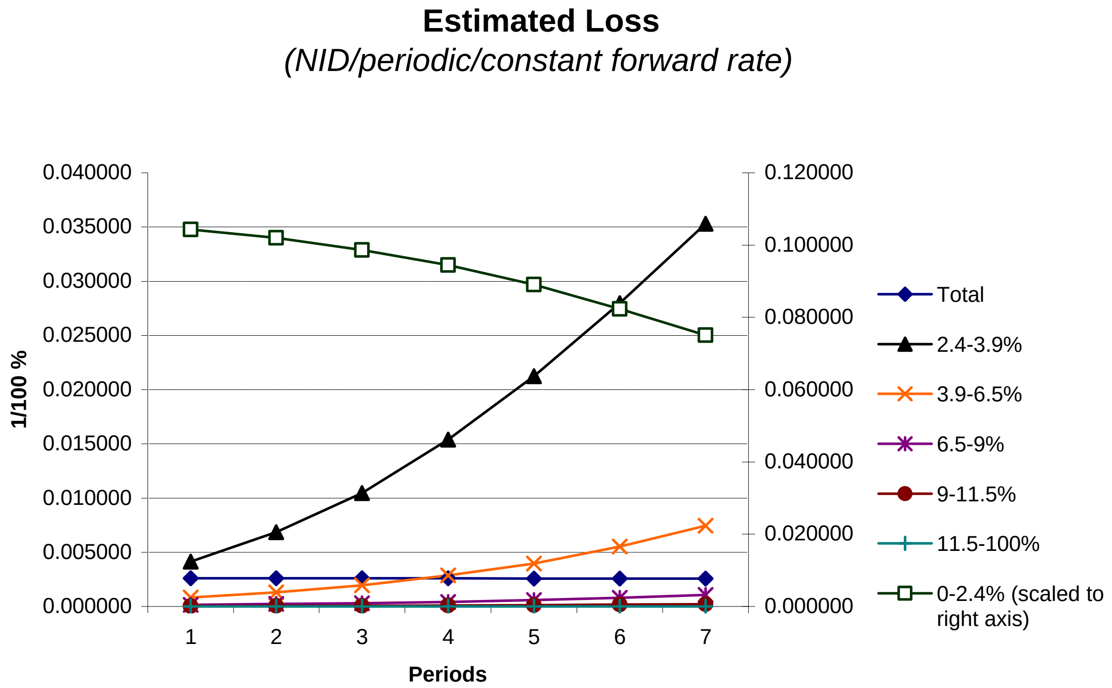


Figure A3. *Term structure of cumulative expected losses in various tranches of a collateral portfolio (normal inverse distribution), depicted on a linear and a logarithmic scale. In the linear version of the loss cascades the first tranche [0-2.4%] scales with the right axis. The most senior tranche has been excluded for the logarithmic case due to negative values.*



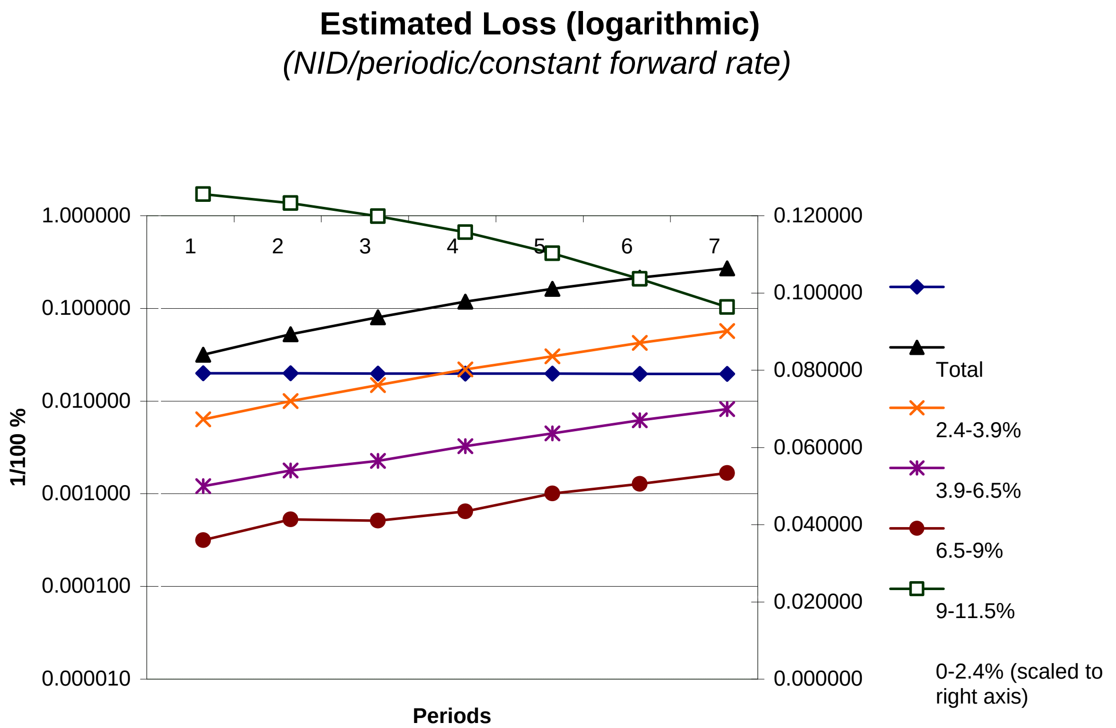
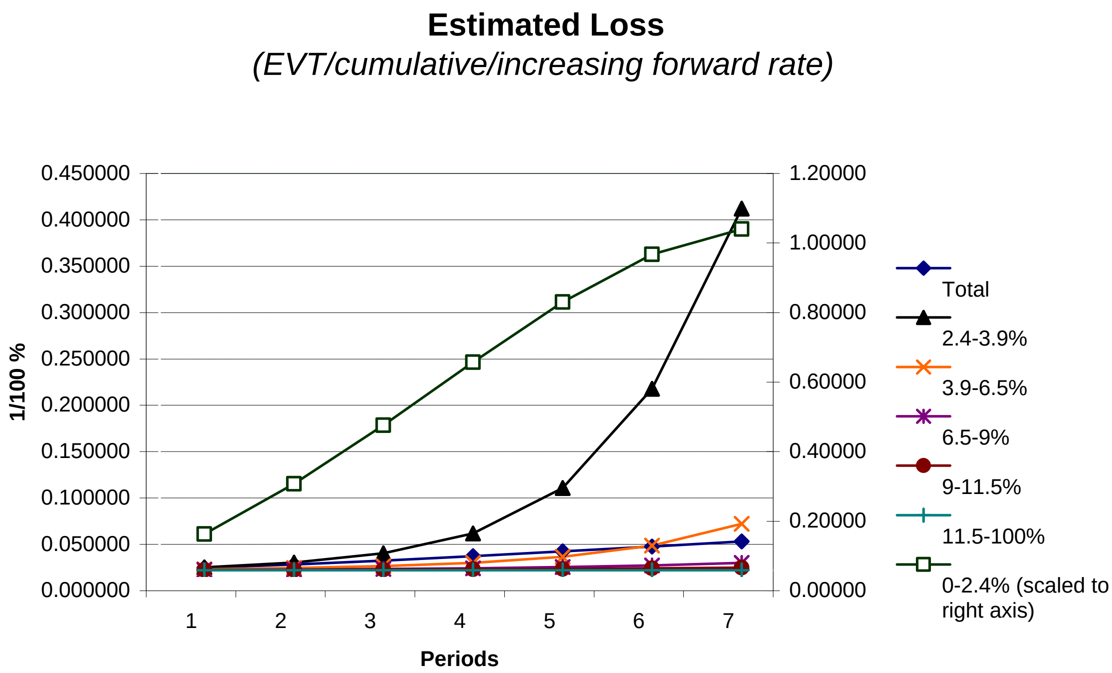


