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Ramsey Monetary Policy and International Relative Prices*

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Abstract:
We analyze welfare maximizing monetary policy in a dynamic two-country model with price stickiness and imperfect competition. In this context, a typical terms of trade externality affects policy interaction between independent monetary authorities. Unlike the existing literature, we remain consistent to a public finance approach by an explicit consideration of all the distortions that are relevant to the Ramsey planner. This strategy entails two main advantages. First, it allows an accurate characterization of optimal policy in an economy that evolves around a steady-state which is not necessarily efficient. Second, it allows to describe a full range of alternative dynamic equilibria when price setters in both countries are completely forward-looking and households' preferences are not restricted. In this context, we study optimal policy both in the long-run and along a dynamic path, and we compare optimal commitment policy under Nash competition and under cooperation. By deriving a second order accurate solution to the policy functions, we also characterize the welfare gains from international policy cooperation.

JEL Classification: E52, F41

Keywords: Optimal Monetary Policy, Ramsey planner, Nash equilibrium, Cooperation, sticky prices, imperfect competition

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1 Introduction

In the classic approach to the study of optimal policy in dynamic economies (Ramsey (1927), Atkinson and Stiglitz (1980), Lucas and Stokey (1983), Chari, Christiano and Kehoe (1992)), and in a typical public finance spirit, a Ramsey planner maximizes household's welfare subject to a resource constraint, to the constraints describing the equilibrium in the private sector economy, and via an explicit consideration of all the distortions that characterize both the long-run and the cyclical behavior of the economy.

Recently there has been a resurgence of interest for a Ramsey-type approach in dynamic general equilibrium models with nominal rigidities. Khan, King and Wolman (2003) analyze optimal monetary policy in a closed economy where the relevant distortions are imperfect competition, staggered price setting and monetary transaction frictions. Schmitt-Grohe and Uribe (2003), and Siu (2003) focus on the joint optimal determination of monetary and fiscal policy. The robust conclusion of these studies - that optimal policy is associated to the prescription of stable inflation - is indeed rooted in the principle that the planner tries to eliminate the distortions induced by fluctuations in the aggregate price level, whether stemming from relative price misalignments or from resource costs of resetting prices.

In this paper we characterize welfare maximizing monetary policy in a two-country world where financial markets are complete, policymakers act under commitment and compete in a Nash equilibrium. Both economies are characterized by two distortions: output is inefficiently low (due to the presence of monopolistic competitive goods markets) and firms face quadratic costs of adjusting prices. However, and relative to a cooperative setting enforced by a world Ramsey planner, openness per se adds a further inefficiency typical of the outcome under a Nash equilibrium. This inefficiency stems from the monopoly power that each country can exert on its own terms of trade, and therefore from an externality that the policy competition motive necessarily entails.[1]

Relative to the corresponding closed economy literature, a Ramsey-type approach has received much less attention in the analysis of optimal monetary and exchange rate arrangements for open economies. Cooley and Quadrini (2003) analyze monetary policy interaction in a model with perfectly competitive goods markets, flexible prices and limited financial markets participation. Their model is essentially static in nature and highlights the presence of a systematic inflation bias induced by international policy competition. Our framework differs from theirs in the fact that prices are sticky (so that *nominal* exchange rate movements exert an effect on international

[1] The idea that terms of trade spillovers generate an externality and therefore room for international (monetary and/or fiscal) policy coordination is already discussed (although within ad-hoc models) in Canzoneri and Henderson (1991), Persson and Tabellini (1995) and dates back in the trade literature at least to Johnson (1954). Chari and Kehoe (1990) discuss the specific role of terms of trade distortions for optimal fiscal policy in a two-country general equilibrium model. More recently, see Corsetti and Pesenti (2000), Tille (2001), Benigno and Benigno (2003), Sutherland (2002).

relative prices), goods markets are imperfectly competitive and agents operate in a fully dynamic environment.

A Ramsey-type approach has also been employed in a certain stream of the so-called New Open Economy Macroeconomics literature (which instead typically features nominal rigidities and imperfect competition). This is the case - for instance - in the work of Benigno and Benigno (2003b), Corsetti and Pesenti (2002), and Devereux and Engel (2003). However, although elegant, these are stylized frameworks in which the analysis of optimal policy is simplified by the assumption that prices (or wages) are predetermined one-period. Such an assumption is restrictive, for it typically generates a Lucas-type aggregate supply curve in which the forward-looking nature of inflation is neglected, and along with it the channel through which the anticipation of future policy conduct comes to play a role.[2] Our work differs from the aforementioned contributions in that it employs optimizing producers' price setting decisions that are forward-looking, thereby rendering the corresponding optimal policy problem inherently dynamic.

Our analysis can be summarized in terms of three main contributions. First, we show that policy competition in an international setting leads welfare maximizing but independent policymakers to generally deviate from the prescription of price stability. Intuitively, in an open economy, the wedge between the marginal rate of substitution (between consumption and leisure) and the marginal rate of transformation depends not only on the fact that markups are time-varying (due to monopolistic competition coupled with sticky prices), but also on the dynamic behavior of the terms of trade. Hence each country tries to engineer price level movements to try to tilt relative prices in its own favour. On the other hand, when policy is set in a centralized fashion by a world Ramsey planner, the two countries manage to coordinate their actions in such a way to replicate very closely the equilibrium dynamics that would prevail under purely flexible prices (thereby mimicking closely the outcome of a corresponding closed economy).

Second, and more generally, our approach allows to study optimal policy in dynamic economies that evolve around a steady-state which is not necessarily efficient. In that, it differs crucially from a recurrent approach in the recent New-Keynesian literature that forces another (complementary) policy instrument (e.g., fiscal subsidies) to offset second order effects of stochastic uncertainty on the mean levels of variables.[3] The same approach resorts to a two-step strategy that involves, at first, taking a log-linear approximation of the competitive equilibrium conditions, and then a quadratic approximation of the correct households' utility function. In particular, resorting to such an approximation method in an open economy requires specific assumptions on preferences, such as log-utility and unitary elasticity of substitution between goods produced in different countries. Yet

[2] It is by now well understood that this entails a major consequence in that it neglects the sense in which (time consistent) discretionary policies are suboptimal in dynamic environments with forward-looking price (and/or wage) decisions (Woodford, 2003).

[3] See, for instance, Rotemberg and Woodford (1997), Woodford (2003), Benigno and Benigno (2003), Clarida, Gali and Gertler (2002).

precisely these assumptions already constrain the form of the optimal policy to coincide, somewhat artificially, with the one that implements the flexible price allocation. Furthermore, if not satisfied, the same conditions do not allow to study each country's policymaker's problem independently, forcing to ignore those equilibria that emerge under policy competition and to restrict the analysis only to the world planner's policy design problem.[4]

Third, we argue that, in this framework, welfare gains from cooperation, although positive, are small. To reach this conclusion, once the efficiency conditions of the corresponding optimal policy problem have been characterized, we resort to second order approximation methods (in the neighborhood of the specified Ramsey steady-state).[5] This is required to account for the fact that when business cycle fluctuations are centered around a distorted steady state stochastic volatility affects the first moments of those variables that may be critical for the household's welfare evaluation.[6]

The remainder of the paper is organized as follows. Section 2 and 3 describe respectively the economic environment and the features of the equilibrium. Section 4 derives the form of the constraints that are relevant to the planner's policy problem. Section 5 analyzes optimal policy under commitment. Section 6 explores the welfare gains from cooperation. Section 7 concludes.

2 The Model

The world economy consists of two countries, that we label them Home and Foreign. Each economy is populated by infinite-lived agents, whose total measure is normalized to unity.

2.1 Domestic Households

Let's denote by $C_t \equiv [(1-\alpha)^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}}]^{\frac{\eta-1}{\eta}}$ a composite consumption index of domestic and imported bundles of goods, where α is the balanced-trade steady state share of imported goods (i.e., an inverse measure of home bias in consumption preferences), and $\eta > 0$ is the elasticity of substitution between domestic and foreign goods. Each bundle is composed of imperfectly substitutable varieties (with elasticity of substitution $\varepsilon > 1$). Optimal allocation of expenditure

[4]More recently, Benigno and Woodford (2003) show (within a closed economy model) how to preserve a quadratic approximation of the household's welfare objective in the case in which the economy fluctuates around a non-efficient steady-state. This per se requires taking a second order approximation also of (some of) the underlying equilibrium conditions. Benigno and Benigno (2003a) and Pappa (2003) apply this approximation method to a two-country optimal policy dynamic model.

[5]Incidentally, one may want to notice that this entails a strategy which exactly reverses the logic of the approximation method described above, and largely employed in the recent literature.

[6]For the development and the application of second order approximation methods for welfare evaluation see Bergin and Tchakarov (2003), Schmitt-Grohe and Uribe (2003), Kolmann (2002), Kim and Kim (2002), Sims (2001).

within each variety of goods yields:

$$C_{H,t}(i) = \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon} C_{H,t} \; ; C_{F,t}(i) = \left(\frac{P_{F,t}(i)}{P_{F,t}}\right)^{-\varepsilon} C_{F,t} \tag{1}$$

where $C_{H,t} \equiv \int_0^1 [C_{H,t}(i)^{\frac{\epsilon-1}{\epsilon}} di]^{\frac{\epsilon}{\epsilon-1}}$ and $C_{F,t} \equiv \int_0^1 [C_{F,t}(i)^{\frac{\epsilon-1}{\epsilon}} di]^{\frac{\epsilon}{\epsilon-1}}$.

Optimal allocation of expenditure between domestic and foreign bundles yields:

$$C_{H,t} = (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t ; \; C_{F,t} = \alpha \left(\frac{P_{F,t}}{P_t}\right)^{-\eta} C_t \tag{2}$$

where $P_t \equiv [(1-\alpha)P_{H,t}^{1-\eta} + \alpha P_{F,t}^{1-\eta}]^{\frac{1}{1-\eta}}$ is the CPI index.

We assume the existence of complete markets for state-contingent money claims expressed in units of domestic currency.[7] Let $s^t = \{s_0, ...s_t\}$ denote the history of events up to date t, where s_t is the event realization at date t. The date 0 probability of observing history s^t is given by $\rho(s^t)$. The initial state s^0 is given so that $\rho(s^0) = 1$. Agents maximize the following expected discounted sum of utilities over possible paths of consumption and labor:

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t U(C_t, N_t) \right\} \tag{3}$$

where where $E_0\{\}$ denotes the mathematical expectations operator conditioned on s_0, and N_t denotes labor hours.[8] We assume that period utility is separable in its arguments. At the beginning of time t the households receive a nominal labor income of $W_t N_t$. To insure their consumption pattern against random shocks at time t they decide to spend $\nu_{t+1,t} B_{t+1}$ in nominal state contingent securities where $\nu_{t,t+1} \equiv \nu(s^{t+1}|s^t)$ is the pricing kernel of the state contingent portfolio. Each state contingent asset B_{t+1} pays one unit of domestic currency at time $t+1$ and in state s^{t+1}. Hence the sequence of budget constraints, after considering the optimal expenditure conditions (1) and (2), assumes the following form:

$$P_t C_t + \sum_{s^{t+1}} \nu_{t+1,t} B_{t+1} \leq W_t N_t + \tau_t + B_t + \int_0^1 \Gamma_t(i) \tag{4}$$

[7] Given that, in our setting, the law of one price holds continually, the unit of denomination of the payoffs of state-contingent assets is not strictly relevant. Alternatively, e.g., in the case in which deviations from the law of one price are due to consumer currency pricing, as in Devereux and Engel (2003), the distinction between nominal and real payoffs would be relevant for the specification of the equilibrium.

[8] Hence the expression for lifetime utility is equivalent to writing

$$\sum_{t=0}^{\infty} \sum_{s^t} \beta^t U(C(s^t)) \rho(s^t)$$

where $\rho(s^t) = \rho(s_t|s_0)$.

where τ_t are government net transfers of domestic currency and $\Gamma_t(i)$ are the profits of monopolistic firm i, whose shares are owned by the domestic residents.[9] Households choose the set of processes $\{C_t, N_t\}_{t=0}^{\infty}$ and bonds $\{B_{t+1}\}_{t=0}^{\infty}$, taking as given the set of processes $\{P_t,\, W_t,\, \upsilon_{t+1,t}\}_{t=0}^{\infty}$ and the initial wealth B_0 so as to maximize (3) subject to (4).

For any given state of the world, the following set of efficiency conditions must hold:

$$U_{c,t}\frac{W_t}{P_t} = -U_{n,t} \tag{5}$$

$$\beta\frac{P_t}{P_{t+1}}\frac{U_{c,t+1}}{U_{c,t}} = \nu_{t+1,t} \tag{6}$$

$$\lim_{j\to\infty} E_t\left\{\nu_{t+j,t}B_{t+j}\right\} = 0 \tag{7}$$

where $U_{\ ,t}$ defines the first order derivative of utility with respect to its argument $\varkappa = C, N$. Our separability assumption implies $U_{cn,t} = U_{nc,t} = 0$. Equation (5) equates the CPI-based real wage to the marginal rate of substitution between consumption and leisure. Optimality requires that the first order conditions (5), (6) and the no-Ponzi game condition (7) are simultaneously satisfied.[10]

The conditional expected return on the state contingent asset is given by $R_{n,t}$, so that, by arbitrage, it holds

$$R_{n,t}^{-1} \equiv E_t\left\{\nu_{t+1,t}\right\}$$

2.2 Law of One Price and Foreign Demand

We assume throughout that the *law of one price* holds, implying that $P_F(i) = \mathcal{E}\ P_F^*(i)$ for all $i \in [0,1]$, where $\mathcal{E}$ is the nominal exchange rate, i.e., the price of foreign currency in terms of home currency, and $P_F^*(i)$ is the price of foreign good i denominated in foreign currency. Let's denote by B^F foreign households' holdings of the state contingent bond denominated in domestic currency. The budget constraint of the foreign representative household will read:

$$P_t^*C_t^* + \sum_{s^{t+1}}\nu_{t+1,t}\frac{B_{t+1}^F}{\mathcal{E}_t} \leq W_t^*N_t^* + \tau_t^* + \frac{B_t^F}{\mathcal{E}_t} + \int_0^1 \Gamma_t^*(i) \tag{8}$$

The efficiency condition for bonds' holdings is

$$\beta\frac{P_t^*\mathcal{E}_t}{P_{t+1}^*\mathcal{E}_{t+1}}\frac{U_{c^*,t+1}^*}{U_{c^*,t}^*} = \nu_{t+1,t} \tag{9}$$

[9] Each domestic household owns an equal share of the domestic monopolistic firms.

