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Alexander Muermann[1] and Stephen H. Shore[2]

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JEL Classification: D82, G13

Keywords: Strategic Trading, Manipulation, Spot Market Power

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Abstract

When a spot market monopolist has a position in a corresponding futures market, he has an incentive to deviate from the spot market optimum to make this position more profitable. Rational futures market makers take this into account when setting prices. We show that the monopolist, by randomizing his futures market position, can strategically exploit his market power at the expense of other futures market participants. Furthermore, traders without market power can manipulate futures prices by hiding their orders behind the monopolist's strategic trades. The moral hazard problem stemming from spot market power thus provides a venue for strategic trading and manipulation that parallels the adverse selection problem stemming from inside information.

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1 Introduction

For many goods, spot markets with market power coexist with competitive futures markets. When a spot market monopolist participates in a futures market, this participation leads to a moral hazard problem in the spot market. In particular, he has an incentive to deviate from the monopoly optimum in order to make his futures market position more profitable. For example, if a monopolist producer of oil holds a short position in an oil futures contract, he will profit if the price of oil goes down. This gives him an incentive to produce more oil than he otherwise might in order to reduce spot market prices and make his futures position more profitable. When rational futures market participants observe the monopolist's position, they will take the impact of this position on subsequent spot prices into account when setting futures prices. When they cannot observe the monopolist's position perfectly they must make rational inferences about the monopolist's position and take these into account when setting prices.[1]

In this paper, we explore strategic trading and manipulation in futures markets when market positions cannot be inferred perfectly. Spot market power allows the monopolist to trade strategically – randomly taking a position in the futures market and then moving spot prices to make that position profitable. This creates an opportunity for those without market power to engage in futures market manipulation – taking a position in a derivatives market and then mimicking the monopolist's futures trading to move futures market prices to make the derivatives position profitable. The literature on market microstructure deals extensively with the effects of asymmetric information when the positions of informed traders cannot be observed perfectly. This paper argues that the moral hazard problem created by spot market power parallels the adverse selection problem created by inside information.

In Section 2, we show how the monopolist can exploit his market power to trade strategically. When he is able to hide his futures market position within the aggregate order flow, he will randomize his orders and then set spot prices to make his futures market position more profitable. This makes hedging more expensive for those who may be the monopolist's counterparty. Spot market power thus discourages futures

[1] This paper takes Anderson's (1990) advice. He surveys the literature on futures trading with perfect inference when the underlying market is imperfectly competitive and suggests in his conclusion:

> "The theoretical development that would be most interesting would be to reconsider some of these models described above under conditions of asymmetric information. In particular, the models reviewed have made the assumption (at least implicitly) that the futures positions of powerful agents are observed so that forecasts of future cash prices can take this into account. In practice, futures positions of agents are likely to be imperfectly observable." (p. 246-247)

market participation for agents without market power and provides a venue for a spot market monopolist to increase expected profits by trading strategically.[2]

This section shows that results similar to those in Kyle's (1985) "noise trader" model are obtained when there is spot market power instead of inside information. In our model, there are no informed traders with private information about future prices *at the time* trading takes place. Instead, the monopolist can set spot market prices *after* trading takes place. In contrast to the "noise traders" in the Kyle model who act mechanically, in our model agents without market power respond optimally to the monopolist's presence in the futures market by reducing their futures market participation (see Spiegel and Subrahmanyam (1992) for the analogous extension of the Kyle model). Pirrong (1995, 2001) shows that a trader who can buy or sell an arbitrarily large number of futures contracts is able to influence the price at liquidation by demanding or selling too many units of the commodity in the delivery market. He can profit in equilibrium from the artificially high or low spot market price if he randomizes his order flow to hide behind the order flow of "noise traders" and if the supply curve in the delivery market is upward sloping. While the randomized strategy in Pirrong parallels the one of the monopolist in this paper, we explicitly model decisions in the spot market, endogenize the initial futures position of the strategic agent and the response of hedgers.

In Section 3, we extend Section 2 by showing how traders can move (i.e. "manipulate") futures prices even when they do not have market power. If a futures market manipulator takes a position in the derivatives market, he has an incentive to move subsequent prices to make his initial position profitable. He will be able to move prices if market participants believe that his subsequent trades may have been submitted by the monopolist. While these trades are unprofitable, their cost is outweighed by the benefit of moving prices to make the initial position more profitable.

Past research has shown that markets can be manipulated if some agents have private information about prices (see e.g. Hart (1977), Jarrow (1992), Allen and Gale (1992), Kumar and Seppi (1992)). For example, Kumar and Seppi, develop a model in which uninformed manipulators are able to profit in the futures market because spot market makers are unable to differentiate the manipulator's order flow from the informed trader's order flow. We show that monopoly power serves a similar function.

[2] Despite the importance of oil price risk faced by many industry sectors, the futures market on oil is relatively illiquid; in particular, the trading volume of longer-term futures contract is very low. Our paper suggests that this may stem from the imperfectly competitive nature of the spot market for oil.

2 Strategic Trading by Spot Market Monopolists

In this section, we show how spot market monopolists can exploit their market power by trading strategically in the futures market. This trade discourages futures market participation since traders fear that the monopolist may be their counterparty or the counterparty of another trader with a similar position. The monopolist will exert spot market power to make his futures position more profitable, thereby reducing the profits of his counterparties.[3]

This section builds on the work of Kyle (1985), who shows that agents with inside information can profitably exploit their informational advantage by hiding behind the order flow of uninformed "noise traders". In our model, the aggregate hedging demand of agents without market power is stochastic just as the number of "noise traders" is stochastic in the Kyle model.

The monopolist can increase his profits because, when setting futures market prices, market makers cannot fully take into account take the impact of the monopolist's unobserved futures market position on expected spot prices. While the monopolist's expected spot market profit is reduced by deviating from the monopoly optimum, his expected profit in the futures market more than makes up for it. Since market makers earn zero expected profits, the monopolist's expected futures market profits imply expected futures market losses for other market participants. This increased cost deters these agents from hedging price risk as much as they otherwise might. While we consider only the case of monopoly – when there is exactly one agent with spot market power – to obtain greater analytic tractability, similar logic will apply in an oligopolistic setting.

2.1 Model Setup

We envision a model with one good and two periods, $t = 1, 2$. The good is produced only in the second period and sold in the spot market. The cost of production is normalized to zero. Demand is uncertain and realizes in between the two periods. In addition, there is a competitive futures market.

There are three types of agents in this market. First, there is a spot market monopolist. The monopolist

[3] Storage may reduce the ability of the monopolist to trade strategically. When storage is inexpensive, agents without market power may purchase and store the good in anticipation of higher prices in the future. This limits the ability of the monopolist to raise prices, as excess capacity will prevent prices from increasing. In this sense, storage serves the same function as durability in the durable-goods monopoly problem of Coase (1972). Like durability, storage erodes market power by providing a venue for the monopolist to compete against his future self. Here, we assume that storage costs are high enough that no storage takes place in equilibrium, so that monopoly power is not eroded.

sets the spot price (and therefore quantity) to maximize expected profits. We assume that the monopolist is risk-neutral, so that he has no incentive to participate in the futures market unless he can increase expected profits by doing so. Second, there are competitive risk-neutral market makers who observe the aggregate order flow and set futures prices accordingly. Third, there are risk-averse agents whose payoff depends on the price realized in the spot market. They have an incentive to participate in the futures market because doing so allows them to hedge spot price risk. We assume that the number of these agents is stochastic and unobservable.