Figure A4. *Term structure of periodic expected losses in various tranches of a collateral portfolio (normal inverse distribution), depicted on a linear and a logarithmic scale. In the linear version of the loss cascades the first tranche [0-2.4%] scales with the right axis. The most senior tranche has been excluded for the logarithmic case due to negative values.*



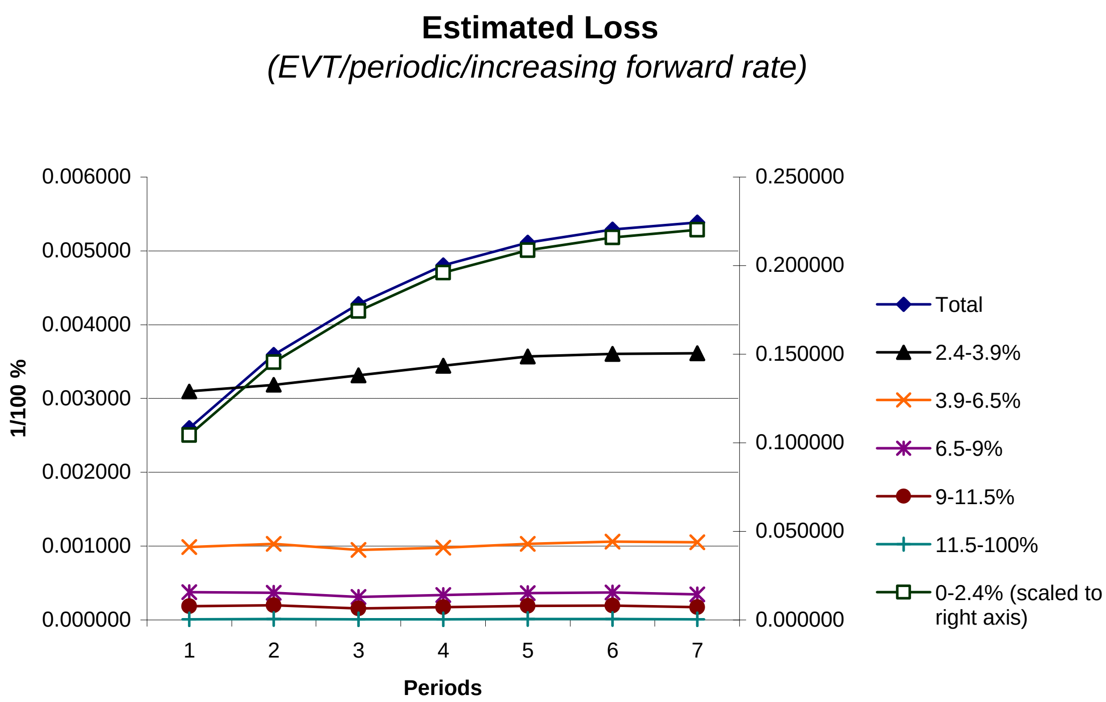
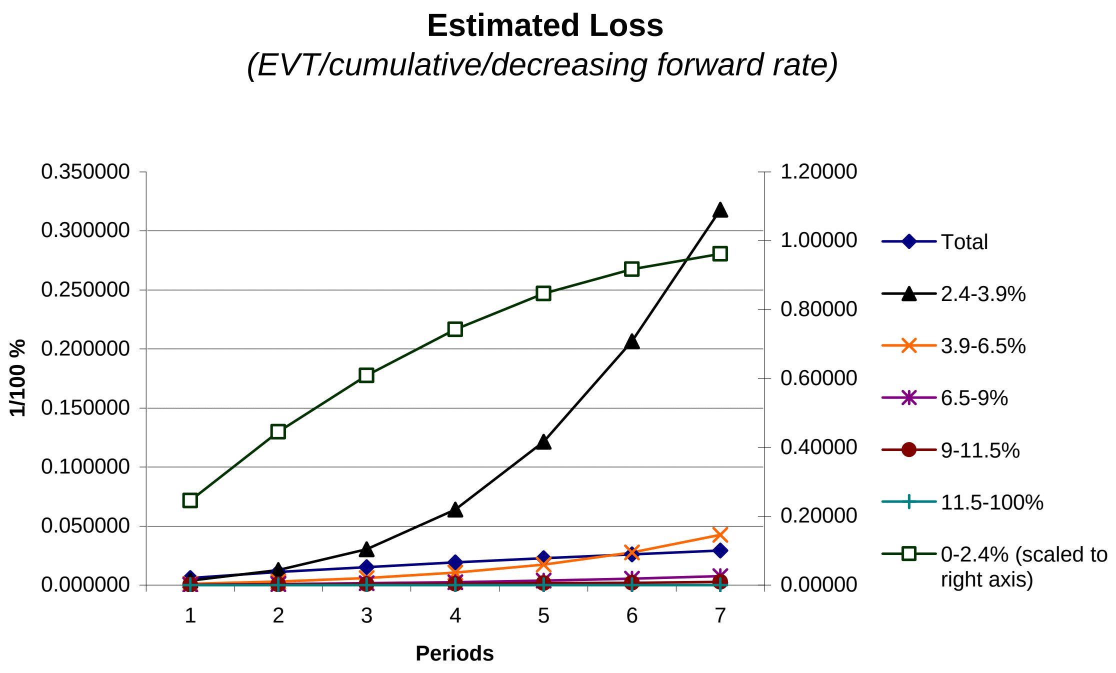