[10] Notice that we do not introduce money explicitly, but rather think of it as playing the role of nominal unit of account. For the sake of simplicity, this allows us to abstract from an additional distortion stemming from the presence of transactions frictions. See Khan, King and Wolman (2003) for an analysis in which transactions frictions interact with monopolistic competition and price staggering in a welfare maximizing monetary policy problem.

Foreign demand for domestic variety i must satisfy:

$$
\begin{aligned}
C^*_{H,t}(i) &= \left(\frac{P^*_{H,t}(i)}{P^*_{H,t}}\right)^{-\varepsilon} C^*_{H,t} \\
&= \left(\frac{P^*_{H,t}(i)}{P^*_{H,t}}\right)^{-\varepsilon} \alpha^* \left(\frac{P^*_{H,t}}{P^*_t}\right)^{-\eta} C^*_t
\end{aligned}
\tag{10}
$$

The remaining efficiency conditions characterizing the foreign economy are then exactly symmetric to the ones of the domestic economy described above.

2.3 Domestic Producers

Each monopolistic firm i produces a homogenous good according to:

$$Y_t(i) = A_t \; N_t(i) \tag{11}$$

The cost minimizing choice of labor input implies:

$$\frac{W_t}{P_{H,t}} = mc_t A_t \tag{12}$$

where mc denotes the real marginal cost. Changing output prices is subject to some costs. We follow Rotemberg (1982) and model the cost of adjusting prices for each firm i equal to:

$$\psi_t(i) \equiv \frac{\theta}{2}\left(\frac{P_{H,t}(i)}{P_{H,t}} - 1\right)^2 \tag{13}$$

where the parameter θ measures the degree of price stickiness. The higher the θ the more sluggish is the adjustment of nominal prices. If $\theta = 0$ prices are flexible. The cost of price adjustment renders the domestic producer's pricing problem dynamic. Each producer chooses the price $P_{H,t}(i)$ of variety i to maximize its total market value:

$$E_t \left\{ \sum_{t=0}^{\infty} \beta^t \lambda_t \; \frac{D_t(i)}{P_{H,t}} \right\} \tag{14}$$

subject to the constraint

$$Y_t(i) \leq \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon} (C_{H,t} + C^*_{H,t}) \tag{15}$$

where $\beta^t \lambda_t$ measures the marginal utility value to the representative producer of additional profits expressed in domestic currency, and where

$$\frac{D_t(i)}{P_{H,t}} \equiv \frac{P_{H,t}(i)Y_t(i)}{P_{H,t}} - \frac{W_t}{P_{H,t}}N_t - \frac{\theta}{2}\left(\frac{P_{H,t}(i)}{P_{H,t-1}(i)} - 1\right)^2$$

The first order condition of the above problem reads

$$\begin{aligned} 0 &= \lambda_t \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\epsilon} \frac{C_t^W}{P_{H,t}} \left((1-\varepsilon) + \varepsilon \frac{W_t}{A_t P_{H,t}} \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-1}\right) \\ &\quad -\lambda_t \theta \left(\frac{P_{H,t}(i)}{P_{H,t-1}(i)} - 1\right) \frac{1}{P_{H,t-1}(i)} \; + \; \beta\lambda_{t+1}\theta \left(\frac{P_{H,t+1}(i)}{P_{H,t}(i)} - 1\right) \frac{P_{H,t+1}(i)}{P_{H,t}(i)^2} \end{aligned} \tag{16}$$

Let's define $\widetilde{p}_{H,t} \equiv \frac{P_{H,t}(i)}{P_{H,t}}$ as the relative price of domestic variety i and $\pi_{H,t} \equiv \frac{P_{H,t}}{P_{H,t-1}}$ as the gross domestic *producer inflation rate*. It is useful to see that the above condition can be rewritten as

$$\begin{aligned} 0 &= \lambda_t C_t^W \widetilde{p}_{H,t}{}^{-\varepsilon} \left((1-\varepsilon) + \varepsilon \frac{W_t}{A_t P_{H,t}}\right) - \\ &\quad \lambda_t \theta \left(\pi_{Ht,1} \frac{\widetilde{p}_{H,t}}{\widetilde{p}_{H,t-1}} - 1\right) \frac{\pi_H, t}{\widetilde{p}_{H,t-1}} \\ &\quad + \beta\lambda_{t+1}\theta \left(\pi_{H,t+1} \frac{\widetilde{p}_{H,t+1}}{\widetilde{p}_{H.t}} - 1\right) \pi_{H,t+1} \frac{\widetilde{p}_{H,t+1}}{\widetilde{p}_{H,t}{}^2} \end{aligned} \tag{17}$$

3 Equilibrium in the Home Economy

We focus our attention on a *symmetric* equilibrium where all domestic producers charge the same price, adopt the same technology and therefore choose the same demand for labor. This implies that $\widetilde{p}_{H,t} = 1$, $N_t(i) = N_t$, $\Gamma_t(i) = \Gamma_t$ for all i, t.

In such an equilibrium equation (17) will simplify to

$$\lambda_t \pi_{H,t}(\pi_{H,t} - 1) = \beta E_t \left\{\lambda_{t+1}\pi_{H,t+1}(\pi_{H,t+1} - 1)\right\} + \frac{\lambda_t \varepsilon A_t N_t}{\theta}\left(mc_t - \frac{\varepsilon - 1}{\varepsilon}\right) \tag{18}$$

The total net supply of bonds must satisfy

$$B_t + B_t^F = 0$$

Market clearing for domestic variety i must satisfy:

$$\begin{aligned} Y_t(i) &= C_{H,t}(i) + C^*_{H,t}(i) + \psi_t(i) \\ &= \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon} \left[\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} (1-\alpha)C_t + \left(\frac{P^*_{H,t}}{P^*_t}\right)^{-\eta} \alpha^* C^*_t\right] + \psi_t(i) \end{aligned} \tag{19}$$

for all $i \in [0,1]$ and t. Plugging (19) into the definition of aggregate output $Y_t \equiv \left[\int_0^1 Y(i)^{1-\frac{1}{\varepsilon}}\, di\right]^{\frac{\varepsilon}{\varepsilon-1}}$, and recalling that $P_{H,t} = \mathcal{E}_t P^*_{H,t}$, we can express the resource constraint as

$$A_t N_t = \left(\frac{P_{H,t}}{P_t}\right)^{-\eta} (1-\alpha)\, C_t + \left(\frac{P_{H,t}}{\mathcal{E}_t P^*_t}\right)^{-\eta} \alpha^* C^*_t + \psi_t \tag{20}$$

4 Deriving the Relevant Constraints

As mentioned before, the optimal policy is determined by a monetary authority that maximizes the discounted sum of utilities of all agents given the constraints that characterize the competitive economy. Our next task is to select the relations that represent the relevant constraints in the planner's optimal policy problem. This amounts to describing the competitive equilibrium in terms of a minimal set of relations involving only real allocations, in the spirit of the primal approach described in Lucas and Stokey (1983). There is a difference, though, between that classic approach and the one followed here, which stems from the impossibility, in the presence of sticky prices, of reducing the planner's problem to a maximization only subject to a single implementability constraint. Khan, King and Wolman (2003) and Schmitt-Grohe and Uribe (2002) face a similar problem in their analysis of dynamic optimal policy problems in the presence of price stickiness.

4.1 Resource and Budget Constraints

Let's begin by analyzing the domestic goods market equilibrium condition (20). This can be rewritten as

$$\begin{aligned} A_t N_t &= (1-\alpha)\, C_t \Phi_t^\eta + Q_t^\eta \Phi_t^\eta \alpha^* C^*_t + \psi_t \\ &= \Phi_t^\eta \left((1-\alpha)\, C_t + \left(\frac{\kappa U^*_{c^*,t}}{U_{c,t}}\right)^\eta \alpha^* C^*_t + \frac{\theta}{2} (\pi_{H,t} - 1)^2 \right) \end{aligned} \tag{21}$$

Symmetrically, the resource constraint in the Foreign country will read:

$$A^*_t N^*_t = (\Phi^*_t)^\eta \left((1-\alpha^*)\, C^*_t + \left(\frac{U_{c,t}}{\kappa U^*_{c^*,t}}\right)^\eta \alpha C_t + \frac{\theta}{2} \left(\pi^*_{F,t} - 1\right)^2 \right) \tag{22}$$

Next we turn to the budget constraint of the Home consumers. By substituting the government budget constraint (which implies $\tau_t = 0$ for all t) into equation (4), imposing (7) and iterating, one obtains (in units of domestic currency):

$$B_0 + \sum_{t=0}^{\infty} \sum_{s^t} z_{0,t} \left[W_t N_t + \Gamma_t\right] = \sum_{t=0}^{\infty} \sum_{s^t} z_{0,t} P_t C_t \tag{23}$$

where the price system $z_{0,t}$ is obtained after iteration of equation (6) and can be expressed as (for each possible state s_t)

$$z_{0,t} = \beta^t \rho_t \frac{U_{c,t}}{P_t} \frac{P_0}{U_{c,0}} \tag{24}$$

Notice, next, that aggregate real profits can be written as:

$$\frac{\Gamma_t}{P_t} = \frac{(1 - mc_t) A_t N_t - \frac{\theta}{2}(\pi_{H,t} - 1)^2}{\Phi_t} \tag{25}$$

where $\Phi_t \equiv \frac{P_t}{P_{H,t}}$ is the *CPI to PPI ratio*

By summing over all possible states s_t in equation (24), substituting (25), (5) and (44) into (23), we obtain the present value budget constraint for domestic households (expressed in real terms):

$$\widetilde{B}_0 + E_0 \sum_{t=0}^{\infty} \beta^t U_{c,t} \left[\frac{A_t N_t - \frac{\theta}{2}(\pi_{H,t} - 1)^2}{\Phi_t} \right] = E_0 \sum_{t=0}^{\infty} \beta^t U_{c,t} C_t \tag{26}$$

where $\widetilde{B}_0 \equiv \frac{B_0}{P_0} U_{c,0}$. This equation states that the sum of initial financial wealth and expected present discounted net income must match the expected presented discounted value of consumption.

We proceed in a similar fashion for the Foreign household. The price system $z_{0,t}^F = \nu_{1,0}\nu_{2,1}..\nu_{t,t-1}$, with $\nu_0 = 1$, expressed in units of domestic currency and obtained from the forward iteration of (9) can be written:

$$z_{0,t}^F = \left(\beta^t \rho_t \frac{U_{c^*,t}^*}{P_t^*} \frac{P_0^*}{U_{c^*,0}^*} \right) \frac{\mathcal{E}_0}{\mathcal{E}_t} \equiv z_{0,t}^* \frac{\mathcal{E}_0}{\mathcal{E}_t} = z_{0,t} \tag{27}$$

Equating with (24) implies the following condition

$$\kappa \frac{U_{c^*,t}^*}{U_{c,t}} = \frac{\mathcal{E}_t P_t^*}{P_t} \equiv Q_t \tag{28}$$

where Q_t is the real exchange rate and $\kappa \equiv \frac{\mathcal{E}_0 P_0^* U_{c,0}}{P_0 U_{c^*,0}^*}$ is a parameter capturing the initial cross-country distribution of wealth.[11] Below we discuss how this parameter signals the underlying risk-sharing arrangement between the two countries.

By taking conditional expectations of both sides of (27) and proceeding with similar substitutions to the ones operated in the Home case we obtain[12]

[11]See also Chari, Kehoe and McGrattan (2003).

[12]In particular one should note that

$$\widetilde{B}_0^* = \frac{B_0^F}{\mathcal{E}_0} \frac{U_{c,0}\ \kappa^{-1}}{P_0}$$

Since equilibrium requires $B_0^F = -B_0$ we obtain $\widetilde{B}_0^* = -\widetilde{B}_0 \kappa^{-1}$.

$$-\widetilde{B}_0\kappa^{-1} + E_0 \sum_{t=0}^{\infty} \beta^t U^*_{c^*,t} \left[\frac{A^*_t N^*_t - \frac{\theta}{2}(\pi^*_{F,t} - 1)^2}{\Phi^*_t} \right] = E_0 \sum_{t=0}^{\infty} \beta^t U^*_{c^*,t} C^*_t \tag{29}$$

4.2 Risk-sharing and PPP

Consider the domestic household maximizing (3) subject to (26). Efficiency requires

$$\beta^t U_{c,t} = \Omega \; z_{0,t} P_t \tag{30}$$

where Ω is the Lagrange multiplier on constraint (26). Notice that this mutiplier is *constant* across time and states. Symmetrically, for the Foreign household we have:

$$\begin{aligned} \beta^t U^*_{c,t} &= \Omega^* \; z^*_{0,t} P^*_t \\ &= \Omega^* \left(\frac{z^F_{0,t}}{\mathcal{E}_0} \right) \mathcal{E}_t P^*_t \end{aligned} \tag{31}$$

By combining (30), (31) and (28), and applying the normalization $\mathcal{E}_0 = 1$, one can write the risk-sharing parameter in terms of relative shadow values of net income:

$$\kappa = \frac{\Omega}{\Omega^*} \tag{32}$$

This allows the following definition of risk sharing:

Definition 1. *Complete international asset markets lead to perfect risk-sharing when the shadow value of the household's present value budget constraint is equalized across countries. From (32), this in turn requires that* $\kappa = 1$.

The risk sharing arrangement has implications on how the marginal utilities of consumption are linked across countries. By combining (26) with (21) and (28), and assuming (for the pure sake of simplicity) that the initial level of wealth is zero (so that $B_0 = B^*_0 = 0$) one can write

$$\sum_{t=0}^{\infty} \sum_{s^t} \frac{\beta^t U_{c,t}}{\Omega} \left\{ \Phi_t^{\eta-1} \left((1-\alpha) C_t + \left(\kappa \frac{U^*_{c,t}}{U_{c,t}} \right)^{\eta} \alpha^* C^*_t \right) - C_t \right\} = 0$$

and similarly for Foreign

$$\sum_{t=0}^{\infty} \sum_{s^t} \frac{\beta^t U^*_{c,t}}{\Omega^*} \left\{ (\Phi^*_t)^{\eta-1} \left((1-\alpha^*) C^*_t + \left(\kappa \frac{U^*_{c,t}}{U_{c,t}} \right)^{-\eta} \alpha C_t \right) - C^*_t \right\} = 0$$

Next we assume cross-country *symmetry*, so that $\alpha = \alpha^*$. It is important to notice that this does *not* necessarily imply that PPP holds, unless we make the further restrictive assumption of *absence of home bias,* which entails $\alpha = \alpha^* = \frac{1}{2}$.[13]

Using (32) one can solve for κ, obtaining:

$$\kappa = \frac{\sum_{t=0}^{\infty} \sum_{s^t} \beta^t U_{c,t} \left\{ \Phi_t^{\eta-1} Z_t - C_t \right\}}{\sum_{t=0}^{\infty} \sum_{s^t} \beta^t U_{c,t}^* \left\{ \left(\Phi_t^*\right)^{\eta-1} Z_t^* - C_t^* \right\}} \tag{33}$$

where $Z_t \equiv \left((1-\alpha) C_t + \left(\kappa \frac{U_{c,t}^*}{U_{c,t}} \right)^{\eta} \alpha C_t^* \right)$ and $Z_t^* \equiv \left((1-\alpha) C_t^* + \left(\kappa \frac{U_{c,t}^*}{U_{c,t}} \right)^{-\eta} \alpha C_t \right)$.