The timing of events is as follows. First, nature chooses a number of risk-averse agents. Then in $t = 1$, the monopolist and the risk-averse agents simultaneously submit futures market orders. Observing the aggregate order flow, the sum of these orders, market makers set futures prices equal to expected spot prices. Next, demand is realized and in $t = 2$, the monopolist chooses spot market quantity to maximize profits. Figure 1 provides a timeline.

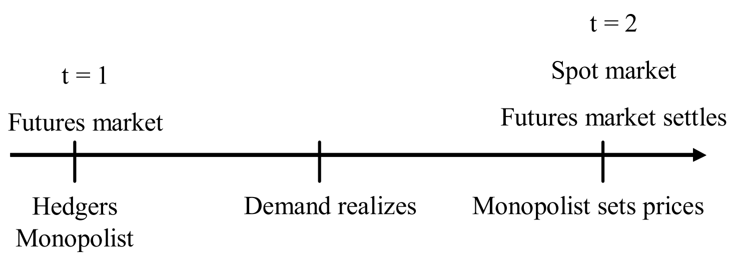

Figure 1

We make the following assumptions:

- The demand curve is linear, so that spot prices are given by $P = a - bQ$, where a is stochastic and $b > 0$.[4]

- All risk-averse agents are identical and the number of such agents, N, is stochastic and uniformly distributed on $[0, 1]$.

- Each risk-averse agent has profits that are linear in the spot market price, i.e. $\pi^n(P) = c_0 + c_1 P$, with

[4] The choice of a linear demand function is for analytic tractability. While a much broader class of functions will obtain similar results, not all demand functions will obtain the same results. In particular, convex demand curves will provide an even stronger incentive for the monopolist to strategically trade in the futures market as large changes in the spot price lead to relatively small changes in monopoly profits. Concave demand curves provide a weaker incentive for strategic trading.

$c_1 < 0$, so that higher spot prices imply lower profits.[5]

- Any risk-averse agent is too small to affect aggregate order flow and thus takes prices as given.

In the initial period, the monopolist can enter into a futures contract with payoff $P - k$ per contract, where k is the futures price. The monopolist chooses a number of contracts C^m. Given C^m and realization of demand, a, the monopolist sets spot market price and quantity to maximize profits

$$\begin{aligned} \pi &= C^m (P - k) + PQ \\ &= C^m (a - bQ - k) + (a - bQ) Q. \end{aligned} \tag{1}$$

The spot market FOC is

$$\frac{\partial \pi}{\partial Q} = -bC^m + a - 2bQ = 0.$$

Note that the SOC is satisfied, yielding an optimal quantity and price

$$\begin{aligned} Q^* &= \frac{1}{2b} (a - bC^m); \\ P^* &= \frac{1}{2} (a + bC^m). \end{aligned} \tag{2}$$

Each risk-averse agent chooses a number of contracts C^n. This number will be determined optimally based on their preferences. The total number of contracts submitted by these agents, NC^n, is stochastic. Market makers only observe the aggregate order flow, $NC^n + C^m$. They have beliefs about the order flow submitted by the monopolist and the risk-averse agents and set the futures price, k, accordingly.

2.2 Equilibrium with Strategic Trading

In this setup, we look for perfect Bayesian equilibria in the futures market given optimal subsequent behavior in the spot market. We assume a set of actions and beliefs for all agents and explore whether any agent has an incentive to deviate. This section explores equilibria in which the monopolist hides his futures market participation by randomizing the order flow he submits. When the monopolist submits a positive (negative)

[5] The linear functional form is used for tractability. Any profit function that provides hedging motives will yield similar results. In particular, an upward sloping profit function, $c_1 > 0$, would imply risk-averse agents taking a short instead of a long position, but to a lesser extend than they would in the absence of strategic trading by the spot market monopolist.

order flow – with plans to drive up (down) spot prices to make this position profitable – market makers are unsure about the order submitted by the monopolist. This imperfect inference allows the monopolist to receive favorable futures market prices, at the expense of other agents in the market.

In this setting, a perfect Bayesian equilibrium consists of

1. beliefs held by market makers about C^n and the distribution of $\tilde{C}^m$, and a price schedule, $k(.)$ for which market makers earn zero expected profits,

2. beliefs held by the monopolist about $k(.)$ and C^n, and a set of possible values for C^m where each yields the same expected profit given those beliefs, and no other values for C^m yield higher expected profits,

3. beliefs held by the risk-averse agents about $k(.)$ and the distribution of $\tilde{C}^m$, and a value of C^n that maximizes expected utility given those beliefs, and

4. off–path beliefs held by market makers about the monopolist's order flow when the observed aggregate order flow is inconsistent with their beliefs – given prices set competitively based on these beliefs, the monopolist will not choose to submit an off–path order flow quantity.

The beliefs of all agents must be consistent with one another, and with the actions of other agents.

2.2.1 Beliefs and Prices of Market Makers

There are many sets of beliefs that market maker could hold about the monopolist's futures market participation that imply that the monopolist's order flow cannot be perfectly inferred from the aggregate order flow. Here, we look for an involving the simplest set of such beliefs. Suppose market makers believe that each risk-averse agent submits an order C^n and that the monopolist randomizes between $+x$ and $-x$ with equal probability where $0 \leq x < \frac{1}{2}C^n$. Based on their beliefs, they set actuarially fair prices. Off–path, we assume that market makers set prices based on the most punitive beliefs.

The aggregate order flow, $\theta \equiv C^m + C^n N$, can indicate that the monopolist has successfully hidden, that he has been caught for sure with having submitted $+x$ or $-x$ given market maker beliefs, or that aggregate order flow is inconsistent with market makers' beliefs. We categorize the aggregate order flow into the following five groups and specify the price schedules for all possible values of θ.[6]

$$
\begin{aligned}
A1.\ k(\theta) &= \frac{1}{2}E[a] + \frac{1}{2}b\theta \text{ if } \theta > x + C^n \\
A2.\ k(\theta) &= \frac{1}{2}E[a] + \frac{1}{2}bx \text{ if } -x + C^n < \theta \leq x + C^n \\
A3.\ k(\theta) &= \frac{1}{2}E[a] \text{ if } x \leq \theta \leq -x + C^n \\
A4.\ k(\theta) &= \frac{1}{2}E[a] - \frac{1}{2}bx \text{ if } -x \leq \theta < x \\
A5.\ k(\theta) &= \frac{1}{2}E[a] + \frac{1}{2}b(\theta - C^n) \text{ if } \theta < -x
\end{aligned}
\tag{3}
$$

In ranges $A1$ and $A5$, market makers know that the monopolist submitted an order flow inconsistent with market makers' expectations. In range $A1$, it must have been the case that $C^m > x$, and they assume that $N = 0$. In range $A5$, it must have been the case that $C^m < -x$, and they assume $N = 1$. Prices are set accordingly. In ranges $A2$ and $A4$, market makers believe that the monopolist submitted $+x$ and $-x$, respectively. Prices are set accordingly. In range $A3$, the monopolist hides successfully within the aggregate order flow. In this region, market makers believe that $-x$ and $+x$ are equally likely.

Prices are set competitively. In other words, if the monopolist and risk-averse agents take actions that conform to the beliefs of market makers, then no market maker will have an incentive to deviate. Note that the market makers' behavior takes as given the order flow of each risk-averse agent, C^n. Next, we examine optimal behavior on the part of the monopolist given the beliefs and price schedule of market makers.