Figure A5. *Expected loss in tranches for a deteriorating (increasing forward rate of default) portfolio given an underlying extreme value distribution, where the first loss position [0-2.4%] is scaled to the right axis.*



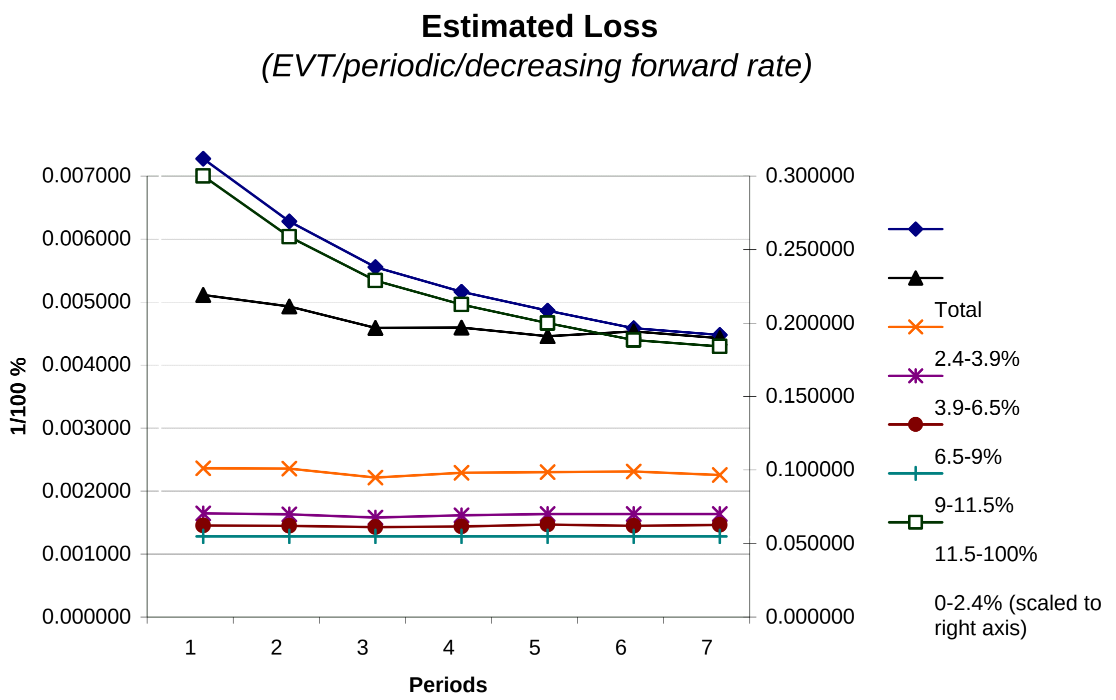
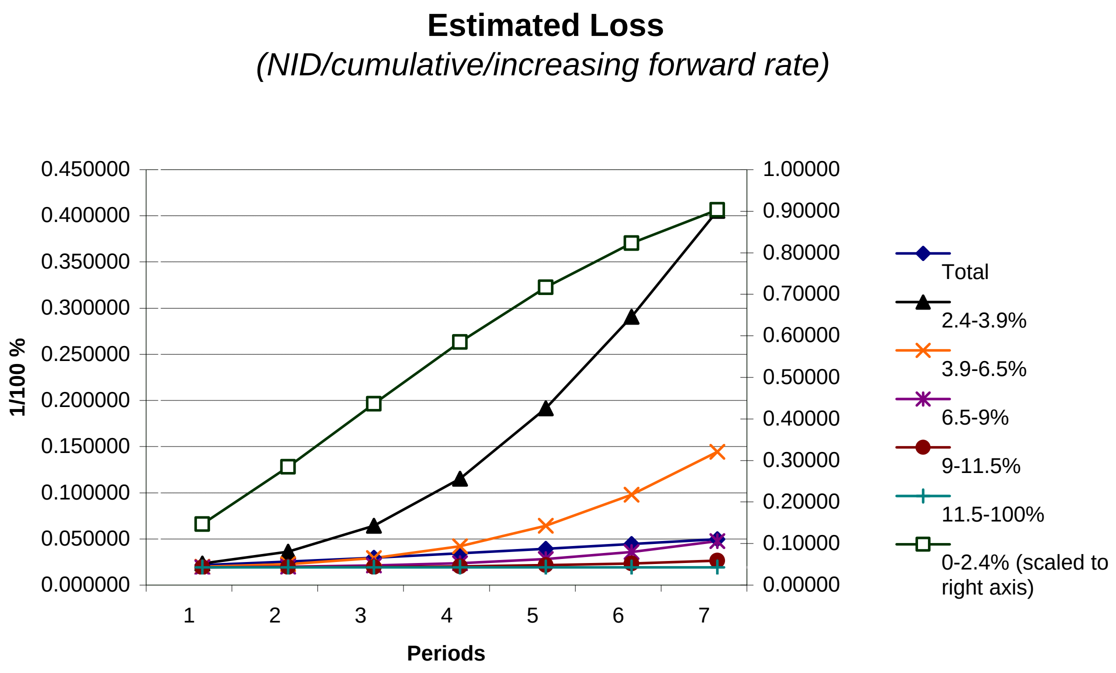