Hence $\kappa = 1$ (perfect risk-sharing) requires

$$\sum_{t=0}^{\infty} \sum_{s^t} \beta^t U_{c,t} \left\{ \Phi_t^{\eta-1} Z_t - C_t \right\} = \sum_{t=0}^{\infty} \sum_{s^t} \beta^t U_{c,t}^* \left\{ \left(\Phi_t^*\right)^{\eta-1} Z_t^* - C_t^* \right\} \tag{34}$$

Notice that the last expression does not necessarily imply a perfect equalization of the marginal utilities of consumption. The latter property follows only in the particular case of absence of home bias, which in turn implies that PPP holds. In this case, and recalling (28), condition $\kappa = 1$ requires $U_{c,t} = U_{c,t}^*$, and therefore $C_t = C_t^*$. It is then easy to verify that (34) also implies $\Phi_t = \Phi_t^*$ for all t.

4.3 Relative Prices and Price Setting Constraints

Below we define a series of relationships linking real quantities to the relevant relative prices in our framework. The terms of trade is the relative price of imported goods:

$$\mathcal{T}_t \equiv \frac{P_{F,t}}{P_{H,t}} \tag{35}$$

It can be related to the CPI-PPI ratio as follows

$$\Phi_t \equiv \frac{P_t}{P_{H,t}} = \left[(1-\alpha) + \alpha \mathcal{T}_t^{1-\eta} \right]^{\frac{1}{1-\eta}} \equiv g(\mathcal{T}_t) \tag{36}$$

with $g' > 0$. The terms of trade and the real exchange rate are linked through the following expression:

$$\begin{aligned} \mathcal{T}_t &= \frac{P_{F,t}\ \Phi_t}{P_t} \\ &= Q_t\ \frac{\Phi_t}{\Phi_t^*} \end{aligned} \tag{37}$$

[13] One can easily verify this by manipulating the CPI expression and substituting conditions $P_{H,t} = \mathcal{E}_t P_{H,t}^*$ and $P_{F,t} = \mathcal{E}_t P_{F,t}^*$, which are implied by the fact that the law of one price holds.

where

$$\Phi_t^* \equiv \frac{P_t^*}{P_{F,t}^*} = [(1-\alpha^*) + \alpha^* \mathcal{T}_t^{\eta-1}]^{\frac{1}{1-\eta}} \equiv g^*(\mathcal{T}_t) \tag{38}$$

with $g^{*'} < 0$.

We now wish to rewrite the relative prices Φ_t and Φ_t^* as a function or real allocations only. By combining (36), (37), (38) and (28), and recalling that α^*one can write

$$\Phi_t = \left(\frac{(1-\alpha^*) - \alpha \left(\kappa \frac{U_{c,t}^*}{U_{c,t}} \right)^{1-\eta}}{1-(\alpha+\alpha^*)} \right) \equiv h(C_t, C_t^*) \tag{39}$$

and symmetrically

$$\Phi_t^* = \left(\frac{(1-\alpha) - \alpha^* \left(\kappa \frac{U_{c,t}^*}{U_{c,t}} \right)^{\eta-1}}{1-(\alpha+\alpha^*)} \right) \equiv h^*(C_t, C_t^*) \tag{40}$$

Notice that when $\eta = 1$, (39) reduces to $\Phi_t = \Phi_t^* = 1$ for all t. In this particular case which corresponds to Cobb-Douglas consumption preferences, the policy competition motive on the terms of trade vanishes.

The CPI level can be linked to the domestic price level and aggregate consumption as follows: $P_t = P_{H,t}\ \Phi_t$. Let's then define gross *CPI inflation* as $\pi_t \equiv \frac{P_t}{P_{t-1}}$. This is related to domestic producer inflation and aggregate relative consumption as follows:

$$\pi_t = \pi_{H,t} \frac{\Phi_t}{\Phi_{t-1}} \tag{41}$$

The condition on optimal bond investment can then be rearranged accordingly. By taking conditional expectations of (6) we obtain

$$U_{c,t} = \beta E_t \left\{ R_t U_{c,t+1} \right\} \tag{42}$$

where

$$R_t = E_t \left\{ \frac{R_t^n P_t}{P_{t+1}} \right\} \tag{43}$$

is the CPI-based gross real interest rate.

Next we need to rearrange the optimality conditions for the production sector. This requires, at first, to express the real marginal cost and the real wage in terms of aggregate real quantities. Hence by combining (5) and (12) we can write

$$mc_t = -\frac{U_{n,t}}{U_{c,t}A_t}\Phi_t \tag{44}$$

This implies that the aggregate condition for optimal pricing (18) can be rewritten as

$$\begin{aligned} U_{c,t}\pi_{H,t}(\pi_{H,t}-1) &= \beta U_{c,t+1}E_t\left\{\pi_{H,t+1}(\pi_{H,t+1}-1)\right\} + \\ & \frac{U_{c,t}\varepsilon A_t N_t}{\theta}\left(-\frac{U_{n,t}}{U_{c,t}A_t}\Phi_t - \frac{\varepsilon-1}{\varepsilon}\right) \end{aligned} \tag{45}$$

An analogous condition will hold in Foreign:

$$\begin{aligned} U^*_{c^*,t}\pi^*_{F,t}(\pi^*_{F,t}-1) &= \beta U^*_{c^*,t}E_t\left\{\pi^*_{F,t+1}(\pi^*_{F,t+1}-1)\right\} + \\ & \frac{U^*_{c^*,t}\varepsilon A^*_t N^*_t}{\theta}\left(-\frac{U^*_{n,t}}{U^*_{c^*,t}A^*_t}\Phi^*_t - \frac{\varepsilon-1}{\varepsilon}\right) \end{aligned} \tag{46}$$

In the following, we formulate a proposition that establishes a mapping between the minimal form expressed above (summarized by conditions (21), (26), (45), for Home, and (22), (29), (46) for Foreign) and the set of allocations describing the (imperfectly) competitive equilibrium in the world economy.

Proposition 1. **(Part A)** *For a given initial symmetric wealth level* $\widetilde{B}_0$, *any equilibrium allocation* $\{C_t,\, Y_t,\, N_t,\, mc_t,\, Q_t,\, \Phi_t,\, \pi_{H,t},\, C_{F,t},\, C_{H,t}\}_{t=0}^{\infty}$ *satisfying equations (2),* (4), *(5)-(7), (12), (18), (20), the risk-sharing condition (28), along with a symmetric set of conditions holding for Foreign, also satisfies equations* (21), (26), (45), (22), (29) and (46). **(Part B)** *By reverse, using allocations* $\{C_t,\, N_t,\, \pi_{H,t}\}_{t=0}^{\infty}$ *and* $\{C^*_t,\, N^*_t,\, \pi^*_{F,t}\}_{t=0}^{\infty}$ *that satisfy equations (45), (21), (26) and (46), (22), (29), it is possible to construct all the remaining real allocations, nominal variables and policy instruments for Home and Foreign.*

Proof. See Appendix A.

5 Optimal Monetary Policy under Commitment

We now turn to the specification of the optimal policy problem in a dynamic context. We assume that *ex-ante commitment* is feasible. In this section we take full advantage of our characterization of the equilibrium conditions (in each country) in terms of a minimal set of relations involving only the choice of allocations for consumption and labor input along with the inflation instrument.

A distinctive feature of our Ramsey analysis is that we allow the relevant distortions characterizing the economy to remain explicit, *both in the short and in the long-run.*[14] This implies that

[14] See King and Wolman (1999) for a closed economy analog.

the policymaker in each country lacks a set of fiscal instruments necessary to achieve the first best allocation. Each economy is in fact characterized by three distortions. The first two, market power and price stickiness, are common to both the closed and the open economy version of our model. The price stickiness distortion, summarized by the quadratic term in inflation in the resource constraint, is obviously minimized at zero net inflation (i.e., $\pi_{H,t} = 1$ for all t). On the other hand, the market power distortion, stemming from the level of activity being inefficiently low, calls for the monetary authority to try to expand output and consumption.

What in general distinguishes the analysis of an open economy (and as also emphasized in Corsetti and Pesenti (2000)), is the presence of an additional inefficiency. This stems from the possibility for each country, in the presence of rigid nominal prices, of strategically affecting the terms of trade, and in turn try to increase its level of consumption for any given level of labor effort. This externality creates per se room for policy competition, and for the possibility of gains from cooperative policies. The interesting aspect concerns the extent to which such policy competition motive may lead each policymaker to try to deviate from the prescription of price stability that would typically characterize optimal policy in the closed economy version of our model.

5.1 Nash Competition

We begin by assuming that the policymaker in each country sets policy independently taking as given policy actions in the other country.

Definition 2. *Let's define* $\mathcal{U}(C_t, N_t, \pi_{H,t}, \Omega) \equiv U(C_t, N_t) + \Omega \left[U_{c,t} \left(C_t - \frac{A_t N_t - \frac{\theta}{2}(\pi_{H,t}-1)^2}{\Phi_t} \right) \right]$ *where* Ω *is the multiplier on constraint (26). Let* $\{\lambda_{p,t}, \lambda_{f,t}\}_{t=0}^{\infty}$ *represent sequences of Lagrange multipliers on the constraints (45) and (21) respectively. Let* $\widetilde{B}_0$ *be given. Then for given allocations* $\{C_t^*\}_{t=0}^{\infty}$ *and stochastic processes* $\{A_t, A_t^*\}_{t=0}^{\infty}$*, plans for the control variables* $\{C_t, \pi_{H,t}, N_t\}_{t=0}^{\infty}$*, and for the costate variables* $\{\lambda_{p,t}, \lambda_{f,t}\}_{t=0}^{\infty}$ *and* Ω*, represent a first best constrained allocation if they solve the following maximization problem:*

Choose $\Lambda_t^n \equiv \{\lambda_{p,t}, \lambda_{f,t}\}_{t=0}^{\infty}$ and $\Xi_t^n \equiv \{C_t, \pi_{H,t}, N_t\}_{t=0}^{\infty}$ to

$$Min_{\{\Lambda_t^n\}_{t=0}^{\infty}} \; Max_{\{\Xi_t^n\}_{t=0}^{\infty}} \; E_0\{ \sum_{t=0}^{\infty} \beta^t E_t\{\mathcal{U}(C_t, N_t, \pi_{H,t}, \Omega) \tag{47}$$

$$+\lambda_{p,t} \left[U_{c,t}\pi_{H,t}(\pi_{H,t}-1) - \beta U_{c,t+1}\pi_{H,t+1}(\pi_{H,t+1}-1) + \frac{U_{c,t}\varepsilon A_t N_t}{\theta} \left(\frac{U_{n,t}\;\Phi_t}{U_{c,t}A_t} + \frac{\varepsilon-1}{\varepsilon} \right) \right]$$
$$+\lambda_{f,t} \left[A_t N_t - (1-\alpha)\; C_t \Phi_t^{\eta} - \kappa^{\eta} \left(\frac{U^*_{c^*,t}}{U_{c,t}} \right)^{\eta} \Phi_t^{\eta} \alpha^* C_t^* - \frac{\theta}{2} \left(\pi_{H,t} - 1 \right)^2 \right] \}\} \;-\; \Omega \widetilde{B}_0$$

A series of observations on the nature of this policy problem are in order. Notice, first, that the distinctive feature of the commitment problem under Nash competition is that the Home policymaker does not internalize that the relative price $\Phi_t = h(C_t, C_t^*)$ depends also on the level of consumption in Foreign. It is key to our analysis that the relative price Φ_t enters pervasively in the behavioral relationships characterizing the optimal policy problem.

Second, it is of independent interest to notice that the present value budget constraint must be part of the policy maximization problem. In fact, and unlike the closed economy case, it is not implicitly satisfied by a combination of the government budget constraint and of the resource feasibility constraint. This dimension characterizes specifically the policy maximization problem in an open economy as opposed to the corresponding closed-economy case.

Third, in the following we assume that, prior to policy implementation, the initial wealth is inelastically supplied (in fact, $\widetilde{B}_0 = 0$ is given) and that policy is chosen *taking the initial risk-sharing arrangement as given.*[15] This has the crucial implication that, already in the Nash problem, each policymaker is in fact facing the same present value budget constraint.

Finally, it is important to notice that, as a consequence of the initial stock of wealth $\widetilde{B}_0$ being exogenously supplied, the multiplier Ω is taken as given in each policymaker's maximization problem. In other words, the initial stock of wealth does not depend on the anticipation about the future implementation of policy.[16]

5.1.1 Non-recursivity and Initial Conditions

As a result of the constraint (45) exhibiting future expectations of control variables, the maximization problem as spelled out in (47) is intrinsically non-recursive.[17] As first emphasized in Kydland and Prescott (1980), and then developed by Marcet and Marimon (1999), a formal way to rewrite the same problem in a recursive stationary form is to enlarge the planner's state space with additional (pseudo) costate variables. Such variables, that we denote χ_t and χ_t^* for Home and Foreign respectively, bear the crucial meaning of tracking, along the dynamics, the value to the planner of committing to the pre-announced policy plan. Another aspect concerns the specification of the law of motion of these lagrange multipliers. For in our case the forward-looking Phillips curve constraint features a simple one period expectation, the same costate variables have to obey the laws of motion:[18]

[15] We believe this is a realistic assumption given that the two policymakers are acting under commitment.

[16] See Schmitt-Grohe and Uribe (2003) for a small open economy model in which the probability of future "policy reform" is not negligible and therefore the determination of $\widetilde{B}_0$ is endogenous.