2.2.2 Beliefs and Actions of Monopolist

The monopolist takes as given the order flow of risk-averse agents, C^n, as well as the futures price schedule, $k(\cdot)$, set by market makers given the aggregate order flow. Since the monopolist is risk-neutral and market makers are rational, the monopolist will not participate in the futures market in any equilibrium in which

[6] If $x = 0$, prices are set as:

$$
\begin{aligned}
A1.\ k(\theta) &= \frac{1}{2}E[a] + \frac{1}{2}b(\theta) \text{ if } \theta > C^n \\
A3.\ k(\theta) &= \frac{1}{2}E[a] \text{ if } 0 \leq \theta \leq C^n \\
A5.\ k(\theta) &= \frac{1}{2}E[a] + \frac{1}{2}b(\theta - C^n) \text{ if } \theta < 0
\end{aligned}
$$

he does not try to disguise his order flow. When he does not participate, his expected profits are

$$E[\pi | C^m = 0] = \frac{1}{4b} E\left[a^2\right].$$

On the other hand, if the monopolist finds it optimal to randomize in a way consistent with market makers' beliefs, he must earn the same expected profits whether he submits an order flow $+x$ or $-x$. Otherwise, he would only play one of the strategies and his actions would be incompatible with market makers' beliefs. Given the futures price schedule, $k(.)$, we now find the optimal behavior on the part of the monopolist.

Proposition 1 *Given that market makers set $k(.)$ as in* (3)*, the monopolist will maximize expected profits by submitting either $C^m = +x$ or $-x$, where $0 \leq x < \frac{1}{4}C^n$. The monopolist's expected profits will be*

$$\begin{aligned} E[\pi | x] &= E[\pi | -x] = \frac{1}{4b} E\left[a^2\right] + bx^2 \left(\frac{1}{4} - \frac{x}{C^n}\right) \\ &> E[\pi | C^m = 0] \text{ for } x > 0. \end{aligned}$$

Proof. See Appendix A.1. ∎

When market makers set $k(.)$ consistent with the belief that the monopolist randomizes between $+x$, and $-x$, the monopolist will find it optimal to act consistently with those beliefs. Note that there are many possible equilibria, one for each x. In an equilibrium in which $x = 0$, the monopolist does not participate in the futures market. For larger x, the monopolist profits in the futures market at the expense of risk-averse agents.

2.2.3 Beliefs and Actions of Risk-Averse Agents

The risk-averse agents know that it is optimal for the monopolist to hide his futures position within the aggregate order flow by randomizing C^m and then set spot market prices optimally given this futures position. Their preferences are represented by a concave utility function, u. To reduce their exposure to spot market price risk, a given risk-averse agent will participate in the futures market by purchasing C units of the futures contract. C^n is the number of contracts purchased by the average risk-averse agent in the market and is set optimally by each agent according to the following optimization problem:

$$\begin{aligned} C^{n*} &= \arg\max_{C} E\left[u\left(C\left(P^{*}-k\right)+c_{0}+c_{1}P^{*}\right)\right] \\ &= \arg\max_{C} E\left[u\left(C\left(\frac{1}{2}\left(a+bC^{m}\right)-k\right)+c_{0}+c_{1}\left(\frac{1}{2}\left(a+bC^{m}\right)\right)\right)\right] \end{aligned} \tag{4}$$

where $k(.)$ is set consistent with (3). As before, risk-averse agents believe that C^m can take on two values, $+x$ and $-x$, with equal probability. We have shown above that for a given C^n there exist equilibria with $0 \leq x < \frac{1}{4}C^n$.

When a risk-averse agent wants to hedge, this provides him with information that the expected aggregate hedging demand is high. He then acts rationally taking this information into account. Since risk-averse agents are identical, none is more or less likely to hedge than any other. Therefore, the distribution of the number of hedgers, conditional on a given agent wanting to hedge, is $f(N) = 2N$.[7]

First, we examine optimal hedging in the absence of the monopolist, i.e. if $x = 0$. In this case,

$$C^{n*} = \arg\max_{C} E\left[u\left(C\left(\frac{1}{2}a-\frac{1}{2}E[a]\right)+c_{0}+c_{1}\frac{1}{2}a\right)\right].$$

The FOC is then

$$E\left[\left(\frac{1}{2}a-\frac{1}{2}E[a]\right)u'\left(C\left(\frac{1}{2}a-\frac{1}{2}E[a]\right)+c_{0}+c_{1}\frac{1}{2}a\right)\right]=0.$$

Note that the SOC is satisfied. For $C^{n*} = -c_1$, the FOC is satisfied and it is a global maximum. Without the monopolist's participation in the futures market, it is optimal for risk-averse agents to eliminate all risk. We now examine optimal hedging when the monopolist participates in the futures market, i.e. when $x > 0$.

Proposition 2 *In an equilibrium in which the monopolist participates in the futures market with order flow $+x$ and $-x$ with equal probability where $0 < x < \frac{1}{4}C^n$, risk-averse agents maximizing (4) will participate in the futures market, though will participate less than they would if the monopolist did not participate, i.e. $0 < C^{n*} < -c_1$.*

Proof. See Appendix A.2. ■

[7] Similarly, agents without hedging needs would update their beliefs about the expected number of hedgers accordingly. These agents will participate in the futures market to exploit their information. This will mitigate but not eliminate the effect we discuss. If all agents do not take into account the information contained in their own hedging demand, these agents will not believe that they face unfavorable prices on average, and will not reduce their hedging demand. However, their expected profits will be lower if they hold these naïve beliefs.

If risk-averse agents believe that the monopolist trades strategically in the futures market, they are concerned that the monopolist will hold an opposite position and move spot prices against them. A given risk-averse agent knows that he is more likely to want to hedge precisely at the wrong times as he is more likely to hedge when aggregate order flow from risk-averse agents is large. In this case, either the monopolist also submits a large order flow and is spotted – in which case futures prices are set fairly – or the monopolist submits a small order flow and hides successfully – in which case the monopolist gains at the risk-averse agents' expense. This makes hedging more expensive for agents without market power and thus discourages their participation in the futures market.

The following proposition shows that a perfect Bayesian equilibrium exists with the beliefs and actions as specified above.

Proposition 3 *Given the market structure described in Subsection 2.1, there exists a perfect Bayesian equilibrium in which futures market prices are set as in (3), risk-averse agents each submit an order of* C^n*, where* $0 < C^n < -c_1$*, and the monopolist submits an order flow of either* $+x$ *or* $-x$ *with equal probability, where* $0 < x < \frac{1}{4}C^n$*.*

Proof. *Proposition 1 shows that the monopolist has no incentive to deviate from this equilibrium. Proposition 2 shows that risk-averse agents have no incentive to deviate from this equilibrium. Market makers earn zero profits and none has an incentive to offer another price schedule.* ■

Here, a spot market monopolist is able to increase profits by trading strategically in the futures market. The monopolist takes a futures market position randomly, then deviates from the spot market monopoly optimum to move spot market prices and make this position more profitable. If the monopolists futures market position were perfectly observable, market makers would set futures market prices anticipating these actions. In this case, the monopolist would not want to participate in the futures market since doing so would decrease expected spot market profits without increasing expected futures market profits. However, when there are other traders in the market, the futures market position submitted by the monopolist cannot be perfectly inferred by observing the aggregate order flow. In this case, market makers set prices based on the rational belief that the orders they receive could have come from either the monopolist or from other agents without market power. As a result, trades submitted by the monopolist move prices less than they would had they been observable. Just as an informed trader in the Kyle model profits at the expense of "noise traders", the monopolist earns positive expected profits in the futures market at the expense of the other market participants. This makes futures market participation expensive, and reduces the optimal participation of risk-averse agents.