Figure A6. *Expected loss in tranches for an improving (decreasing forward rate of default) portfolio given an underlying extreme value distribution, where the first loss position [0-2.4%] is scaled to the right axis.*



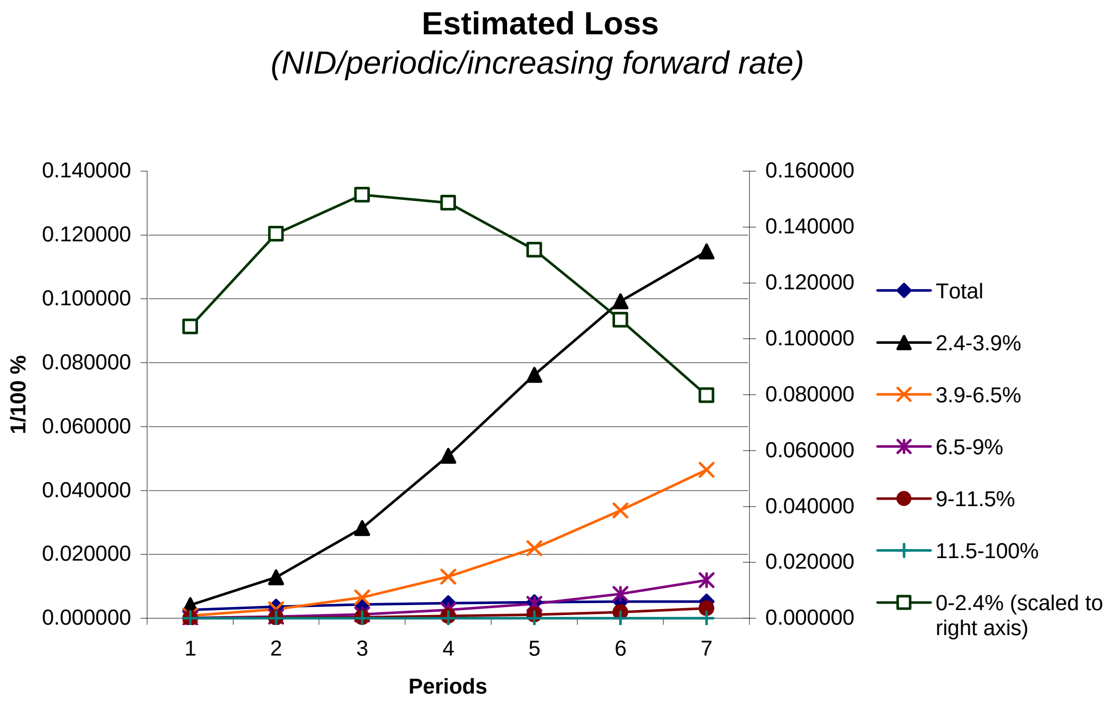
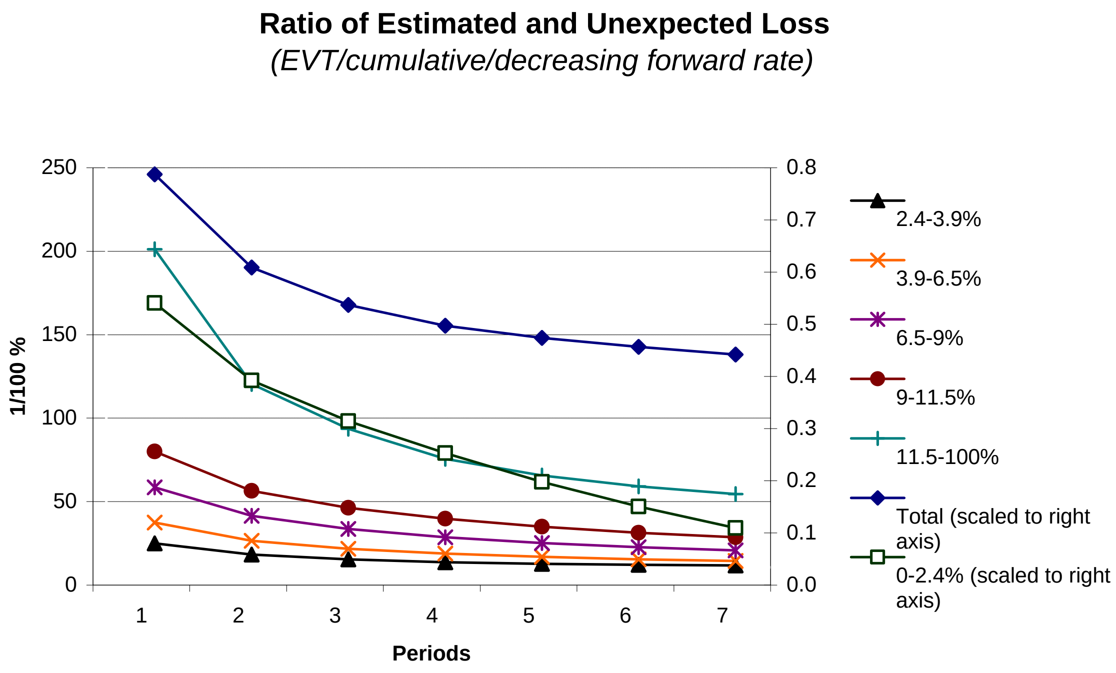