[17] See Kydland and Prescott (1977), Calvo (1978). As such the system does not satisfy per se the principle of optimality, according to which the optimal decision at time t is a time invariant function only of a small set of state variables.

[18] The laws of motion of the additional costate variables would take a more general form if the expectations horizon in the forward looking constraint(s) featured a more complicated structure, as, for instance, in the case of constraints in present value form. See Marcet and Marimon (1999).

$$\chi_{t+1} = \lambda_{p,t} \tag{48}$$

$$\chi^*_{t+1} = \lambda^*_{p,t} \tag{49}$$

A particularly important point concerns the definition of the appropriate initial conditions for χ_t and χ^*_t. Marcet and Marimon (1999) show that for the modified (recursive) Lagrangian in (47) to generate a global optimum under time zero commitment it must hold:

$$\chi_0 = 0 = \chi^*_0 \tag{50}$$

The above condition states that there is no value to the policy planner, in either country and as of time zero, attached to prior commitments. Commitment, in this context, bears exactly the meaning that while it would be technically feasible for the planner (in each country) to satisfy (50) for all $t > 0$, it would also be suboptimal to do so.

In *Appendix B* we show how to reformulate the optimal plan in an equivalent recursive stationary form. First order conditions for time $t \geq 1$ for the choice of C_t, N_t and $\pi_{H,t}$ imply respectively:

$$\begin{aligned} 0 = {} & U_{c,t} + U_{cc,t}\ \pi_{H,t}(\pi_{H,t}-1)\ (\lambda_{p,t}-\chi_t) + \frac{\lambda_{p,t}N_t}{\theta}\left(\epsilon\ U_{n,t}\Phi_{c,t} + (\varepsilon-1)A_tU_{cc,t}\right) \\ & -\lambda_{f,t}\ \left[(1-\alpha)\left(\Phi_t^{\eta} + \eta C_t\Phi_t^{\eta-1}\Phi_{c,t}\right) - \alpha^* C^*_t\kappa^{\eta}U^{*\eta}_{c,t}\left(\eta\Phi_t^{\eta-1}\Phi_{c,t}U_{c,t}^{-\eta} - \eta U_{c,t}^{-\eta-1}U_{cc,t}\Phi_t^{\eta}\right)\right] \\ & -\Omega\left[(A_tN_t - \frac{\theta}{2}(\pi_{H,t}-1)^2)\left(U_{cc,t}\Phi_t^{-1} - \Phi_{c,t}\Phi_t^{-2}U_{c,t}\right) - (U_{cc,t}C_t + U_{c,t})\right] \end{aligned} \tag{51}$$

$$0 = U_{n,t} + \frac{\lambda_{p,t}\varepsilon\Phi_t}{\theta}\left(U_{n,t} + N_tU_{nn,t}\right) + \lambda_{p,t}\frac{\varepsilon-1}{\theta}U_{c,t}A_t + \lambda_{f,t}A_t - \Omega\frac{U_{c,t}A_t}{\Phi_t} \tag{52}$$

$$0 = U_{c,t}(2\pi_{H,t}-1)\left(\lambda_{p,t}-\chi_t\right) - \theta(\pi_{H,t}-1)\left(\lambda_{f,t} - \Omega\frac{U_{c,t}A_t}{\Phi_t}\right) \tag{53}$$

The system of efficiency conditions for Home is completed by the law of motion (48), the initial condition (50) and by the constraints (45) and (21) holding with equality. Notice also that first order conditions evaluated at time $t = 0$ differ for what concerns equation (51), which must feature the additional term $-\Omega\frac{B_0}{P_0}U_{cc,0}$.[19]

Once defined a completely symmetric problem for the *policy maker in Foreign*, we can state the following definition of a Nash equilibrium:

Definition 3 *(Nash equilibrium under commitment) The set of processes* $\Lambda^n_t \equiv \{\lambda_{p,t}, \lambda_{f,t}\}_{t=0}^{\infty}$, $\Lambda^{n*}_t \equiv \{\lambda^*_{p,t}, \lambda^*_{f,t}\}_{t=0}^{\infty}$, $\Xi^n_t \equiv \{C_t,\ \pi_{H,t},\ N_t\}_{t=0}^{\infty}$, $\Xi^{n*}_t \equiv \{C^*_t,\ \pi^*_{F,t},\ N^*_t\}_{t=0}^{\infty}$ *and the multiplier* Ω *fully*

[19]Which, in particular, disappears under the particular assumption of $B_0 = 0$.

describe a Nash equilibrium under commitment if they solve the system of equations (51) - (53), equations (45), (21), (26) holding with equality, along with a similar set of conditions jointly holding for Foreign.

5.1.2 Nash-Optimal Inflation Rate in the Long-Run

To determine the long-run optimal inflation rate associated to the Nash-game described above, one needs to solve the steady state version of the set of efficiency conditions (51)-(53). In the language of the Ramsey-Cass-Koopmans model this amounts to computing the *modified golden rule* steady state, or, in other words, the *unconstrained* long-run optimal inflation rate. This contrasts with the *golden rule* inflation rate, which would correspond to the one that maximizes households' instantaneous utility under the constraint that the steady state conditions are imposed ex-ante. It is well known that in dynamic economies with discounted utility the two concepts of long-run optimal policy cannot coincide.

In *Appendix C* we characterize the system of equations that describes the long-run steady state associated to the optimal policy problem under Nash competition. Under the assumption $\alpha = \alpha^*$, and of zero initial wealth ($B_0 = 0$), the solution to the steady-state of the Nash game is symmetric and features $\pi_H = \pi_F^* = 1$, $\Phi = \Phi^* = 1$, $N = N^*$, and $C = C^*$, along with a non-zero value for the multiplier Ω. Hence the steady state of the solution to the Nash-optimal policy indicates that, if unconstrained, *both* policymakers would choose to set the economy along a path that would lead to a long-run net inflation rate of zero. The intuition for this result is simple. One can view the modified golden rule as the long-run state of the economy when the discount rate β converges to 1. In this case the steady state version of the Phillips curve relation (45) is vertical, and the policymaker of either country cannot exert any effect on markups by setting inflation rates different from zero.

5.1.3 Optimal Stabilization Policy around the Long-Run Steady-State

We are now in the position to analyze the dynamic features of the optimal commitment policy under Nash competition. In this section we interpret optimal policy in the sense of *optimal stabilization* in response to shocks. To this end, we proceed in the following way. After characterizing (for both countries) the stationary allocations associated to the deterministic steady state of the first order conditions (51)-(53) (and symmetric ones for Foreign), we compute a log-linear approximation of the respective policy functions in the neighborhood of the same long-run steady state.[20]

[20] From a methodological point view, it may be of independent interest that at this stage,since the Nash-optimal allocation has been already characterized, we can limit ourselves to employ standard log-linear approximation methods to describe the policy function. On the other hand, when later computing relative conditional welfare of alternative policy equilibria, we will have to resort to a second order approximation of the same policy function. This is necessary to account for the natural effect of stochastic volatility on the first moments of critical variables, as well as for the

The spirit of this exercise deserves some further comments. In concentrating on (log-linear) dynamics in the neighborhood of the steady state associated to the efficiency conditions of the Nash-optimal policy, we deviate from the initial requirement (50) in the fact that we set the initial value of the lagged lagrange multipliers equal to their deterministic steady state values, i.e., $\chi_0 = \overline{\chi}_0$; $\chi_0^* = \overline{\chi}_0^*$. It is important to understand that this strategy, as in Khan, King and Wolman (2003), corresponds to focusing on a particular dimension. Namely, optimal stabilization policy in response to bounded shocks that hit in the neighborhood of the long-run steady state. This amounts to implicitly assuming that such a steady state has been already reached after the implementation of the optimal policy plan as of time zero.[21]

5.1.4 Parameterization

In conducting our simulations we employ the following form of the period utility: $U(C_t, N_t) = \frac{1}{1-\sigma}C_t^{1-\sigma} - \frac{1}{1+\gamma}N_t^{1+\gamma}$. The time unit is meant to be quarters. The discount factor β is equal to 0.99. The degree of risk aversion σ is 1, the inverse elasticity of labor supply γ is equal to 3. As a benchmark value (see below for a discussion) we set $\eta = 2$. As in Bergin and Tchakarov (2003), and consistent with estimates by Ireland (2001), we set the price stickiness parameter θ equal to 50. The elasticity ε between varieties produced by the monopolistic sector is 6. The (inverse) degree of home bias α, identified by the share of foreign imported goods in the domestic consumption basket, is set to a default value of 0.4. This implies the existence of a *mild home bias*, which is assumed to be symmetric across countries ($\alpha = \alpha^*$).

5.1.5 Response to Productivity Shocks: Nash-Optimal vs. Inflation Targeting

Figure 1 compares impulse responses of selected variables to a one percent rise in Home productivity in the case of *Nash-commitment* with the same responses under (domestic) *inflation targeting*. The figure is illustrative of the inefficiency associated to policy competition in our context. Since productivity is higher in Home, the adjustment to the equilibrium requires an increase in the demand of domestic relative to foreign goods. This is achieved by means of a terms of trade depreciation, captured by a rise in the CPI to PPI ratio Φ. Recall, in fact, that $\Phi_t = \Phi(\mathcal{T}_t)$, i.e., the same ratio is a (positive) function of the terms of trade. The only equilibrium is the one in which the same terms of trade depreciation is achieved via an increase in prices in both countries, Home and Foreign. In fact, and due to risk sharing, both countries face the incentive to increase prices to tilt the terms of trade in their own favor, thereby achieving a relatively higher real income

transitional dynamics that characterize the economy in its adjustment towards the long-run steady-state associated to the optimal policy.

[21] To rephrase it, this corresponds to assuming that the economy has been evolving around such a steady state for a sufficiently long period of time.

and consumption for any given level of labor effort. However, since Home is the country in which productivity is relatively higher, the increase in domestic (producer) prices falls short the increase in foreign (producer) prices. This explains why, for a given nominal adjustment in the exchange rate, *the terms of trade depreciate more in a Nash equilibrium* relatively to the inflation targeting case. In the resulting dynamics, since aggregate consumption must rise equally in both countries due to risk sharing, the rise in employment exceeds the one that obtains under inflation targeting.

It is also interesting to notice that a Nash equilibrium generates a dynamic behavior of the price level that resembles the one in response to a cost-push shock. The novel aspect of our results is that the same dynamics are obtained in response to a productivity shock, which is not aimed per se (like in many recent New Keynesian studies of optimal monetary policy) to induce the artificial effect of exogenously drifting the economy away from the efficient allocation. The fact that productivity shocks are a source of price variability under the optimal policy is here an endogenous outcome of the competition on international relative prices.[22]

Figure 2 illustrates how the incentive to generate price movements vary with a critical parameter, namely the elasticity of substitution between domestic and foreign goods. The figure displays impulse responses (under Nash-commitment) of the same selected variables to a productivity shock for alternative values of $\eta = [1, 2, 3]$. The first case nearly corresponds to the benchmark case of Cobb-Douglas preferences typically employed in the linear-quadratic approach to the study of optimal policy for open economies. The literature lacks a consensus on the value of this parameter. Harrigan (1993) and Trefler and Lai (1999) suggest an empirical value as high as 5. Collard and Dellas (2002) derive an estimated value of 2.5. In their quantitative (theoretical) studies, Backus, Kehoe and Kydland (1992) explore a range of η between 0 and 5. Chari et al (2002) set $\eta = 1.5$, while Bergin and Tchakarov (2003) set $\eta = 5$. Overall, there seems to exist both empirical and theoretical support for the hypothesis that the value of η lies above unity. The figure highlights the coincidence of the Nash-optimal response with a close-to-price stability strategy only in the particular case of $\eta = 1$. In this knife-edge case, the income effect of the required terms of trade depreciation (given the relatively higher productivity in Home) balances the incentive to switch expenditure towards Home goods.[23] In general, the higher the elasticity of substitution, the larger (at the margin) the incentive for the policymaker to induce a strategic rise in the (producer) price level to try to generate a relative appreciation of its own residents' real income and purchasing power for any given level of labor effort.

[22] For an analysis of the optimal policy setting in response to this type of shocks see Woodford (2003) and Clarida et al. (1999). For open economy models with one-period predetermined prices see Sutherland (2001).

[23] See also Benigno and Benigno (2003b). Another way of seeing this is that, from equations (39) and (40), both Φ_t and Φ_t^* cease to play any role in the determination of the equilibrium in the particular case of $\eta = 1$.

5.2 Cooperation

Under cooperation, a social planner explicitly recognizes the channel of interdependence that works through the relative prices Φ_t and Φ_t^*. Below we define the world Ramsey planner problem, under the assumption that the same planner aims at maximizing the *average* level of utility of the two countries. We also assume that both countries receive equal weight in the planner's objective function.

Let's define the *world Ramsey period utility objective* as:

$$\mathcal{U}_t^w(C_t, C_t^*, N_t, N_t^*, \pi_{H,t}, \pi_{F,t}^*, \Omega^w) \equiv \left\{ \frac{U(C_t, N_t) + U_t^*(C_t^*, N_t^*)}{2} \right\} +$$

$$\Omega^w \left[U_{c,t} \left(C_t - \frac{A_t N_t - \frac{\theta}{2}(\pi_{H,t} - 1)^2}{\Phi_t} \right) + U_{c^*,t}^* \left(C_t^* - \frac{A_t^* N_t^* - \frac{\theta}{2}(\pi_{F,t}^* - 1)^2}{\Phi_t^*} \right) \right]$$

where Ω^w is a constant multiplier on the sum of the constraints (26) and (29). Then the Ramsey maximization problem can be defined as follows.