3 Futures Market Manipulation under Spot Market Power

The last section showed that a spot market monopolist can profitably exploit spot market power in the futures market. This section documents that even those without market power can profit in the futures market when another agent has spot market power. This section relies on the insight of Kumar and Seppi (1992), that agents without inside information can manipulate a market if they are mistaken for agents with inside information. Here, we show that the same can be said of market power: agents without market power who hold futures market positions can use later trading to manipulate prices to make the original position profitable when market makers believe they might be the monopolist.

3.1 Model Setup with Manipulators

3.1.1 Timing and Markets

While the model developed Section 2 has markets in only two periods, the model in this section requires trade in three periods. $t = 1, 2$ mirror our earlier setup; here, we add an initial period, $t = 0$, in which agents trade contracts whose payoffs are contingent on futures prices in the next period. Presenting the markets in reverse chronological order:

$t = 2$: There is a spot market at time $t = 2$. As before, production in this period is controlled by a monopolist, who faces a linear demand curve, i.e. spot prices are given by $P = a - bQ$, where a, $b > 0$.[8] The cost of production is zero.

$t = 1$: There is a futures market at time $t = 1$, characterized by linear cash-settled contracts based on the spot price in the next period, with payoff $P - k_1$ per contract.

[8] While the assumption of linear demand is critical to obtain simple analytic results, the same intuition obtains with a convex demand curve. While losing analytic tractability, these demand functions have the advantage that the monopolist has a strict benefit from participating in the futures market. When demand is linear, the increased profits in the futures market that come with futures market participation are exactly offset by lower spot market monopoly profits.

$t = 0$: There is a futures market at time $t = 0$, characterized by linear cash-settled contracts based on the futures strike price in the next period, with payoff $k_1 - k_0$ per contract.[9]

3.1.2 Actors

The model involves four types of actors:

1. Noise traders submit a stochastic order flow, C_0^n, at $t = 0$ and they do not participate at $t = 1$. We assume that C_0^n is uniformly distributed on $[C^{n-}, C^{n+}]$. The assumption that noise traders participate only in the initial period is for expositional simplicity and is not necessary to obtain these results.[10]

2. Monopolist submits an order flow, C_1^m, at $t = 1$, and then sets prices and quantities optimally at $t = 2$. To simplify the problem, the monopolist is assumed not to participate in the futures market at $t = 0$.

3. Manipulator (denoted by the letter h to refer to "hiders") submits an order flow, C_0^h, at $t = 0$, and C_1^h, at $t = 1$. We impose the following liquidity constraint $\left|C_0^h\right| \leq W < \frac{1}{2}\left(C^{n+} - C^{n-}\right)$.[11]

4. Market makers, as before, are risk-neutral and act competitively to set strike prices k_0 and k_1. Market makers observe aggregate order flow $\theta_1 \equiv C_1^m + C_1^h$ at $t = 1$ and $\theta_0 \equiv C_0^h + C_0^n$ at $t = 0$, and make rational inferences about the positions of various agents and their impact on contract payoffs. Therefore, $k_1 = E\left[P^*|\theta_1\right]$ and $k_0 = E\left[k_1|\theta_0\right]$.

[9] Note that a futures contract whose payoff is based on the price of another futures contract is unusual. However, there are many options whose payoff is based on a futures contract. While we use a linear futures contract and not an options contract at $t = 0$ for analytical tractability, our result that manipulative trading exists in equilibrium is robust to changes in the contractual structure. Furthermore, many futures markets based on the spot price of a storable commodity are effectively a future on a future, as storability links current spot and forward prices.

[10] For analytical tractability, we assume that these traders act mechanically, as "noise traders" do in the Kyle model. Introducing optimal behavior on their part as in Section 2 would not change our result that manipulative trading is possible in equilibrium.

[11] Similar to Kumar and Seppi (1992), the manipulator will find it optimal to take an unbounded position. We follow Kumar and Seppi by imposing a wealth constraint to obtain an equilibrium.

Figure 2 provides a timeline showing which agents participate in each market.

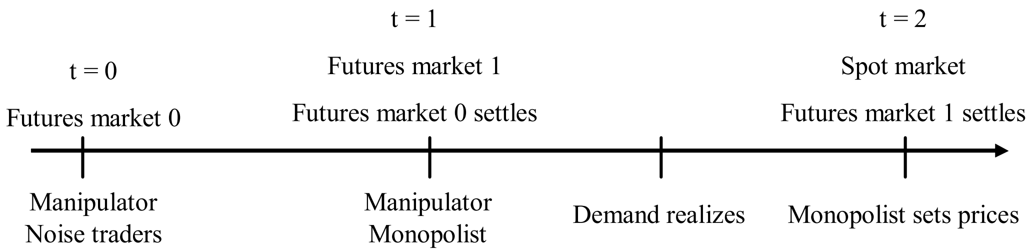

Figure 2

The monopolist is willing to participate in the futures market for the same strategic reason outlined in Section 2. He earns profits by setting spot market prices to make his futures market position profitable. While the monopolist's spot market profit at $t = 2$ is lower than it would be had he not participated in the futures market, futures market profits in $t = 1$ are high enough (at least weakly) to offset these reduced profits.

The manipulator is willing to accept expected losses in the futures market at $t = 1$ for the same reason that the monopolist is willing to accept lower expected profits in the spot market at $t = 2$. Just as the monopolist sets spot prices at $t = 2$ to make his futures market position at $t = 1$ profitable, the manipulator trades in the futures market at $t = 1$ in order to move futures prices, thereby making his futures market position at $t = 0$ profitable. Just as the monopolist earns expected profits at the expense of the manipulator at $t = 1$, the manipulator earns expected profits at the expense of noise traders at $t = 0$.

3.1.3 Spot Market Prices

For a given futures market position, the monopolist's profit is given by (1) so that prices and quantities are set optimally according to (2). Optimal profit will then be

$$\pi = -C_1^m k_1 + \frac{1}{4b}\left(a + bC_1^m\right)^2.$$

As in Section 2, futures market participation causes the monopolist to deviate from the spot market monopoly optimum. He moves prices to make the futures market position profitable.

3.2 Equilibrium with Manipulation

Here, we propose a perfect Bayesian equilibrium with manipulation:

$t = 0 :$ The manipulator randomizes between $+x$ and $-x$ with equal probability where

$$x = \min\left(W, \frac{2}{3}\left(C^{n+} - C^{n-} - W\right)\right).$$

Market makers set

$$k_0 = \frac{1}{2}a$$

regardless of the aggregate order flow submitted.