Figure A7. *Expected loss in tranches for a deteriorating (increasing forward rate of default) portfolio given an underlying normal inverse distribution, where the first loss position [0-2.4%] is scaled to the right axis.*



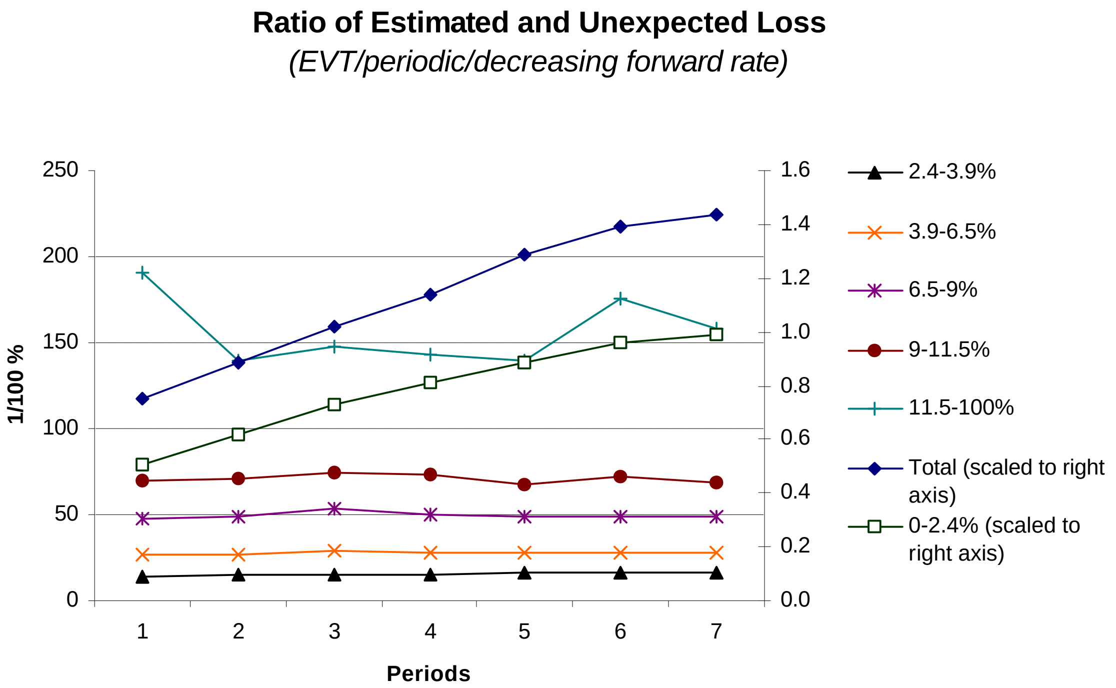
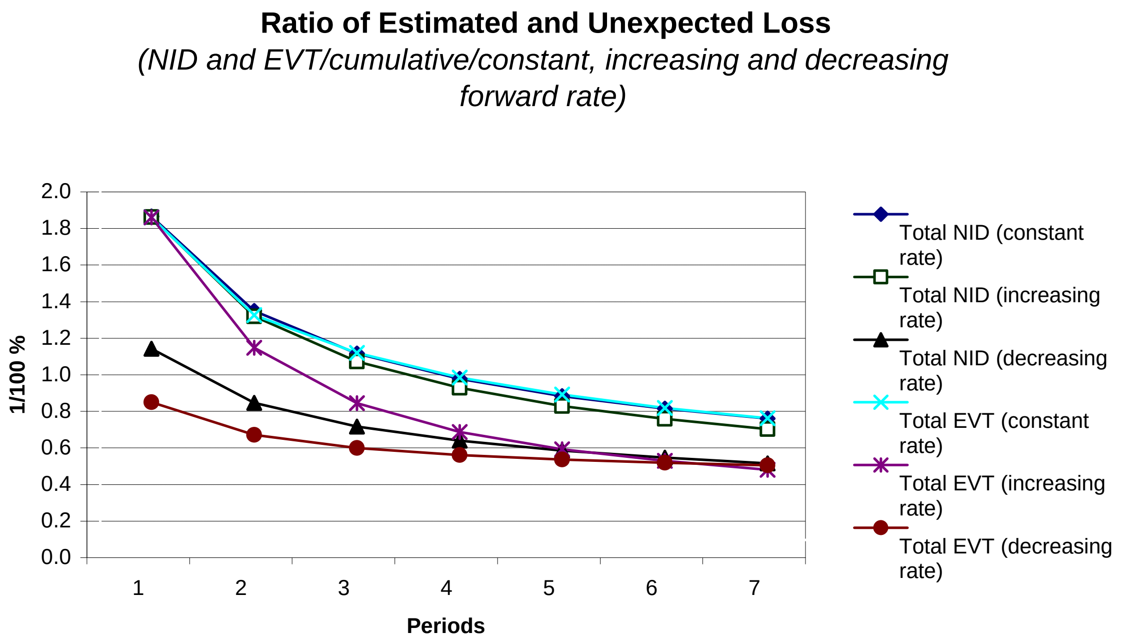


Figure A8. *Term structure of the* $s_{\tilde{L}_j^k} / \tilde{L}_j^k$ *ratio for all tranches of the collateral portfolio on the basis of uniform default characteristics of tranches (EVT) for a decreasing forward rate of default (cumulative and periodic loss).*