Definition 4. *Let* $\left\{\lambda_{p,t},\, \lambda_{f,t},\, \lambda_{p,t}^*,\, \lambda_{f,t}^*\right\}_{t=0}^{\infty}$ *represent sequences of Lagrange multipliers on the constraints (45), (46), (21) and (22) respectively. Let* $\widetilde{B}_0$ *be given and* $\kappa = 1$. *For any given stochastic processes* $\{A_t,\, A_t^*\}_{t=0}^{\infty}$, *plans for the control variables* $\{C_t,\, \pi_{H,t},\, N_t,\, C_t^*,\, \pi_{F,t}^*,\, N_t^*\}$, *and for the costate variables* $\{\lambda_{p,t},\, \lambda_{f,t},\, \lambda_{p,t}^*,\, \lambda_{f,t}^*\}_{t=0}^{\infty}$ *and* Ω^w, *represent a first best constrained allocation if they solve the following maximization problem:*

Choose $\Lambda_t^c \equiv \left\{\lambda_{p,t}, \lambda_{f,t}, \lambda_{p,t}^*, \lambda_{f,t}^*\right\}_{t=0}^{\infty}$ and $\Xi_t^c \equiv \{C_t,\, \pi_{H,t},\, N_t\}_{t=0}^{\infty}$ to

$$Min_{\{\Lambda_t^c\}_{t=0}^{\infty}}\ Max_{\{\Xi_t^c\}_{t=0}^{\infty}}\ E_0\{ \sum_{t=0}^{\infty} \beta^t E_t\{\mathcal{U}_t^w(C_t, C_t^*, N_t, N_t^*, \pi_{H,t}, \pi_{F,t}^*, \Omega^w) \tag{54}$$

$$+\lambda_{p,t} \left[U_{c,t}\pi_{H,t}(\pi_{H,t} - 1) - \beta U_{c,t+1}\pi_{H,t+1}(\pi_{H,t+1} - 1) + \frac{U_{c,t}\varepsilon A_t N_t}{\theta} \left(\frac{U_{n,t}\ \Phi_t}{U_{c,t}A_t} + \frac{\varepsilon - 1}{\varepsilon} \right) \right]$$

$$+\lambda_{f,t} \left[A_t N_t - (1-\alpha)\ C_t \Phi_t^{\eta} - \kappa^{\eta} \left(\frac{U_{c^*,t}^*}{U_{c,t}} \right)^{\eta} \Phi_t^{\eta} \alpha^* C_t^* - \frac{\theta}{2} \left(\pi_{H,t} - 1 \right)^2 \right]$$

$$+\lambda_{p,t}^* \left[U_{c^*,t}^*\ \pi_{F,t}^*(\pi_{F,t}^* - 1) - \beta U_{c^*,t+1}^* \pi_{F,t+1}^*(\pi_{F,t+1}^* - 1) + \frac{U_{c^*,t}^*\ \varepsilon A_t^* N_t^*}{\theta} \left(\frac{U_{n,t}^*\ \Phi_t^*}{U_{c^*,t}^* A_t^*} + \frac{\varepsilon - 1}{\varepsilon} \right) \right]$$

$$+\lambda_{f,t}^* \left[A_t^* N_t^* - (1-\alpha^*)\ C_t^* \left(\Phi_t^*\right)^{\eta} - \kappa^{-\eta} \left(\frac{U_{c,t}}{U_{c^*,t}^*} \right)^{\eta} \left(\Phi_t^*\right)^{\eta} \alpha C_t - \frac{\theta}{2} \left(\pi_{F,t}^* - 1 \right)^2 \right] \}\} \ -\ 2\Omega^w \widetilde{B}_0$$

We defer to *Appendix D* the description of the first order conditions corresponding to this plan. The discussion on the non-recursivity structure of the problem follows exactly the logic applied above to the re-definition of the Nash-commitment policy setup. In practice, this will entail specifying an equivalent recursive stationary program in the new world planner's state space defined by $\{A_t, A_t^*,\ \chi_t,\ \chi_t^*\}$.

5.2.1 Ramsey Steady-State

A deterministic Ramsey steady state is a set of allocations $\{C, C^*, N, N^*, \pi_H, \pi_F^*, \Omega^w\}$ that solves the steady-state version of the efficiency conditions associated to the program under Definition 3. In *Appendix E* we characterize such system of equations. Under the assumption of zero initial net wealth ($B_0 = 0$), this steady state has a symmetric solution in which $\pi_H = \pi_F^* = 1$. Hence, and exactly like in the steady state version of the efficiency conditions of the Nash problem analyzed above, the unconstrained long run optimal inflation policy is associated with price stability in both countries.

5.2.2 Optimal Response to Shocks around the Ramsey Steady-State: Nash vs. Cooperation

Figure 3 displays impulse responses to a normalized one percent increase in home productivity and compare selected variables under *Nash-commitment* versus *Cooperation-commitment*. Under policy cooperation, the planner coordinate the responses of both policy makers to achieve the required terms of trade depreciation only by means of a nominal exchange rate depreciation. In other words, it is optimal for the Ramsey planner to have both countries targeting very closely the flexible price allocation. This results in a dampened dynamic of the CPI-PPI ratio Φ under cooperation. The crucial aspect is that this is now compatible with a smooth path of the price level (the response of the price level, measured in percent deviation from steady state, barely deviates from zero) and with a smoother response of employment, for any given variation in consumption (in turn equalized across countries).

It is also interesting to notice, under the Ramsey cooperative regime, that while the response of the price level resembles the one that would obtain in a closed economy under the optimal policy (see for instance King and Wolman, 1997), so does not the response of employment. For an intuition, consider the (equilibrium) real marginal cost equation (44). As already emphasized, the closed economy version of that equation obtains in the case $\Phi_t = 1$. Hence, in a closed economy, it is optimal, in response to a rise in productivity, to fully absorb the rise in productivity by means of an equal increase in consumption (and output), while keeping employment constant. In an open economy, the equilibrium requires a rise in Φ_t (i.e., a real depreciation). Hence, coordinating on stable prices (i.e., constant real marginal costs), requires a rise in consumption which is smaller than the one in productivity and, under the benchmark parameterization, also a rise in employment.[24]

[24]The size of the rise in employment will be, in turn, a function of the elasticity η (with a smaller η implying a smaller rise) and of the labor supply elasiticity $\frac{1}{\gamma}$ (with a higher elasticity requirying a smaller rise in employment).

6 Welfare Analysis and Dynamic Features of the Ramsey Policy

We now turn to a characterization of the dynamic properties of the alternative policy regimes analyzed so far.[25] We illustrate our numerical analysis in terms of cyclical properties of selected variables and welfare levels associated to each policy arrangements. We report results under three alternative parameter scenarios: 1) *High home bias,* in which the value of $\alpha = \alpha^*$ is set to 0.1; 2) *Small home bias* in which $\alpha = \alpha^* = 0.45$; 3) Low elasticity of substitution, in which $\eta = 0.1$.

Some observations on the computation of welfare are in order. First, one cannot safely rely on standard first order approximation methods to compare the relative welfare implied by each policy arrangement. In fact, in an economy like ours, in which distortions exert an effect both in the short and in the long-run, stochastic volatility affects both first and second moments of those variables that are critical for household's welfare. Since in a first order approximation of the model's solution the expected value of a variable is always equal to its non-stochastic steady state, the effects of volatility on mean values of variables is by construction neglected. Hence policy arrangements can be correctly ranked only by means of a higher order approximation of the policy functions.[26]

This last observation suggests also that our welfare metric needs to be correctly chosen. In particular one needs to focus on the *conditional* expected discounted utility of the representative agent. This is necessary exactly to take into account of transitional effects from the deterministic to the different stochastic steady states respectively uimplied by each alternative policy arrangement.

We proceed in the following steps. First, we compute a second order approximation of the policy function(s) around the long-run deterministic steady-state implied by each policy regime under scrutiny.[27] Then we assume that both economies are subject to a stationary distribution of

[25] More in line with the present analysis is the one of Kollmann (2003) and Bergin and Tchakarov (2003), who study optimizing linear interest rate rules and perform welfare calculations based on a second order approximation of the model's equilibrium conditions. Our paper differs crucially in that it characterizes equilibrium allocations under the Ramsey policy, without restricting the form of the policy function to the arguments of a pre-specified (log-linear) interest rate rule.

[26] See Kim and Kim (2002) for an analysis of the inaccuracy of welfare calculations based on log-linear approximations in dynamic open economies. See Kim et al. (2003) and Schmitt-Grohe and Uribe (2004) for a more general discussion.

[27] The set of optimality conditions of the Ramsey plan can be described as follows:

$$E_t \; f(Y_{t+1}, Y_t, X_{t+1}, X_t) = 0 \tag{55}$$

where E_t denotes the mathematical expectations operator, conditional on information available at time t, Y_t is the vector of endogenous non-predetermined variables and $X_t \equiv [x_{1,t}, x_{2,t}]$ is the state vector. Here $x_{1,t}$ denotes the vector of (pseudo) co-state variables $[\chi_t, \chi_t^*]$, while $x_{2,t}$ is the vector of exogenous variables $[A_t, A_t^*]$ which follows a stochastic process.

$$x_{2,t+1} = F x_{2,t} + \overline{\eta}\sigma\varepsilon_{t+1}; \quad \varepsilon_t \sim i.i.d.N(0, \Sigma) \tag{56}$$

The scalar σ and $\overline{\eta}$ are known parameters. The solution to the model is of the form:

$$Y_t = g(X_t, \sigma) \tag{57}$$

(productivity shocks) and generate, for given initial conditions, artificial time series of length $T_p = 500$ periods. We compute mean, standard deviation and implied presented discounted utility for any given random draw. We then iterate the computation $T_n = 1000$ times and average across experiments.[28]

Along the lines of Lucas (1987), the measure of welfare cost (of business cycles) that we associate to each policy is the proportional upward shift in the consumption process that would be required to make the representative household indifferent between its random consumption allocation and a nonrandom consumption allocation with the same mean. Hence such measure is defined as the fraction Δ that satisfies the following equality between conditional utilities:

$$E_0 \sum_{t=0}^{\infty} \beta^t U((1+\Delta)C_t, N_t) = \sum_{t=0}^{\infty} \beta^t U(E_0 (C_t, N_t)) \tag{59}$$

In *Table 1* we report second moments of selected variables under Nash competition and Cooperation. In particular, we report the cyclical properties of the CPI to PPI ratio (our key relative price) meant as a proxy of the terms trade. Hence we see that, across parameter scenarios, and relative to cooperation, Nash competition is indeed a source of enhancement of inflation volatility. In particular this happens in a low home bias scenario. Intuitively, under this scenario, economies are more open to trade, and therefore, at the margin, more open to a policy competition motive over their terms of trade. However, deviations from price stability, once measured in terms of second moments of inflation, remain per se rather small under Nash competition and barely different from the regime of Cooperation.

In the same Table we also report the measure Δ of the welfare costs associated to the alternative policy scenarios. Hence it is clear that cooperation delivers welfare gains relative to a Nash competition arrangement. However, such welfare numbers remain quite small. In absolute terms, and across all policy scenarios, the upward shift in consumption needed to make the household indifferent between a random and a nonrandom allocation range between a minimum of *0.0113 percent* (achieved under Cooperation with high home bias) to a maximum of *0.0157 percent* (achieved under Nash competition with low home bias). In relative terms, welfare gains from Cooperation are also rather small. This result, however, is hardly surprising. That both policies were delivering

$$X_{t+1} = h(X_t, \sigma) + \bar{\eta}\sigma\varepsilon_{t+1} \tag{58}$$

where Y_t is the vector of control variables, equation (57) is the policy function and equation (58) is the transition function. We compute a second order expansion of the functions $g(x_t, \sigma)$ and $h(x_t, \sigma)$. Schmitt-Grohe and Uribe (2002) show, crucially, that, up to a second order, the coefficients of the policy functions attached to terms that are linear in the state vector x_t are independent of the size of the volatility of the shock(s). To evaluate numerically the first and second order derivatives of the policy functions we employ the Matlab codes compiled by Schmitt-Grohe and Uribe, available at the website http://www.econ.duke.edu/~grohe/.

[28] We employ a standard parameterization for the innovations to the productivity processes, and assume $Var(\varepsilon_t^a) = Var(\varepsilon_t^{a^*}) = (0.01)^2$, with persistence $\rho^a = \rho^{a^*} = 0.9$.

very similar dynamics in response to the same distribution of shocks was already apparent from our previous impulse response analysis.

It is of some interest, however, to see that, in our exercise, such gains may depend on the comparative statics on two critical parameters that identify "openness" and that affect the relationship between the terms of trade and the CPI to PPI ratio, namely α and η . Hence our exercise indicate that welfare gains from cooperation are minimized when the elasticity of substitution between domestic and foreign goods η is small, and maximized when the home bias is low (i.e., $\alpha = 0.45$). Intuitively, these two scenarios correspond, respectively, to a case in which the policy competition motive is either reduced in scope or, alternatively, magnified.

7 Golden Rule

So far we have concentrated only on the optimal short-run stabilization policy around a deterministic Ramsey steady-state. This is the steady state associated to the efficiency conditions that describe the optimal policy under commitment (modified golden rule). In this long-run, the policymaker faces no incentive to use the inflation instrument to affect the markup via an exploitation of the Phillips curve. However, if the policymaker was forced to maximize households' utility under the constraint that the steady state conditions are imposed ex-ante, this may lead to the presence of an *inflationary or deflationary bias*. In analogy with the terminology of the neoclassical growth model, and as in King and Wolman (1997), we can define this as the policy maker's *golden rule*.

To understand whether openness coupled with policy competition is responsible for either an inflationary or deflationary bias, let us focus on how the relevant distortions interact in a golden rule steady state. It is first instructive to derive the markup function from the competitive equilibrium of the domestic open economy. By combining the steady-state version of (5) and (12) we can write

$$\frac{-U_n}{U_c} = \frac{1}{\mu}\frac{P_H}{P} = \frac{1}{\mu(\pi_H, N)}\left[\Phi(\mathcal{T})\right]^{-1}$$

where the function $\mu(\pi_H, N)$ derives from the steady-state version of (18) as

$$\mu(\pi_H, N) = \frac{\varepsilon N}{\theta\pi_H(\pi_H - 1)(1-\beta) + (\varepsilon - 1)N} \tag{60}$$

Hence efficiency in any given steady state of the economy requires

$$\mu(\pi_H, N)\Phi(\mathcal{T}) = 1 \tag{61}$$

To gain intuition on how the international relative price distortion (summarized by the wedge $\Phi(\mathcal{T})$ which is increasing in $\mathcal{T}$) interacts with the markup distortion from the view point of a given country, let's assume, for the sake of argument, that producer prices are fully flexible, so that the markup is always equal to a constant value of $\frac{\varepsilon}{\varepsilon-1}$. We therefore temporarily abstract from the price-stickiness distortion. By making use of (36), we can rewrite a relationship between the desired terms of trade and the markup which reads

$$\mathcal{T} = \left(\frac{\alpha}{\mu^{\eta-1} - (1-\alpha)} \right)^{\frac{1}{\eta-1}} \tag{62}$$

Notice that, independently of the values of η and α, $\mathcal{T}$ is always decreasing in μ. One should view equation (62) as an iso-efficiency condition. Hence, and in order to keep the economy at the maximum welfare in steady-state, a higher markup calls for more *appreciated* terms of trade. The intuition is simple. The presence of imperfectly competitive output markets makes desirable to expand output towards the efficient level. While in an closed economy with flexible prices this is always welfare improving, in an open economy the same rise in output requires (in equilibrium) also a depreciation of the terms of trade, which hurts the purchasing power of domestic consumers. Equation (62) shows that, at the margin and for *any given level of foreign consumption*, it is optimal to have the terms of trade *depreciate less,* or equivalently let output expand less relatively to the case of an imperfectly competitive closed economy.