$t = 1 :$ There are three possible subgames (denoted SG1, SG2, and SG3) depending on the aggregate order flow, $\theta_0 = C_0^h + C_0^n$, at $t = 0$:

SG1. If $\theta_0 > C^{n+} - x$ then market makers know that the manipulator must have submitted $C_0^h = +x$. In this case, the monopolist will not participate in the futures market, i.e. $C_1^m = 0$. The manipulator submits the same order as in the previous period, i.e. $C_1^h = C_0^h$, and market makers set the futures price as

$$k_1 = E\left[P^*|\theta_1\right] = \frac{1}{2}a + \frac{1}{2}b\left(\theta_1 - x\right).$$

SG2. If $C^{n-} + x \leq \theta_0 \leq C^{n+} - x$ then the manipulator has successfully hidden his order flow in the previous period. The monopolist randomizes over $C_1^m \in \left\{-\frac{1}{2}x, \frac{1}{2}x\right\}$ with equal probability, and the manipulator sets $C_1^h = \frac{1}{2}C_0^h$. Market makers set the futures price as

$$k_1 = E\left[P^*|\theta_1\right] = \frac{1}{2}a + \frac{1}{4}b\theta_1.$$

SG3. If $\theta_0 < C^{n-} + x$ then market makers know that the manipulator must have submitted $C_0^h = -x$. The monopolist will not participate, i.e. $C_1^m = 0$, and the manipulator submits the same order as in the previous period, i.e. $C_1^h = C_0^h$. The futures price is then set as

$$k_1 = E\left[P^*|\theta_1\right] = \frac{1}{2}a + \frac{1}{2}b\left(\theta_1 + x\right).$$

$t = 2:$ The monopolist sets prices and quantities according to (2).

Proposition 4 *The actions and beliefs described above constitute a perfect Bayesian equilibrium.*

Proof. See Appendix A.3. ■

Financial market manipulation is possible when agents without market power can be mistaken for those with market power.

An agent without market power can profit by taking a random position in the initial futures market at $t = 0$. When this random position is not spotted as in SG2, he has an incentive to move subsequent futures prices at $t = 1$ to make this initial position more profitable. For example, if he takes a long position in the initial futures market, this position becomes profitable if subsequent futures market prices are high. As a result, he has an incentive to take a long position in the subsequent futures market to drive up prices. When market makers observe this long position, they believe it could have been submitted by the monopolist, who would then use his monopoly power to raise spot prices. Therefore, market makers rationally set higher futures prices in response to the long aggregate order flow they observe. Since the manipulator's trade at $t = 1$ moves prices without altering the underlying contract payoff, this trade is unprofitable. By taking a larger position in the initial futures market than in the subsequent one, the profits he earns in the initial futures market by moving subsequent prices exceed his losses from subsequent trading.

When the manipulator's position is identified (SG1 and SG3), his subsequent trades can be inferred perfectly and he earns no profit.

In Section 2, a monopolist's trades could not be differentiated from those submitted by risk-averse agents. He was able to profit because the futures market trades he submitted moved prices by *less* than they would have had they been observable. In this section, a manipulator's trades cannot be differentiated from those of the monopolist. The manipulator is able to profit because the futures market trades he submits move prices by *more* than they would have had they been observable. As in Section 2, the monopolist profits from the manipulator's presence at $t = 1$ since this causes the trades he submits to move prices by *less* than they would otherwise.

4 Conclusions

In this paper, we have shown how monopoly power impacts futures market behavior when futures market participation is not observable. Spot market monopolists will trade in the futures market – trying to hide behind the trades of agents without market power – and then strategically set spot prices to make their futures positions more profitable. This makes hedging expensive, and therefore reduces futures market participation for agents without market power. Agents without market power may manipulate futures prices by hiding behind the trades of the monopolist to make their earlier futures market positions profitable. As in the case of models with an informed trader instead of a monopolist, we have shown that both strategic and manipulative trading can exist in equilibrium.

Many existing futures markets with imperfectly competitive underlying spot markets exhibit very low levels of participation relative to their importance. In particular, markets for longer term contracts tend to be illiquid. For example, the trading activity in futures markets for oil is relatively low. Our paper suggests an explanation based on the imperfectly competitive nature of the oil spot market. Given the moral hazard problems discussed in this paper, several markets – including weather derivatives – have emerged to avoid the inefficiencies caused by market power. Weather derivatives provide an index-hedge against extreme temperatures, and therefore against oil demand risk. Despite large basis risk, these contracts are not susceptible to moral hazard.

A Appendix: Proofs

A.1 Proof of Proposition 1

In this case, when submitting C^m, there are 7 ranges the monopolists order flow, C^m, could be in. These are categorized according to which possible prices, $A1 - A5$, the monopolist could face, depending upon the realization of N:

M1 $C^m > x + C^n$ always $A1$ "caught up off-"

$$\begin{aligned}
E[\pi|C^m] &= -C^m E[k|C^m] + \frac{1}{4b} E\left[(a + bC^m)^2\right] \\
E[\pi|C^m] &= -C^m \int_0^1 \left(\frac{1}{2}E[a] + \frac{1}{2}b(C^m + C^n N)\right) dN + \frac{1}{4b} E\left[(a + bC^m)^2\right] \\
&= \frac{1}{4b} E\left[a^2\right] - \frac{1}{2}bC^m \left(\frac{1}{2}C^m + \frac{1}{2}C^n\right) \\
&< E[\pi|0] = \frac{1}{4b} E\left[a^2\right] \text{ if } C^n > -C^m
\end{aligned}$$

M2 $-x + C^n < C^m \leq x + C^n$ either $A1$ "caught up off-" or $A2$ "caught up on-"

(a) $A1$ if $C^n + x - C^m < C^n N \leq C^n$

(b) $A2$ if $0 \leq C^n N \leq C^n + x - C^m$

$$\begin{aligned}
E[\pi|C^m] &= -C^m E[k|C^m] + \frac{1}{4b} E\left[(a + bC^m)^2\right] \\
E[\pi|C^m] &= -C^m \frac{1}{2}E[a] - C^m \int_{1+\frac{x-C^m}{C^n}}^1 \frac{1}{2}b(C^m + C^n N)\, dN - C^m \int_0^{1+\frac{x-C^m}{C^n}} \frac{1}{2}bx dN + \frac{1}{4b} E\left[(a + bC^m)^2\right] \\
&= \frac{1}{4b} E\left[a^2\right] - \frac{1}{4}bC^m \left(C^m + \frac{(x - C^m)^2}{C^n}\right) \\
&< E[\pi|0] = \frac{1}{4b} E\left[a^2\right] \text{ if } C^m > 0
\end{aligned}$$

M3 $x < C^m \leq -x + C^n$ either $A1$ "caught up off-", $A2$ "caught up on-", or $A3$ "hidden"

(a) $A1$ if $C^n + x - C^m < C^n N \leq C^n$

(b) $A2$ if $C^n - x - C^m \leq C^n N \leq C^n + x - C^m$

(c) $A3$ if $0 \leq C^n N \leq C^n - x - C^m$

$$\begin{aligned}
E\left[\pi|C^m\right] &= -C^m E\left[k|C^m\right] + \frac{1}{4b} E\left[\left(a + bC^m\right)^2\right] \\
E\left[\pi|C^m\right] &= -C^m \frac{1}{2} E\left[a\right] - C^m \int_{1+\frac{x-C^m}{C^n}}^{1} \frac{1}{2} b\left(C^m + C^n N\right) dN - C^m \int_{1-\frac{x+C^m}{C^n}}^{1+\frac{x-C^m}{C^n}} \frac{1}{2} bxdN + \frac{1}{4b} E\left[\left(a + bC^m\right)^2\right] \\
&= \frac{1}{4b} E\left[a^2\right] - \frac{1}{2} bC^m \left(\frac{1}{2} C^m - \frac{1}{2} \frac{x^2 - C^{m2}}{C^n} - x + 2\frac{x^2}{C^n}\right)
\end{aligned}$$

$$\begin{aligned}
\frac{dE\left[\pi|C^m\right]}{dC^m} &= -\frac{1}{2} b \left(C^m - x + \frac{3}{2}\frac{x^2}{C^n} + \frac{3}{2}\frac{C^{m2}}{C^n}\right) \\
\frac{d}{dC^m} E\left[\pi|x < C^m \leq -x + C^n\right] &< 0 \; \forall \; C^m > x
\end{aligned}$$