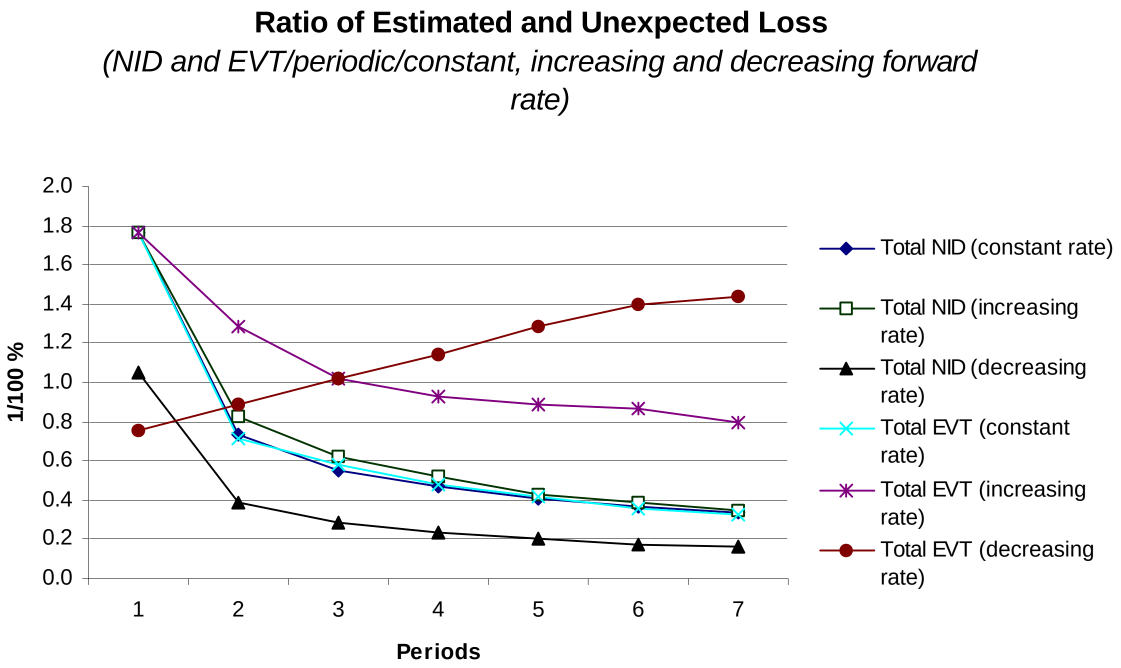
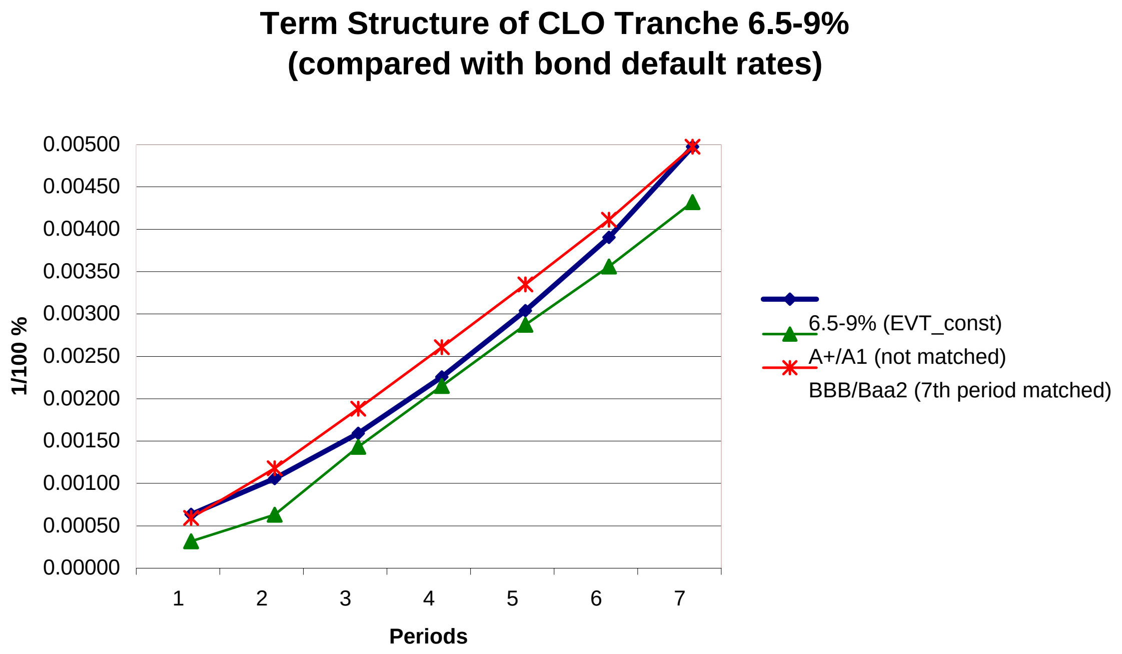


Figure A9. *Term structure of the* $s_{\tilde{L}_j^k} / \tilde{L}_j^k$ *ratio for the entire collateral portfolio on the basis of uniform default characteristics of tranches (on the basis of EVT and NID loss functions) for a constant, increasing and decreasing forward rate of default (cumulative and periodic loss).*