However, such a strategic incentive characterizes also the optimal reaction function in Foreign. To formalize the policy game let's define, *for any given* C^* , the *golden rule* Nash steady state for Home as the triplet

$$\{\pi_H, C, N\}^{gr} \equiv \arg\max\{U(C, N)\} \tag{63}$$

subject to a (steady state) pricing-implementability condition

$$\pi_H(\pi_H - 1)(1-\beta) \leq \frac{\varepsilon N}{\theta} \left(\frac{-U_n \Phi(C, C^*)}{U_c} - \frac{\varepsilon - 1}{\varepsilon} \right) \tag{64}$$

and to a (steady state) feasibility constraint

$$N \leq (1-\alpha)\ C\ \ [\Phi(C, C^*)]^{\eta} + \left(\frac{\kappa U^*_{c^*}}{U_c} \right)^{\eta}\ \ [\Phi(C, C^*)]^{\eta}\, \alpha^* C^* + \frac{\theta}{2} (\pi_H - 1)^2 \tag{65}$$

where it is understood that $C^*$is taken as given from the view point of the policymaker in Home but is instead chosen optimally by the policymaker in Foreign. First order conditions of this problem define Home policymaker's reaction function for any given level of Foreign consumption. An exactly symmetric problem characterizes the reaction function of the policymaker in Foreign.

The solution to the joint system of equations pins down the Nash equilibrium, and is reported in *Figure 4.* The *dashed line* shows the solution of the Nash game for a selected number of variables as a function of the (inverse) home bias parameter α. This parameter is a natural index of openness in our context. In the simulations, we set the vector $[\sigma,\ \gamma,\ \eta] = [1\ ,\ 3,\ 2]$, while we maintain that

$\alpha = \alpha^*$. Hence we see that for $\alpha \to 0$ the rate of (producer) inflation that maximizes steady-state welfare is positive and coincides exactly with the one of the corresponding closed economy with sticky prices and monopolistic competition. As α turns positive, i.e., both economies become open to trade, the desired steady-state inflation rate decreases below the one of the corresponding closed economy. The intuition is simple. As explained above, policy competition calls for a strategic reduction in consumption (relative to the closed economy case) in order to obtain an appreciation of the terms of trade and a reduction in work effort. For any given level of foreign consumption, *and in the absence of any asymmetric shock*, this is welfare improving. However, in a Nash-game where policy objectives collide, terms of trade and work effort remain constant, while consumption ends up being slightly lower. This entails, for any level of openness (as measured by α), a *lower* steady-state rate of inflation relative to a closed economy.

On the other hand, if two policymakers coordinate their actions in order to avoid a strategic manipulation of the terms of trade, they can enjoy higher consumption for any given level of work effort. Under Cooperation, we assume that the planner chooses simultaneously the two triplets $\{\pi_H, C, N\}$ and $\{\pi_F^*, C^*, N^*\}$ in order to maximize average utility $\frac{U(C,N)+U(C^*,N^*)}{2}$, subject to the constraints (64), (65) and to the corresponding equations for Foreign.

Figure 5 (in *solid line*) shows the outcome of the optimal cooperative policy game. Notice that once again the optimal inflation rate lies below the one of the corresponding closed economy. Yet the inflation rate under policy cooperation lies above the one prevailing under Nash competition. Intuitively, under cooperation, the planner induces both policymakers to avoid any strategic reduction in consumption aimed at appreciating the terms of trade. In equilibrium, the terms of trade and work effort are unchanged, and consumption is higher relative to the Nash outcome. This, ceteris paribus, entails also a relatively higher inflation rate.

Is the result of a Nash deflationary bias (relative to Cooperation) robust to parameter values? In fact it is not. Intuitively, and as emphasized, although in a different framework, also by Sutherland (2002) and Benigno and Benigno (2003b), the gains from strategically manipulating the terms of trade should be decreasing in the elasticity of substitution between domestic and foreign goods, for this is a measure of the strength of the expenditure switching effect. We therefore employ an alternative parameterization, with high labor supply elasticity and low international elasticity of substitution. This implies setting $[\sigma, \gamma, \eta] = [1, 1, 0.7]$. We see that in this case there exist values for the degree of openness such that the inflation rate under Nash lies *above* the one under policy cooperation. It is only when the economy becomes extremely open (i.e., the degree of home bias is extremely small) that the Nash deflationary bias result re-emerges.

The above discussion on the golden-rule incentives for an optimal inflation policy can be summarized as follows:

Result 1 *(Open economy bias). In an open economy with price adjustment costs and monopo-*

listic competition, the (producer) inflation rate that maximizes steady-state utility lies monotonically below the one of the corresponding closed economy. This holds under both policy competition and cooperation.

The interesting aspect of Result 1 is that, while in a closed economy with sticky prices and monopolistic competition price stability cannot implement the steady-state maximization of welfare, it can indeed do so when the economy is open, due to the additional effect of the international relative price distortion that pushes the efficient inflation rate downwards.

Result 2 (*Nash bias* **).** *There is no monotonic ranking between the golden rule inflation rate under Nash and the one under Cooperation. Hence policy competition may lead either to a deflationary or inflationary bias depending on the degree of trade openness and on the elasticity of substitution between domestic and foreign goods.*

This result differs from the one of Cooley and Quadrini (2003), who find that policy competition is necessarily associated with an *inflationary* bias. The reason lies in the structure of their model, which features flexible prices and an unambiguos *positive* output effect of real appreciations. In our context, and crucially, prices are sticky (so that monetary authorities can exert a direct effect on the terms of trade) and the effect of international relative price movements on output strictly depends on the value of the elasticity of substitution between domestic and foreign goods. In particular, under our preferred parameterization of $\eta > 1$ (with log utility), real appreciations exert a *negative* effect on output.

8 Conclusions

We have laid out a typical public finance framework for the analysis of welfare maximizing monetary policy within an economy characterized by three distortions: market power, rigidity in the adjustment of producer prices and international terms of trade externality. The main advantage of our approach, relative to the existing literature, is that it allows to characterize optimal policy in a fully dynamic open economy setting while maintaining all the distortions completely spelled out both in the short and in the long run.

Despite the generality of the approach, our modelling framework remains restrictive in three main dimensions. First, in assuming that the law of one price for traded goods holds continually. Second, in allowing households to obtain full risk sharing via international financial markets. Third, in not allowing households to invest in physical capital. Amending on all these features should aim at generating less trivial dynamics of the current account than the ones generated here via the only movements in the trade balance. Such dynamics may be of first order importance for the welfare evaluation along two dimensions. First, they would more critically affect the transition from deterministic to stochastic steady state. Second, they would impinge on the transition from one policy regime to another. For instance from Nash-competition to cooperation, or from the optimal commitment policy to a fixed exchange rate arrangement. Given the flexibility and the rigor of a Ramsey-based approach, all these issues will certainly be the source of new research efforts in international macroeconomics in the near future.

A Proof of Proposition 1

(Part A). The proof follows from the substitutions and the rearrangements of Section 4 that lead to the minimal form. (Part B). For given productivity processes A_t, A_t^* and using the allocations $\{C_t, C_t^*, N_t, N_t^*, \Phi_t,\ \Phi_t^*\}_{t=0}^{\infty}$ satisfying the optimal plan, one can obtain the optimal allocation for the real wage, output and real marginal cost from (5), the aggregate version of (11) and (12), which are symmetric across countries. Using the producer inflation rates $\left\{\pi_{H,t},\ \pi_{F,t}^*\right\}$ obtained by the optimal plan and the relative prices $\{\Phi_t,\ \Phi_t^*\}_{t=0}^{\infty}$ one can obtain the CPI inflation rates from (41) and the analog for Foreign. For given optimal paths $\{C_t, C_t^*\}_{t=0}^{\infty}$ the path for the real exchange rate follows from (28). Given $\{\Phi_t,\ \Phi_t^*\}_{t=0}^{\infty}$, this allows to solve for the terms of trade from (37). By rewriting $C_H = (1-\alpha)\Phi_t^{\eta} C_t$ and $C_{F,t} = \alpha\left(\Phi_t \mathcal{T}_t\right)^{\eta} C_t$ from (2) one obtains consumption demand for domestic and foreign goods. For given consumption, the path for the real interest rate is given by (42), which implies, given CPI inflation, a path for the nominal rate via equation (43).

B The Stationary Dynamic Policy Problem

Below we derive the stationary form of the policy problem under Nash commitment. We illustrate the argument only for the Home policymaker's problem, since the problem in Foreign is exactly symmetric. Let's consider the optimal plan as formulated in equation (47) in the text. By applying the law of iterated expectations and by grouping expectations and multipliers that share the same date one obtains:

$$Min_{\{\Lambda_t\}_{t=0}^{\infty}}\ Max_{\{\Xi_t\}_{t=0}^{\infty}} E_0\{\mathcal{U}(C_0,\ N_0,\ \pi_{H,0},\ \Omega)$$

$$\begin{aligned}
&+\lambda_{p,0}\left[U_{c,0}\pi_{H,0}(\pi_{H,0}-1)+\frac{U_{c,0}\varepsilon A_0 N_0}{\theta}\left(\frac{U_{n,0}\Phi_0}{U_{c,0}A_0}+\frac{\varepsilon-1}{\varepsilon}\right)\right]\\
&+\lambda_{f,0}\left[A_0N_0-(1-\alpha)C_0\Phi_0^{\eta}-\kappa^{\eta}\left(\frac{U^*_{c^*,0}}{U_{c,0}}\right)^{\eta}\Phi_0^{\eta}\alpha^*C_0^*-\frac{\theta}{2}\left(\pi_{H,0}-1\right)^2\right]\\
&+\beta\{U(C_1,N_1)+(\lambda_{p,1}-\beta\lambda_{p,0})(U_{c,1}\pi_{H,1}(\pi_{H,1}-1))+\lambda_{p,1}\left(\frac{U_{c,1}\varepsilon A_1N_1}{\theta}\left(\frac{U_{n,1}\Phi_1}{U_{c,1}A_1}+\frac{\varepsilon-1}{\varepsilon}\right)\right)\\
&+\lambda_{f,1}\left[A_1N_1-(1-\alpha)C_1\Phi_1^{\eta}-\kappa^{\eta}\left(\frac{U^*_{c^*,1}}{U_{c,1}}\right)^{\eta}\Phi_1^{\eta}\alpha^*C_1^*-\frac{\theta}{2}\left(\pi_{H,1}-1\right)^2\right]+...\}\}\ -\ \Omega\widetilde{B}_0
\end{aligned}$$

Notice that this problem is not time-invariant due to the fact that the constraints at time zero lack the term $-\beta\lambda_{p,-1}(U_{c,0}\pi_{H,0}(\pi_{H,0}-1))$. For this reason we amplify the state space to introduce a new (pseudo) costate variable χ_t and define a new policy functional $\mathcal{W}(C_t, N_t, \pi_{H,t}, \chi_t, \Omega) \equiv \mathcal{U}(C_t, N_t, \pi_{H,t}, \Omega) - \chi_t(U_{c,t}\pi_{H,t}(\pi_{H,t}-1))$. We then write the optimal policy plan in the following form:

Choose $\Lambda_t \equiv \{\lambda_{p,t}, \lambda_{f,t}\}_{t=0}^{\infty}$ and $\Xi_t \equiv \{C_t,\ \pi_{H,t},\ N_t\}_{t=0}^{\infty}$ to

$$Min_{\{\Lambda_t\}_{t=0}^{\infty}}\ Max_{\{\Xi_t\}_{t=0}^{\infty}}\ E_0\{\sum_{t=0}^{\infty}\beta^t E_t\{\mathcal{W}(C_t, N_t, \pi_{H,t}, \chi_t, \Omega) \tag{66}$$

$$+\lambda_{p,t}\left[U_{c,t}\pi_{H,t}(\pi_{H,t}-1)+\frac{U_{c,t}\varepsilon A_t N_t}{\theta}\left(\frac{U_{n,t}\ \Phi_t}{U_{c,t}A_t}+\frac{\varepsilon-1}{\varepsilon}\right)\right]$$
$$+\lambda_{f,t}\left[A_t N_t-(1-\alpha)\ C_t\Phi_t^{\eta}-\kappa^{\eta}\left(\frac{U^*_{c^*,t}}{U_{c,t}}\right)^{\eta}\Phi_t^{\eta}\alpha^* C_t^*-\frac{\theta}{2}\left(\pi_{H,t}-1\right)^2\right]\}\}-\Omega\widetilde{B}_0$$

with law of motion for the new costate

$$\chi_{t+1}=\lambda_{p,t}$$

and initial condition

$$\chi_0=0$$

Following Marcet and Marimon (1999), one can show that this new maximization program is now saddle point stationary in the amplified state space $\{A_t,\ \chi_t\}$. First order conditions of this problem exactly replicate conditions (51)-(53) in the text. An exactly symmetric argument is applied to the design of the policy problem in Foreign, which will involve specifying an amplified state space $\{A_t^*,\ \chi_t^*\}$, with law of motion $\chi_{t+1}^*=\lambda_{p,t}^*$ and initial condition $\chi_0^*=0$.