M4 $-x \leq C^m \leq x$ either $A2$ “caught up on-”, $A3$ “hidden”, or $A4$ “caught down on-”

(a) $A2$ if $C^n - x - C^m \leq C^n N \leq C^n$
(b) $A3$ if $x - C^m \leq C^n N \leq C^n - x - C^m$
(c) $A4$ if $0 \leq C^n N < x - C^m$

$$\begin{aligned}
E\left[\pi|C^m\right] &= -C^m \frac{1}{2} E\left[a\right] - C^m \int_{1-\frac{x+C^m}{C^n}}^{1} \frac{1}{2} bxdN + C^m \int_{0}^{\frac{x-C^m}{C^n}} \frac{1}{2} bxdN + \frac{1}{4b} E\left[\left(a + bC^m\right)^2\right] \\
&= \frac{1}{4b} E\left[a^2\right] + bC^{m2} \left(\frac{1}{4} - \frac{1}{C^n} x\right)
\end{aligned}$$

For $0 < x < \frac{1}{4} C^n$ – as in the proposed — $E\left[\pi|C^m\right]$ is maximized at $C^m = x$ and $C^m = -x$.

M5 $x - C^n \leq C^m < -x$ either $A3$ “hidden”, $A4$ “caught down on-”, or $A5$ “caught down off-”

(a) $A3$ if $x - C^m \leq C^n N \leq C^n$
(b) $A4$ if $-x - C^m \leq C^n N < x - C^m$
(c) $A5$ if $0 \leq C^n N < -x - C^m$

By analogy to $M3$, $\frac{d}{dC^m} E\left[\pi|x - C^n \leq C^m < -x\right] > 0$

M6 $-x - C^n \leq C^m < x - C^n$ either $A4$ “caught down on-” or $A5$ “caught down off-”

(a) $A4$ if $-x - C^m \leq C^n N < C^n$
(b) $A5$ if $0 \leq C^n N < 0 - x - C^m$

By analogy to $M2$, $E\left[\pi| - x - C^n \leq C^m < x - C^n\right] < E\left[\pi|0\right]$

M7 $C^m < -x - C^n$ always $A5$ “caught up off-”
By analogy to $M1$,$E\left[\pi|C^m < -x - C^n\right] < E\left[\pi|0\right]$.

Given the values of $E[\pi|C^m]$ given above, $E[\pi]$ is maximized for $C^m = x$ and $C^m = -x$ for $0 < x < \frac{1}{4}C^n$, so that

$$E[\pi|x] = E[\pi|-x] = \frac{1}{4b}E\left[a^2\right] + \frac{1}{4}bx^2 - \frac{b}{C^n}x^3 > E[\pi|0].$$

Therefore, the monopolist is indifferent between submitting $C^m = x$ and $C^m = -x$, and the market makers can rationally believe that the monopolist randomizes between these two values with equal probability. Given these beliefs, prices are set competitively and no market maker has an incentive to change k.

A.2 Proof of Proposition 2

Risk-averse agents maximize the following objective function:

$$\begin{aligned} C^{n*} &= \arg\max_C E\left[u\left(C\left(P^* - k\right) + c_0 + c_1 P^*\right)\right] \\ &= \arg\max_C E\left[u\left(C\left(\frac{1}{2}\left(a + bC^m\right) - k\right) + c_0 + c_1\frac{1}{2}\left(a + bC^m\right)\right)\right] \end{aligned}$$

where the risk averse agent takes as given the order flow submitted by the average risk-averse agent, C^n, and by the monopolist, C^{m*}. The first and second derivative of expected utility are given by

$$\begin{aligned} \frac{\partial Eu}{\partial C} &= E\left[\left(\frac{1}{2}\left(a + bC^m\right) - k\right)u'\left(C\left(\frac{1}{2}\left(a + bC^m\right) - k\right) + c_0 + c_1\frac{1}{2}\left(a + bC^m\right)\right)\right] \\ \frac{\partial^2 Eu}{\partial C^2} &= E\left[\left(\frac{1}{2}\left(a + bC^m\right) - k\right)^2 u''\left(C\left(\frac{1}{2}\left(a + bC^m\right) - k\right) + c_0 + c_1\frac{1}{2}\left(a + bC^m\right)\right)\right] < 0. \end{aligned}$$

Expected utility of risk averse agents is therefore a concave function in C.

$$\begin{aligned} &\frac{\partial Eu}{\partial C} \\ &= \left[\begin{array}{c} \frac{1}{2}\int E\left[\left(\frac{1}{2}\left(a+bx\right) - k\left(x + NC^n\right)\right)u'\left(C\left(\frac{1}{2}\left(a+bx\right) - k\left(x+NC^n\right)\right) + c_0 + c_1\frac{1}{2}\left(a+bx\right)\right)\right]2NdN \\ +\frac{1}{2}\int E\left[\left(\frac{1}{2}\left(a-bx\right) - k\left(-x + NC^n\right)\right)u'\left(C\left(\frac{1}{2}\left(a-bx\right) - k\left(-x+NC^n\right)\right) + c_0 + c_1\frac{1}{2}\left(a-bx\right)\right)\right]2NdN \end{array}\right] \\ &= \left[\begin{array}{c} \frac{1}{2}\left(1 - \left(1 - \frac{2x}{C^n}\right)^2\right)E\left[\left(\frac{1}{2}\left(a+bx\right) - \frac{1}{2}E[a] - \frac{1}{2}bx\right)u'\left(C\left(\frac{1}{2}\left(a+bx\right) - \frac{1}{2}E[a] - \frac{1}{2}bx\right) + c_0 + c_1\frac{1}{2}\left(a+bx\right)\right)\right] \\ +\frac{1}{2}\left(1 - \frac{2x}{C^n}\right)^2 E\left[\left(\frac{1}{2}\left(a+bx\right) - \frac{1}{2}E[a]\right)u'\left(C\left(\frac{1}{2}\left(a+bx\right) - \frac{1}{2}E[a]\right) + c_0 + c_1\frac{1}{2}\left(a+bx\right)\right)\right] \\ +\frac{1}{2}\left(1 - \left(\frac{2x}{C^n}\right)^2\right)E\left[\left(\frac{1}{2}\left(a-bx\right) - \frac{1}{2}E[a]\right)u'\left(C\left(\frac{1}{2}\left(a-bx\right) - \frac{1}{2}E[a]\right) + c_0 + c_1\frac{1}{2}\left(a-bx\right)\right)\right] \\ +\frac{1}{2}\left(\frac{2x}{C^n}\right)^2 E\left[\left(\frac{1}{2}\left(a-bx\right) - \frac{1}{2}E[a] + \frac{1}{2}bx\right)u'\left(C\left(\frac{1}{2}\left(a-bx\right) - \frac{1}{2}E[a] + \frac{1}{2}bx\right) + c_0 + c_1\frac{1}{2}\left(a-bx\right)\right)\right] \end{array}\right] \end{aligned}$$