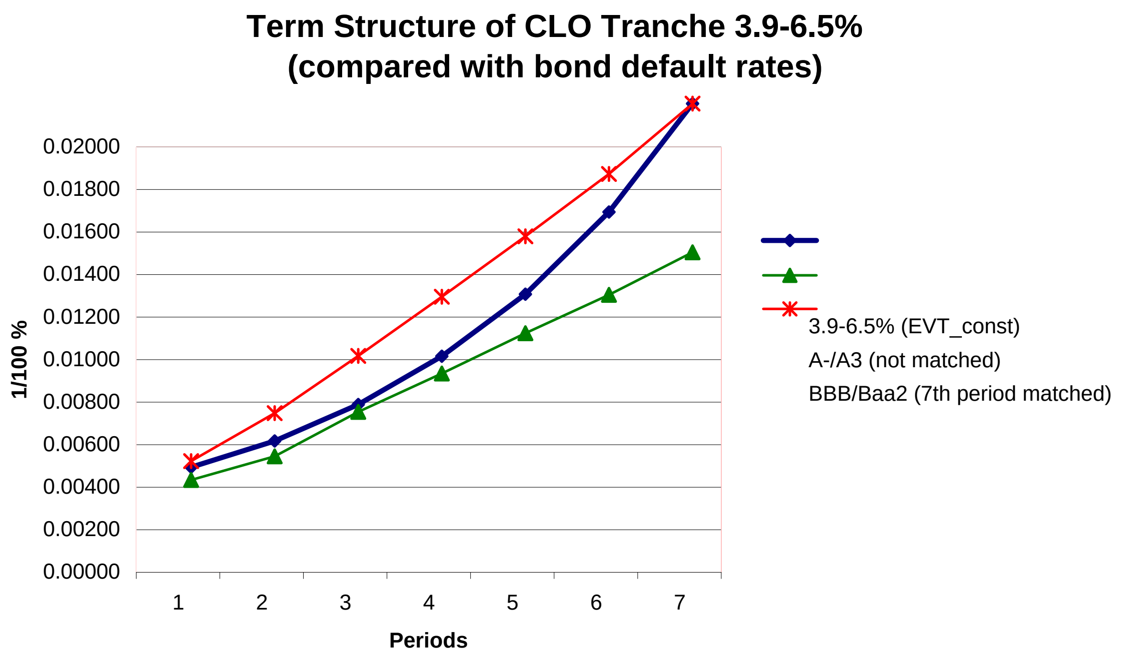
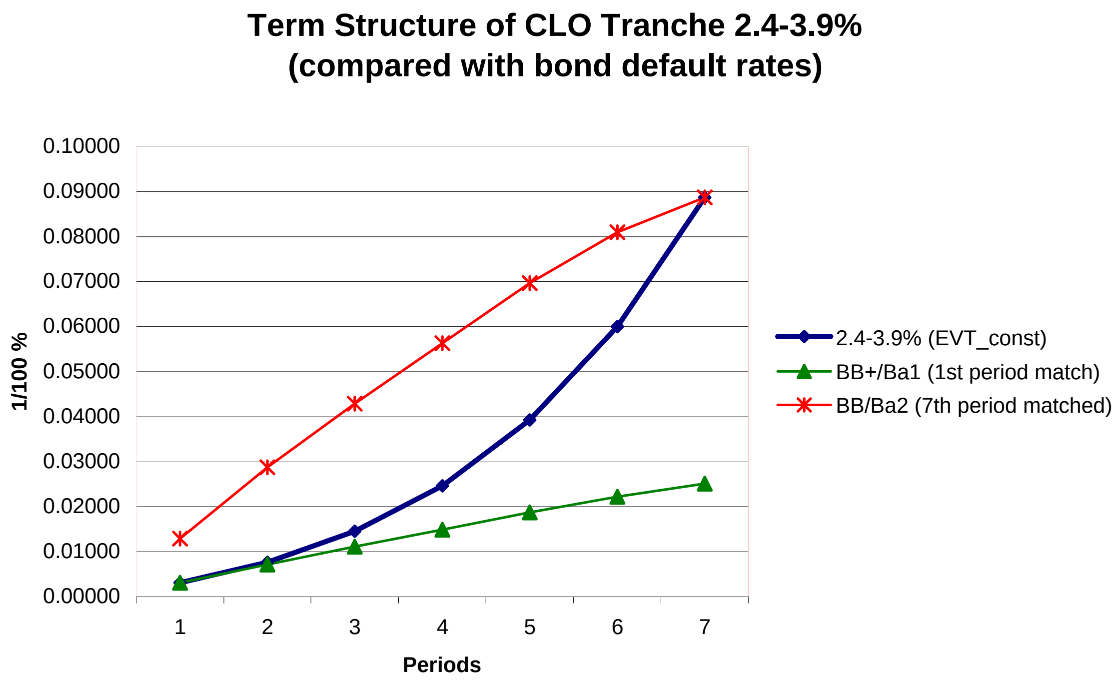


Figure A10. *Term structure of expected losses in the senior "investor tranches" [3.9-6.5%] and [6.5-9%] at a constant forward rate of default of $p=0.0026$ on the basis of an extreme value theory loss function compared with corporate zero-coupon bonds.*



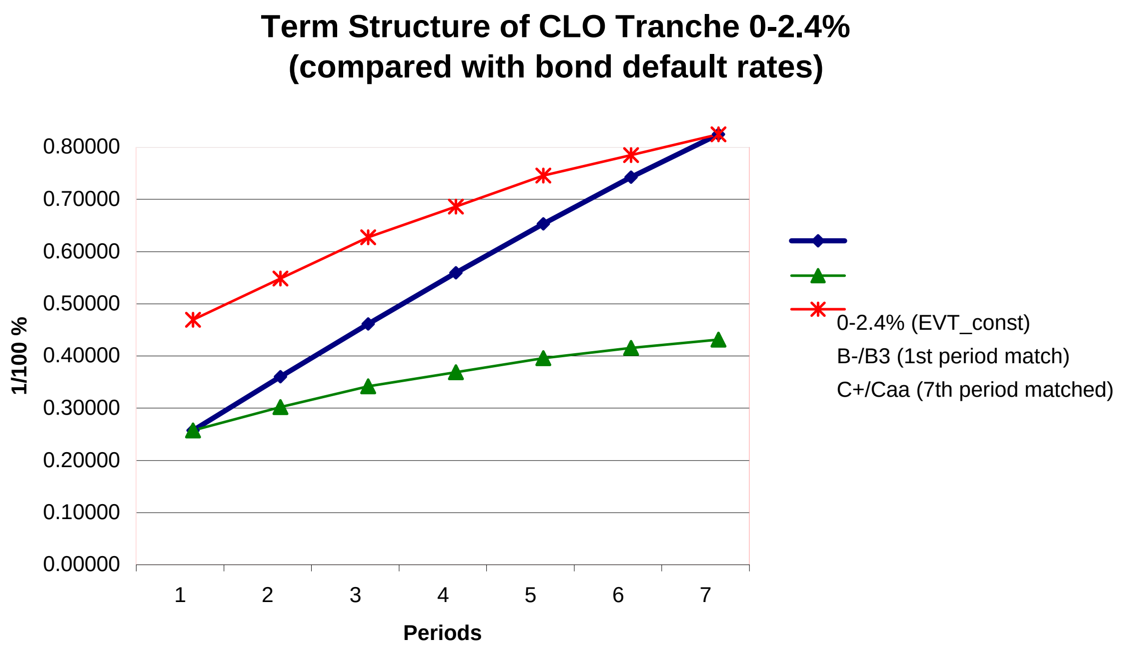


Figure A11. *Term structure of expected losses in the most junior "investor tranche" [2.4-3.9%] and the first loss position ([0-2.4%] tranche) at a constant forward rate of default of p=0.0026 on the basis of an extreme value theory loss function compared with corporate zero-coupon bonds.*