C Steady State of the Nash-Optimal Policy Problem

The steady state version of the efficiency conditions (51)-(53) of the Nash problem is derived by imposing $\lambda_{p,t}=\lambda_{p,t-1}=\lambda_p=\chi$. This implies:

$$\begin{aligned}0 \;=\;& U_c+\frac{\lambda_p N}{\theta}\left(\epsilon\ U_n\Phi_c+(\varepsilon-1)U_{cc}\right)-\lambda_f\ (1-\alpha)\left(\Phi^{\eta}+\eta C\Phi^{\eta-1}\Phi_c\right)\\ &+\lambda_f\ \alpha^* C^*\kappa^{\eta}U_c^{*\eta}\left(\eta\Phi^{\eta-1}\Phi_c U_c^{-\eta}-\eta U_c^{-\eta-1}U_{cc}\Phi^{\eta}\right)\\ &-\Omega\left[(N-\frac{\theta}{2}(\pi_H-1)^2)\left(U_{cc}\Phi^{-1}-\Phi_c\Phi^{-2}U_c\right)-(U_{cc}C+U_c)\right]\end{aligned}$$

$$0=U_n+\frac{\lambda_p\varepsilon\Phi}{\theta}\left(U_n+NU_{nn}\right)+\lambda_p\frac{\varepsilon-1}{\theta}U_c+\lambda_f-\Omega\frac{U_c}{\Phi}$$

$$0=-\theta(\pi_H-1)\left(\lambda_f-\Omega\frac{U_c}{\Phi}\right)$$

The system is completed by the steady state version of (4), (45), (21) along with symmetric equation for Foreign. One can easily verify that the solution is a vector $\left\{\Omega,\ C,\ C^*,\ N,\ N^*,\ \pi_{H,}\ \pi^*_{F,}\ \lambda_{F,}\ \lambda^*_{F,}\ \lambda_{p,}\ \lambda^*_{p,}\right\}$ with $\pi_{H,}=\pi^*_{F,}=1$, $C=C^*$, $N=N^*$, $\Phi=\Phi^*=1$.

D First Order Conditions of the Policy Cooperation Problem

First order conditions for the Ramsey problem under Cooperation at time $t \geq 1$ read:

$$\begin{aligned}
0 \quad = \quad & \frac{1}{2}U_{c,t} + U_{cc,t}\ \pi_{H,t}(\pi_{H,t}-1)\ (\lambda_{p,t}-\chi_t) + \frac{\lambda_{p,t}N_t\varepsilon}{\theta}U_{n,t}\Phi_{c,t} + \lambda_{p,t}\left(\frac{\varepsilon-1}{\theta}\right)A_tN_tU_{cc,t} \\
& -\lambda_{f,t}\ (1-\alpha)\left(\Phi_t^{\eta} + \eta C_t\Phi_t^{\eta-1}\Phi_{c,t}\right) - \lambda_{f,t}\alpha^* C_t^*\kappa^{\eta}\left(U_{c^*,t}^*\right)^{\eta}\left[\eta\Phi_t^{\eta-1}\Phi_{c,t}U_{c,t}^{-\eta} - \eta U_{c,t}^{-\eta-1}U_{cc,t}\Phi_t^{\eta}\right] \\
& +\lambda_{p,t}^*\left(\frac{\varepsilon N_t^* U_{n,t}^*\Phi_{c,t}^*}{\theta}\right) - \lambda_{f,t}^*\left(\eta\left(\Phi_t^*\right)^{\eta-1}\Phi_{c,t}^*\left(1-\alpha^*\right)C_t^*\right) \\
& -\lambda_{f,t}^*\alpha\kappa^{-\eta}\left(U_{c^*,t}^*\right)^{-\eta}\ \left[\eta\left(\Phi_t^*\right)^{\eta-1}\Phi_{c,t}^*C_tU_{c,t}^{\eta} + \left(\Phi_t^*\right)^{\eta}\left(U_{c,t}^{\eta} + \eta U_{c,t}^{\eta-1}U_{cc,t}C_t\right)\right] \\
& +\Omega^w(A_tN_t - \frac{\theta}{2}(\pi_{H,t}-1)^2)\ \left(U_{cc,t}\Phi_t^{-1} - \Phi_{c,t}\Phi_t^{-2}U_{c,t}\right) - \Omega^w\left(U_{cc,t}C_t + U_{c,t}\right) + \\
& +\Omega^w(A_t^*N_t^* - \frac{\theta}{2}(\pi_{F,t}^*-1)^2)\ U_{c^*,t}^*\ \left(-\Phi_{c,t}^*\Phi_t^{*-2}\right)
\end{aligned} \tag{67}$$

$$U_{n,t} + \frac{\lambda_{p,t}\varepsilon\Phi_t}{\theta}\left(U_{n,t} + N_tU_{nn,t}\right) + \lambda_{p,t}\left(\frac{\varepsilon-1}{\theta}\right)U_{c,t}A_t + \lambda_{f,t}A_t + \Omega^w\frac{U_{c,t}A_t}{\Phi_t} = 0 \tag{68}$$

$$U_{c,t}(2\pi_{H,t}-1)\left(\lambda_{p,t}-\chi_t\right) - \theta(\pi_{H,t}-1)\left(\lambda_{f,t} + \Omega^w\frac{U_{c,t}A_t}{\Phi_t}\right) = 0 \tag{69}$$

The expression for $\Phi_{c,t}^* \equiv \frac{\partial \Phi_t^*}{\partial C_t}$ reads:

$$\Phi_{c,t}^* = -\left(\frac{\alpha^*}{1-\alpha^*}\right)\left(\Phi_t^*\right)^{2-\eta}\left(U_{c,t}^*\right)^{\eta-1}\kappa^{\eta-1}\left[\Phi_t^{\eta-2}\Phi_{c,t}U_{c,t}^{-(\eta-1)} - U_{c,t}^{-\eta}U_{cc,t}\Phi_t^{\eta-1}\right]$$

The set of analogous conditions for Foreign variables at time $t \geq 0$ will involve an expression for $\Phi_{c^*,t} \equiv \frac{\partial \Phi_t}{\partial C_t^*}$:

$$\Phi_{c^*,t} = -\left(\frac{\alpha}{1-\alpha}\right)\Phi_t^{2-\eta}\left(U_{c,t}\right)^{\eta-1}\kappa^{1-\eta}\left[\left(\Phi_t^*\right)^{\eta-2}\Phi_{c^*,t}^*\ \left(U_{c,t}^*\right)^{-(\eta-1)} - \left(U_{c,t}^*\right)^{-\eta}U_{cc,t}^*\left(\Phi_t^*\right)^{\eta-1}\right]$$

In addition all constraints must hold with equality. Also, when evaluated at time $t = 0$, condition (67) must feature the additional term $-\Omega_0^w\frac{B_0}{P_0}$.

E Steady State of the Policy Cooperation Problem

Under Cooperation the steady state version of the efficiency conditions (67)-(69) reads:

$$
\begin{aligned}
0 \;=\;& \frac{1}{2}U_c + \frac{\lambda_p N\varepsilon}{\theta}U_n\Phi_c + \lambda_p\left(\frac{\varepsilon-1}{\theta}\right)NU_{cc} \\
& -\lambda_f\,(1-\alpha)\left(\Phi^\eta + \eta C\Phi^{\eta-1}\Phi_c\right) - \lambda_f\alpha^* C^*\kappa^\eta\left(U^*_{c^*}\right)^\eta\left[\eta\Phi^{\eta-1}\Phi_c U_c^{-\eta} - \eta U_c^{-\eta-1}U_{cc}\Phi^\eta\right] \\
& +\lambda_p^*\left(\frac{\varepsilon N^* U_n^*\Phi_c^*}{\theta}\right) - \lambda_f^*\left(\eta\left(\Phi^*\right)^{\eta-1}\Phi_c^*\left(1-\alpha^*\right)C^*\right) \\
& -\lambda_f^*\alpha\kappa^{-\eta}\left(U^*_{c^*}\right)^{-\eta}\;\left[\eta\left(\Phi^*\right)^{\eta-1}\Phi_c^* C U_c^\eta + \left(\Phi^*\right)^\eta\left(U_c^\eta + \eta U_c^{\eta-1}U_{cc}C\right)\right] \\
& +\Omega^w(N - \frac{\theta}{2}(\pi_H-1)^2)\;\left(U_{cc}\Phi^{-1} - \Phi_c\Phi^{-2}U_c\right) - \Omega^w\left(U_{cc}C + U_c\right) + \\
& +\Omega^w(N^* - \frac{\theta}{2}(\pi_F^*-1)^2)\;U^*_{c^*}\;\left(-\Phi_c^*\Phi^{*-2}\right)
\end{aligned}
$$

$$
0 = U_n + \frac{\lambda_p\varepsilon\Phi}{\theta}\left(U_n + NU_{nn}\right) + \lambda_p\left(\frac{\varepsilon-1}{\theta}\right)U_c + \lambda_f + \Omega^w\frac{U_c}{\Phi}
$$

$$
0 = -\theta(\pi_H - 1)\left(\lambda_f + \Omega^w\frac{U_c}{\Phi}\right)
$$

The system is completed by the steady state version of (4), (45), (21) along with symmetric equation for Foreign. One can easily verify that the solution is again a vector $\Big\{\Omega,\ C,\ C^*,\ N,\ N^*, \pi_{H,}\ \pi^*_{F,}\ \lambda_{F,}\ \lambda^*_{F,}\ \lambda$ with $\pi_H = \pi_F^* = 1$, $C = C^*$, $N = N^*$, $\Phi = \Phi^* = 1$.

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Table 1

Volatility and Welfare under Alternative Policy Regimes

	High Home Bias		Low Home Bias		Low Elasticity	
	Nash	**Cooperation**	**Nash**	**Cooperation**	**Nash**	**Cooperation**
Consumption	1.553	1.443	1.834	1.695	1.561	1.491
Labor	0.227	0.196	0.184	0.100	0.155	0.142
PPI Inflation	0.053	0.005	0.072	0.006	0.041	0.009
CPI/PPI Ratio	0.698	0.699	0.240	0.237	1.552	1.541
Welfare cost Δ	0.0122	0.0113	0.0157	0.0146	0.0118	0.0118

Note: Standard deviations are in %. The welfare cost (in %) is the proportional upward shift in the consumption process that would make the representative household indifferent between its random consumption allocation and a nonrandom consumption allocation with the same mean

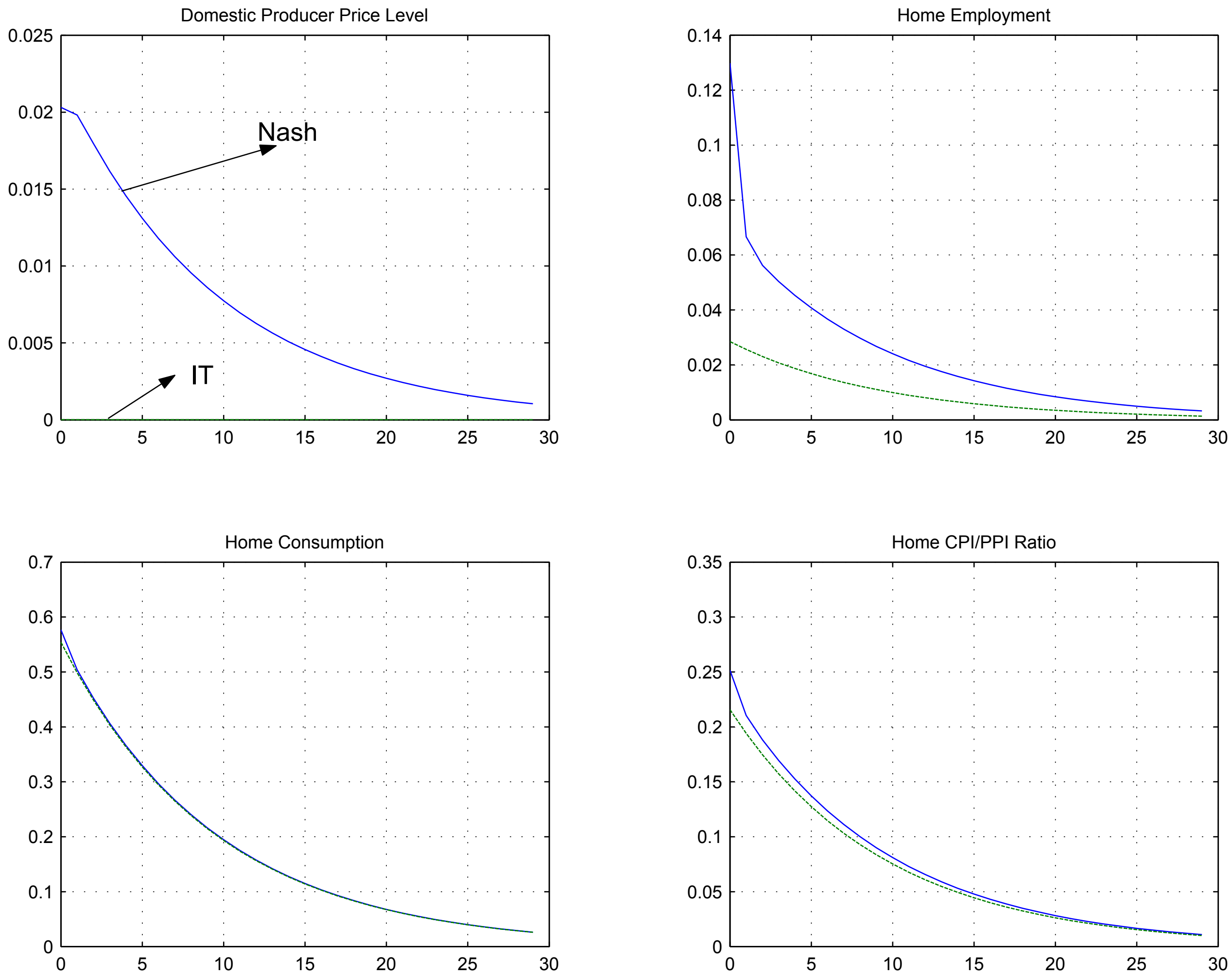

Figure 1. Impulse Responses to a Positive Home Productivity Shock: Nash-Optimal vs. Inflation Targeting