If we set $x = \lambda C^n$ for $0 < \lambda < \frac{1}{4}$ and $C = C^n$, we can define the function $g(.)$ such that

$$\begin{aligned} g(C) &\equiv \frac{\partial Eu}{\partial C}|_{C^n = C} \\ &= \left[\begin{array}{c} \frac{1}{2}\left(1 - \left(1 - 2\lambda\right)^2\right)E\left[\left(\frac{1}{2}a - \frac{1}{2}E[a]\right)u'\left(C\left(\frac{1}{2}a - \frac{1}{2}E[a]\right) + c_0 + c_1\frac{1}{2}\left(a + b\lambda C\right)\right)\right] \\ +\frac{1}{2}\left(1 - 2\lambda\right)^2 E\left[\left(\frac{1}{2}\left(a + b\lambda C\right) - \frac{1}{2}E[a]\right)u'\left(C\left(\frac{1}{2}\left(a + b\lambda C\right) - \frac{1}{2}E[a]\right) + c_0 + c_1\frac{1}{2}\left(a + b\lambda C\right)\right)\right] \\ +\frac{1}{2}\left(1 - \left(2\lambda\right)^2\right)E\left[\left(\frac{1}{2}\left(a - b\lambda C\right) - \frac{1}{2}E[a]\right)u'\left(C\left(\frac{1}{2}\left(a - b\lambda C\right) - \frac{1}{2}E[a]\right) + c_0 + c_1\frac{1}{2}\left(a - b\lambda C\right)\right)\right] \\ +\frac{1}{2}\left(2\lambda\right)^2 E\left[\left(\frac{1}{2}a - \frac{1}{2}E[a]\right)u'\left(C\left(\frac{1}{2}a - \frac{1}{2}E[a]\right) + c_0 + c_1\frac{1}{2}\left(a - b\lambda C\right)\right)\right] \end{array}\right] \end{aligned}$$

First, we evaluate $g(0)$

$$g(0)=E\left[\left(\frac{1}{2}a-\frac{1}{2}E[a]\right)u'\left(c_0+c_1\frac{1}{2}a\right)\right]$$

Note that this expression is positive as $c_1<0$. As a result, when other risk-averse agents do not hedge, any given risk-averse agent can increase utility by hedging. Therefore, there is no in which no hedging occurs unless $c_1=0$, in which case the agents have no incentive to hedge. Next, we evaluate $g(-c_1)$. In this case, we get

$$\begin{aligned} g(-c_1) &= 2\lambda u'\left(c_1\frac{1}{2}E[a]+c_0\right)\frac{1}{2}b\lambda c_1(1-2\lambda) \\ &< 0 \text{ for } c_1<0. \end{aligned}$$

As expected, $g(-c_1)<0$.

This means that g switches signs between $C=0$ and $C=-c_1$. Furthermore, g is a smooth function. Therefore, there must exist a C^* between zero and $-c_1$ such that $g(C^*)=0$. As shown above, expected utility is a concave function in the amount of hedging, C, which implies that the first derivative of expected utility, g, is decreasing in C. The solution C^* to $g(C^*)=0$ is therefore unique. Note that this implies that in the proposed equilibria above there is some hedging by the risk averse agents but hedging is reduced relative to the case of no monopolist participation in the market.

A.3 Proof of Proposition 4

We first examine the optimal behavior given the beliefs about the manipulator's behavior.

A.3.1 Monopolist

We showed that at $t=2$ the monopolist maximizes his profits by setting price and quantity as $P^*=\frac{1}{2}(a+bC_1^m)$ and $Q^*=\frac{1}{2b}(a-bC_1^m)$. At $t=1$, the monopolist maximizes expected profits given the price schedule he faces and the beliefs he holds about the manipulator's trading behavior. His objective function at $t=1$ depends on the aggregate order flow $\theta_0=C_0^h+C_0^n$ at $t=0$.

In SG1, if $\theta_0>C^{n+}-x$, his expected profits are

$$E[\pi]=E\left[-C_1^m\left(\frac{1}{2}a+\frac{1}{2}b\left(C_1^m+C_1^h-x\right)\right)+\frac{1}{4b}\left(a+bC_1^m\right)^2\right].$$

The FOC is

$$\frac{\partial E[\pi]}{\partial C_1^m}=E\left[-\frac{1}{2}b\left(C_1^h-x\right)-\frac{1}{2}bC_1^m\right].$$

Note that given $\theta_0>C^{n+}-x$ and the beliefs about the manipulator's trade at $t=0$ we have $C_1^h=x$, which implies $C_1^m=0$. The SOC is

$$\frac{\partial^2 E[\pi]}{\partial C_1^{m2}}=-\frac{1}{2}b<0.$$

In SG2, if $C^{n-}+x\leq\theta_0\leq C^{n+}-x$, his expected profits are

$$E[\pi]=E\left[-C_1^m\left(\frac{1}{2}a+\frac{1}{4}b\left(C_1^m+C_1^h\right)\right)+\frac{1}{4b}\left(a+bC_1^m\right)^2\right]$$

The FOC is

$$\frac{\partial E[\pi]}{\partial C_1^m} = E\left[-\frac{1}{4} b C_1^h\right].$$

Note that the monopolist believes that the manipulator will randomize between $+\frac{1}{2}x$ and $-\frac{1}{2}x$ with equal probability, so that $E\left[C_1^h\right] = 0$, and $\frac{\partial E[\pi]}{\partial C_1^m} = 0$. Therefore, the monopolist is indifferent between submitting an order flow and therefore willing to submit $C_1^m \in \left\{-\frac{1}{2}x, \frac{1}{2}x\right\}$ with equal probability.[12]

In SG3, if $\theta_0 < C^{n-} + x$, his expected profits are

$$E[\pi] = E\left[-C_1^m\left(\frac{1}{2}a + \frac{1}{2}b\left(C_1^m + C_1^h + x\right)\right) + \frac{1}{4b}\left(a + bC_1^m\right)^2\right]$$

In this case, the FOC is

$$\frac{\partial E[\pi]}{\partial C_1^m} = E\left[-\frac{1}{2}b\left(C_1^h + x\right) - \frac{1}{2}bC_1^m\right].$$

Note that given $\theta_0 < C^{n-} + x$ and the beliefs about the manipulator's trade at $t = 0$ we have $C_1^h = -x$. This implies

$$\frac{\partial E[\pi]}{\partial C_1^m} = -\frac{1}{2}bC_1^m = 0$$

which implies $C_1^m = 0$. The SOC is

$$\frac{\partial^2 E[\pi]}{\partial C_1^{m2}} = -\frac{1}{2}b < 0.$$

We have thus shown that the monopolist has no incentive to deviate from the proposed given the price schedule and his beliefs about the manipulator's actions. Next, we examine the optimal behavior of the manipulator.