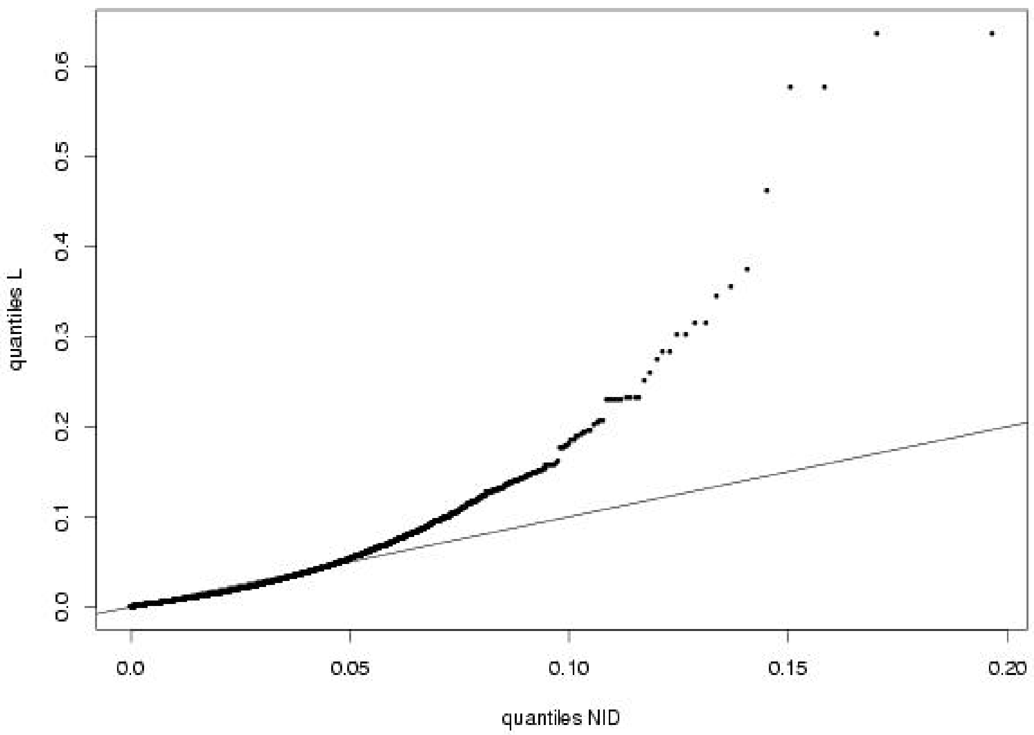


Figure A12. *Comparison of the term structures of expected losses on the basis of the q-q-plot for the simulation of portfolio losses and subsequent loss cascading on the basis of a normal inverse distribution and an extreme value theory loss function.*

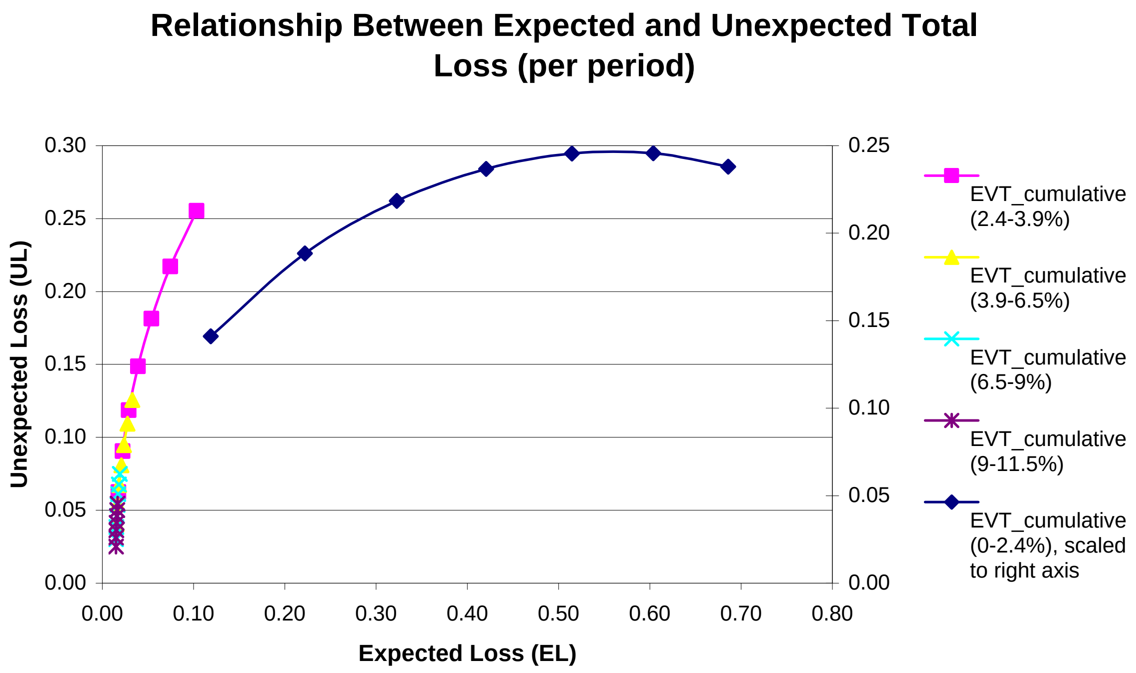


Figure A13. *Relationship between estimated and unexpected losses (per period) on an EVT loss function of credit default*

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