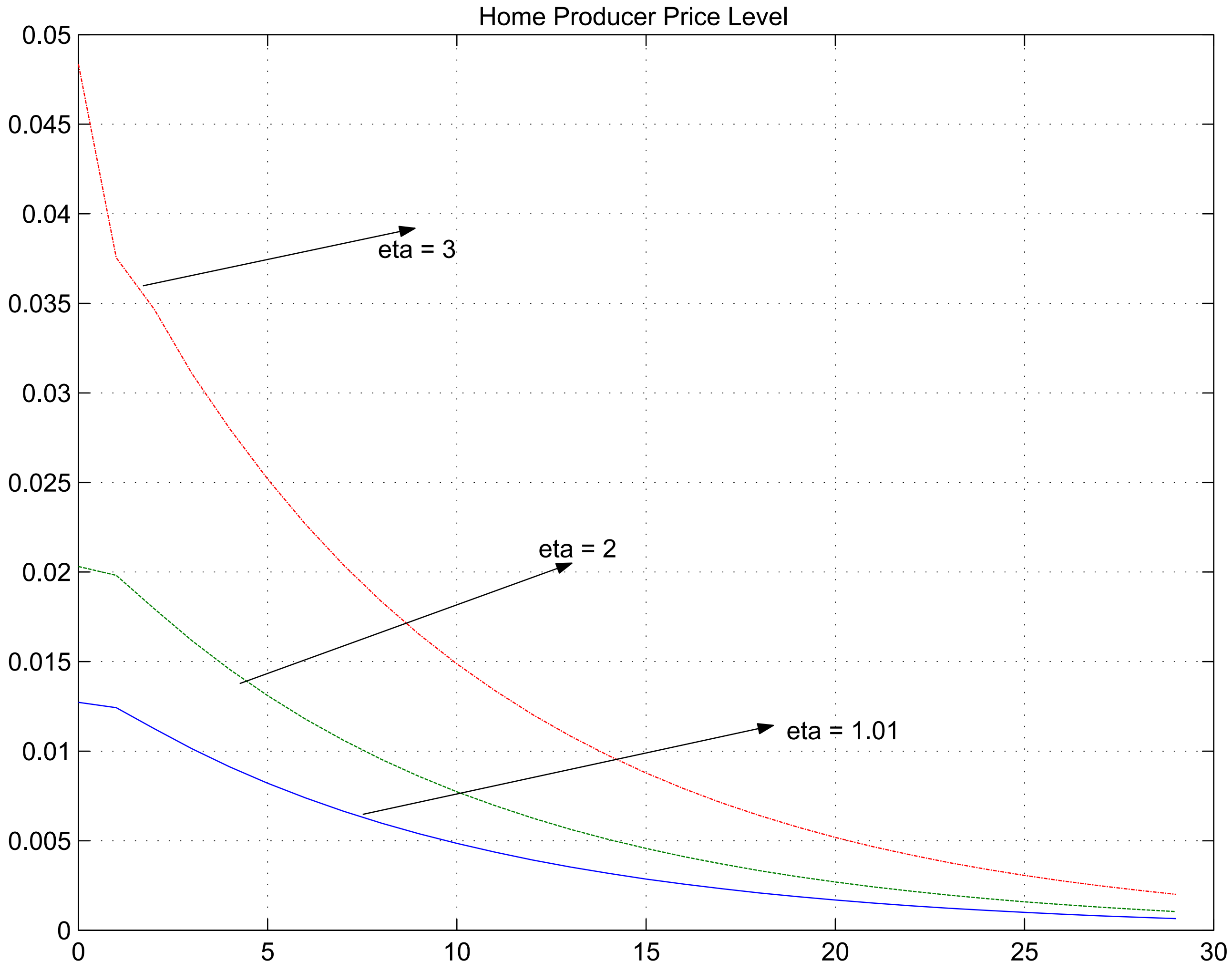

Figure 2. Response to a Productivity Shock under Nash-Optimal: Effect of Varying the Elasticity of Substitution

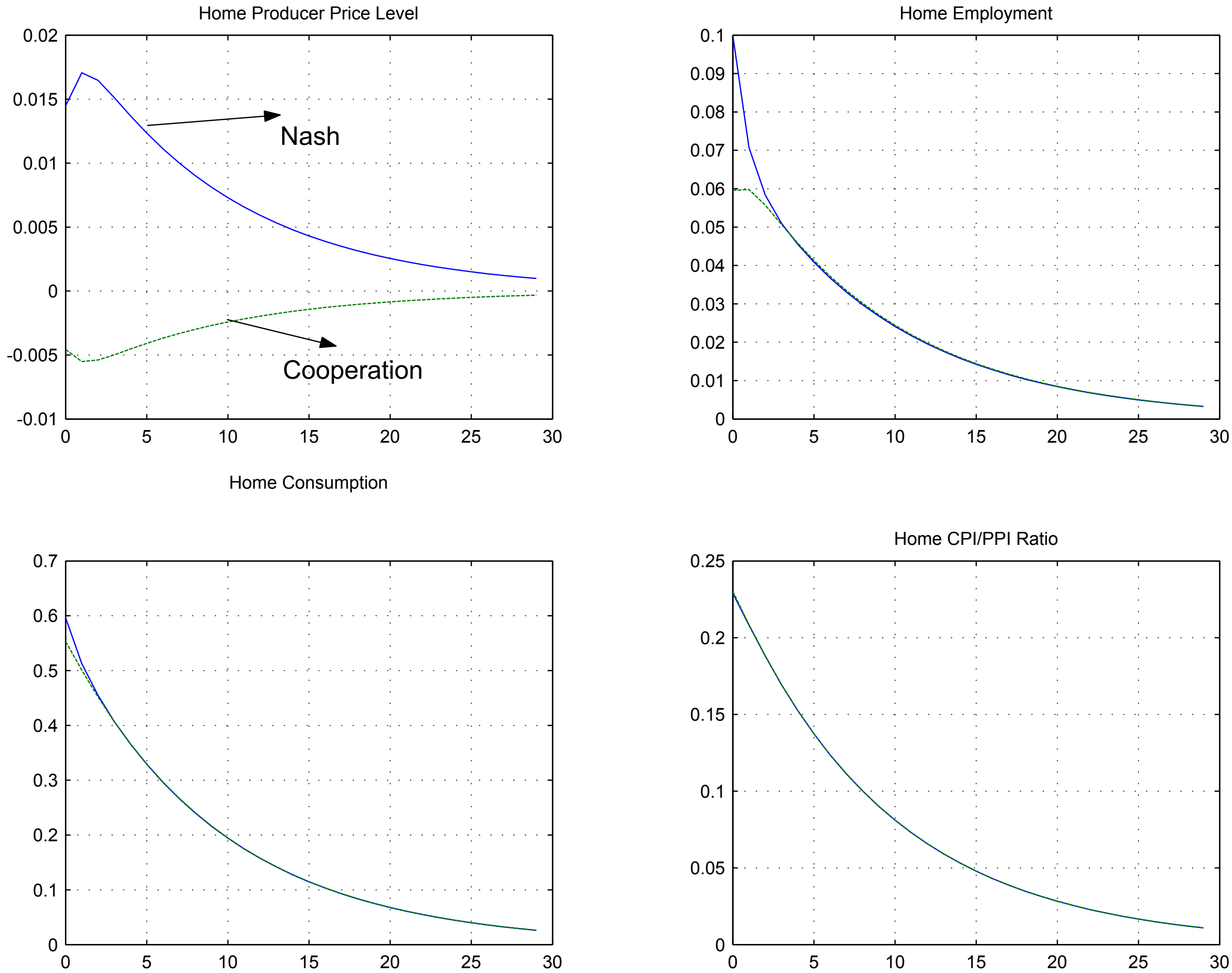

Figure 3. Impulse Responses to a Home Productivity Shock: Nash vs. Cooperation

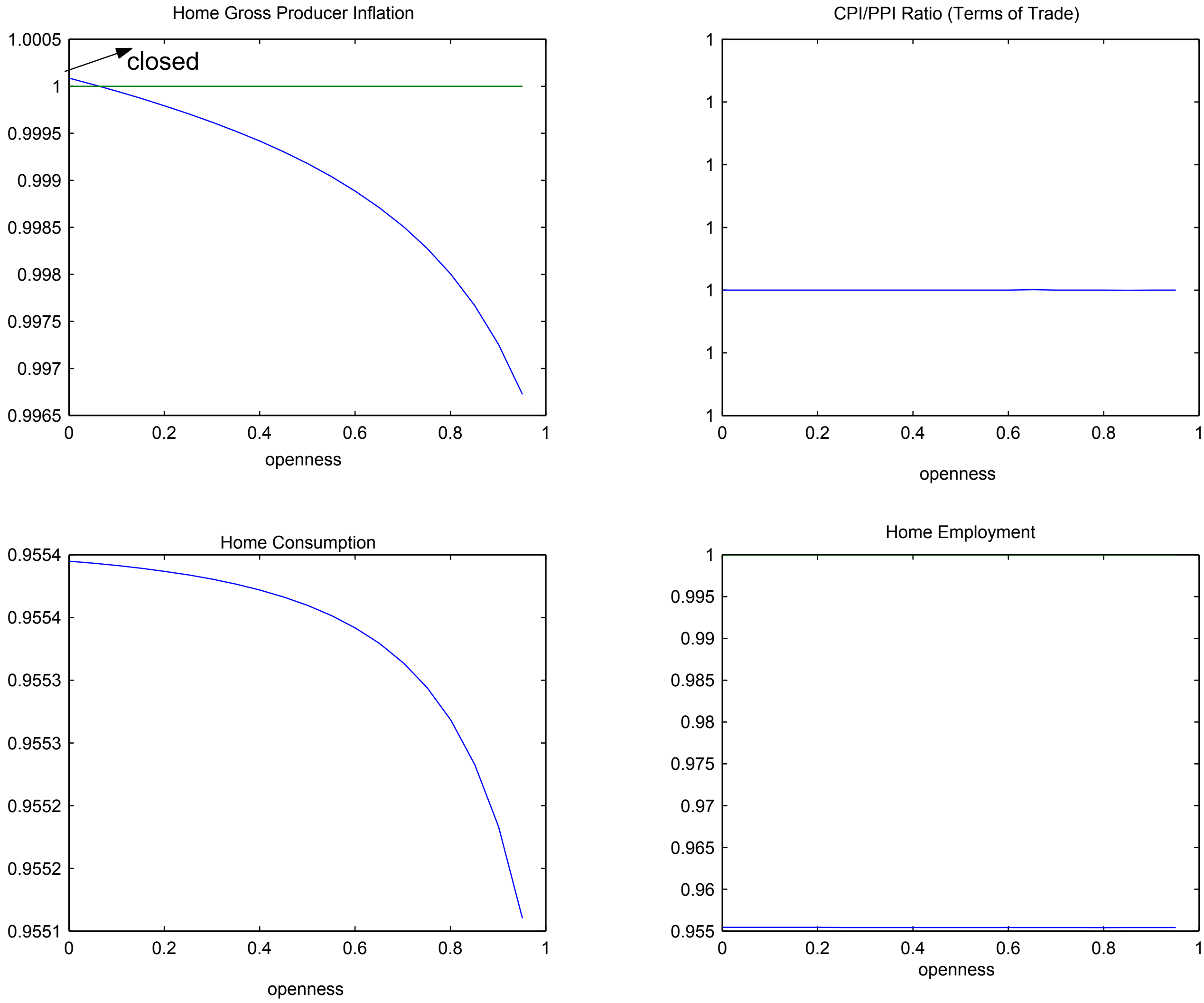

Figure 4. Effect of Varying Openness on the Golden Rule Steady-State under Nash Competition

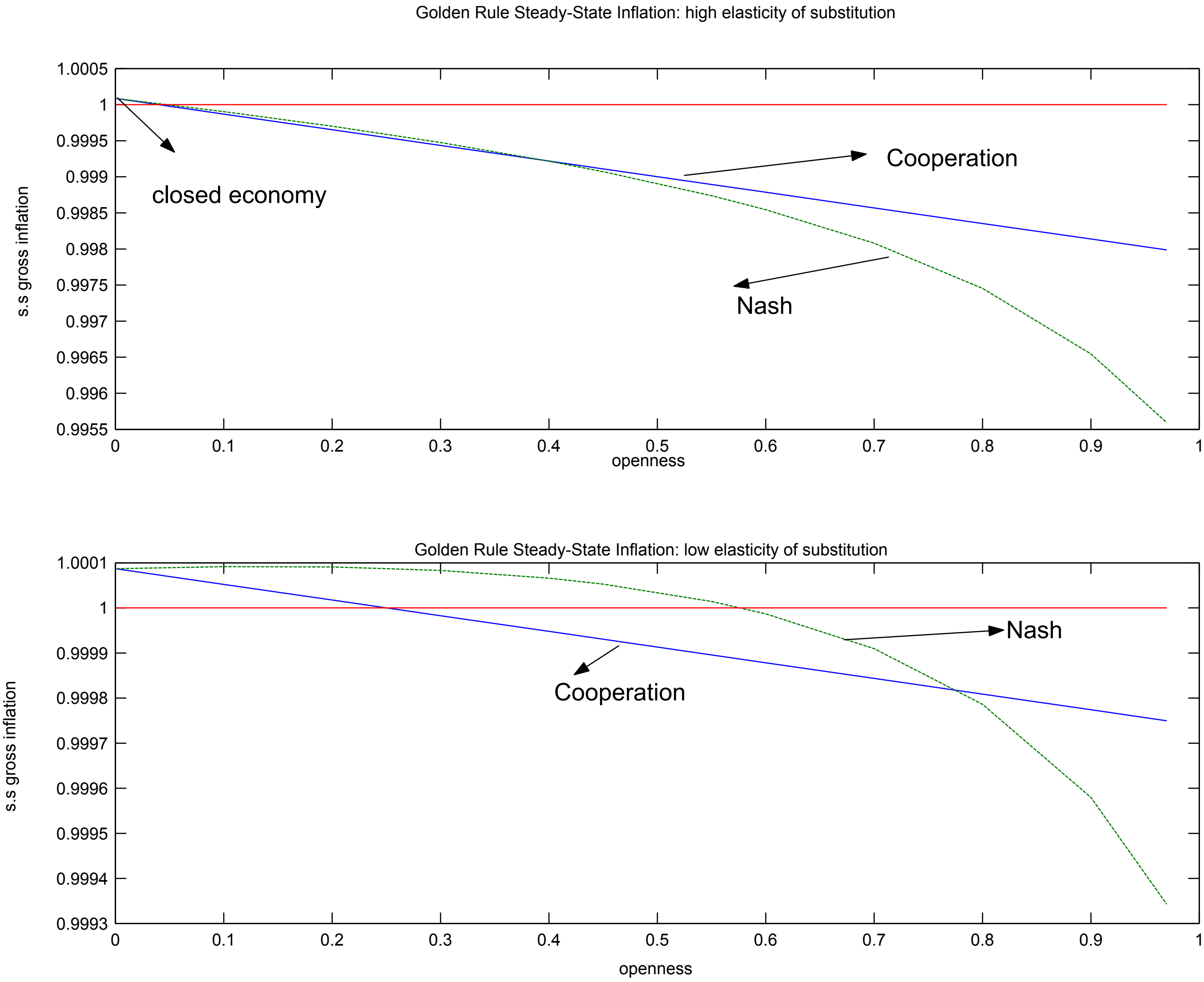

Figure 5. Effect of Varying Openness on the Golden Rule Steady-State Inflation Rate

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