A.3.2 Manipulator

At $t = 1$ the manipulator submits an order flow to maximize his expected profits which depend on the aggregate order flow at $t = 0$. His expected profits are

$$E\left[\pi^h\right] = E\left[C_0^h\left(k_1 - k_0\right) + C_1^h\left(P - k_1\right)\right].$$

In SG1, if $\theta_0 > C^{n+} - x$, his expected profits are

$$\begin{aligned} E\left[\pi^h\right] &= E\left[C_0^h\left(k_1 - k_0\right) + C_1^h\left(P - k_1\right)\right] \\ &= E\left[\frac{1}{2}b\left(C_1^h - x\right)\left(C_0^h - C_1^h\right)\right] \end{aligned}$$

The FOC is

$$\frac{\partial E\left[\pi^h\right]}{\partial C_1^h} = E\left[\frac{1}{2}b\left(C_0^h - 2C_1^h\right) + \frac{1}{2}bW\right] = 0$$

and the SOC is satisfied. This implies $C_1^h = \frac{1}{2}\left(x + C_0^h\right)$. Note that if the manipulator does randomize between $+x$ and $-x$ at $t = 0$, then $\theta_0 > C^{n+} - x$ is only true if $C_0^h = x$. Thus $C_1^h = x$. His profits are then $E\left[\pi^h\right] = 0$.

[12] Here the monopolist is indifferent between participating and not participating in the futures market. This result is obtained because we make the assumptions that there is no noise trading at $t = 1$ and demand is linear. If we either allow for noise trading at $t = 1$ or a convex demand function the monopolist would have a strict incentive to strategically randomize at $t = 1$.

In SG2, if $C^{n-}+x \leq \theta_0 \leq C^{n+}-x$, his expected profits are

$$\begin{aligned} E\left[\pi^h\right] &= E\left[C_0^h\left(k_1-k_0\right)+C_1^h\left(P-k_1\right)\right] \\ &= E\left[\frac{1}{4}bC_0^h\left(C_1^m+C_1^h\right)+\frac{1}{4}bC_1^h\left(C_1^m-C_1^h\right)\right]. \end{aligned}$$

The FOC is

$$\frac{\partial E\left[\pi^h\right]}{\partial C_1^h}=\frac{1}{4}b\left(C_0^h+E\left[C_1^m\right]-2C_1^h\right)=0$$

and the SOC is satisfied. This implies $C_1^h=\frac{1}{2}\left(C_0^h+E\left[C_1^m\right]\right)$. Note that if the manipulator believes that the monopolist randomizes between $+\frac{1}{2}x$ and $-\frac{1}{2}x$ with equal probability at $t=0$, then $C_1^h=\frac{1}{2}C_0^h$. His profits are then

$$E\left[\pi^h\right]=\frac{1}{16}bC_0^{h2}>0$$

In SG3, if $\theta_0<C^{n-}+x$, his expected profits are

$$\begin{aligned} E\left[\pi^h\right] &= E\left[C_0^h\left(k_1-k_0\right)+C_1^h\left(P-k_1\right)\right] \\ &= E\left[\frac{1}{2}b\left(C_1^h+x\right)\left(C_0^h-C_1^h\right)\right]. \end{aligned}$$

The FOC is

$$\frac{\partial E\left[\pi^h\right]}{\partial C_1^h}=E\left[\frac{1}{2}b\left(C_0^h-2C_1^h\right)-\frac{1}{2}bW\right]=0$$

and the SOC is satisfied. This implies $C_1^h=\frac{1}{2}\left(-x+C_0^h\right)$. Again, if the manipulator randomizes between $+x$ and $-x$ at $t=0$, then $\theta_0<C^{n-}+x$ is only true if $C_0^h=-x$. Therefore, $C_1^h=-x$ and his profits are $E\left[\pi^h\right]=0$.

At $t=0$ the manipulator submits an order flow C_0^h to maximize his expected profits given optimal behavior in subsequent periods. His expected profits are given as

$$E\left[\pi^h\right]=E\left[C_0^h\left(k_1-k_0\right)+C_1^h\left(P-k_1\right)\right].$$

If $C_0^h\geq 0$ then

$$\begin{aligned} E\left[\pi^h\right] &= \left[\begin{array}{c} \frac{1}{C^{n+}-C^{n-}}\int_{C^{n+}-x-C_0^h}^{C^{n+}}\left(C_0^h\left(k_1-k_0\right)+C_1^h\left(P-k_1\right)\right)dC_0^n \\ +\frac{1}{C^{n+}-C^{n-}}\int_{C^{n-}}^{C^{n+}-x-C_0^h}\left(C_0^h\left(k_1-k_0\right)+C_1^h\left(P-k_1\right)\right)dC_0^n \end{array}\right] \\ &= \frac{1}{C^{n+}-C^{n-}}\frac{1}{16}bC_0^{h2}\left(C^{n+}-x-C_0^h-C^{n-}\right). \end{aligned}$$

The FOC is

$$\frac{\partial E\left[\pi^h\right]}{\partial C_0^h}=\frac{1}{C^{n+}-C^{n-}}\frac{1}{16}bC_0^h\left(2\left(C^{n+}-C^{n-}\right)-2x-3C_0^h\right).$$

If the FOC is satisfied, then $C_0^h=\frac{2}{3}\left(C^{n+}-C^{n-}-x\right)$. Recall the liquidity constraint, $\left|C_0^h\right|\leq W<\frac{1}{2}\left(C^{n+}-C^{n-}\right)$. In an interior , $x=C_0^h$, so that $C_0^h=\frac{2}{5}\left(C^{n+}-C^{n-}\right)$. The SOC is satisfied in this case. Taking the liquidity constraint into account, it is optimal to set $C_0^h=\min\left(W,\frac{2}{5}\left(C^{n+}-C^{n-}\right)\right)$.

If $C_0^h < 0$ then

$$\begin{aligned} E\left[\pi^h\right] &= \left[\begin{array}{c} \frac{1}{C^{n+}-C^{n-}} \int_{C^{n-}+x-C_0^h}^{C^{n+}} \left(C_0^h \left(k_1-k_0\right)+C_1^h \left(P-k_1\right)\right) dC_0^n \\ +\frac{1}{C^{n+}-C^{n-}} \int_{C^{n-}}^{C^{n-}+x-C_0^h} \left(C_0^h \left(k_1-k_0\right)+C_1^h \left(P-k_1\right)\right) dC_0^n \end{array} \right] \\ &= \frac{1}{C^{n+}-C^{n-}} \frac{1}{16} b C_0^{h2} \left(C^{n+}-C^{n-}-x+C_0^h\right). \end{aligned}$$

The FOC is

$$\frac{\partial E\left[\pi^h\right]}{\partial C_0^h} = \frac{1}{C^{n+}-C^{n-}} \frac{1}{16} b C_0^h \left(2\left(C^{n+}-C^{n-}\right)-2x+3C_0^h\right).$$

By logic parallel to the case where $C_0^h \geq 0$, it is optimal to set $C_0^h = -\min\left(W, \frac{2}{5}\left(C^{n+}-C^{n-}\right)\right)$.

For an interior solution, expected profits are

$$E\left[\pi^h | C_0^h = \pm\frac{2}{5}\left(C^{n+}-C^{n-}\right)\right] = \frac{1}{100} b \left(C^{n+}-C^{n-}\right)^2 > 0;$$

for a corner solution they are

$$E\left[\pi^h | C_0^h = \pm W\right] = \frac{1}{16} b W^2 > 0.$$

Note that the expected profits are the same if the manipulator submits $+x$ or $-x$, so he will be willing to randomize with equal probability.

A.3.3 Market Makers

Market makers beliefs are consistent with the actions of the noise traders, the monopolist, and the manipulator. Since prices are set competitively, no market maker has an incentive to change k.

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