



---

No. 2008/25

**Central Bank Misperceptions and  
the Role of Money in Interest Rate Rules**

Günter W. Beck and Volker Wieland

---





## Center for Financial Studies

The *Center for Financial Studies* is a nonprofit research organization, supported by an association of more than 120 banks, insurance companies, industrial corporations and public institutions. Established in 1968 and closely affiliated with the University of Frankfurt, it provides a strong link between the financial community and academia.

The CFS Working Paper Series presents the result of scientific research on selected topics in the field of money, banking and finance. The authors were either participants in the Center's Research Fellow Program or members of one of the Center's Research Projects.

If you would like to know more about the *Center for Financial Studies*, please let us know of your interest.

Prof. Dr. Jan Pieter Krahen

Prof. Volker Wieland, Ph.D.



CFS Working Paper No. 2008/25

## Central Bank Misperceptions and the Role of Money in Interest Rate Rules\*

Günter W. Beck<sup>1</sup> and Volker Wieland<sup>2</sup>

August 2008

### Abstract:

Research with Keynesian-style models has emphasized the importance of the output gap for policies aimed at controlling inflation while declaring monetary aggregates largely irrelevant. Critics, however, have argued that these models need to be modified to account for observed money growth and inflation trends, and that monetary trends may serve as a useful cross-check for monetary policy. We identify an important source of monetary trends in form of persistent central bank misperceptions regarding potential output. Simulations with historical output gap estimates indicate that such misperceptions may induce persistent errors in monetary policy and sustained trends in money growth and inflation. If interest rate prescriptions derived from Keynesian-style models are augmented with a cross-check against money-based estimates of trend inflation, inflation control is improved substantially.

**JEL Classification:** E32, E41, E43, E52, E58

**Keywords:** Taylor Rules, Money, Quantity Theory, Output Gap Uncertainty, Monetary Policy Under Uncertainty

\* We thank Athanasios Orphanides for providing his data on historical U.S. output gap revisions and Christina Gerberding for the Bundesbank's real-time data set with output gap revisions. Furthermore, Wieland thanks the European Central Bank that he visited as Wim Duisenberg Research Fellow and the Stanford Center for International Development for the hospitality extended while part of this research was accomplished. An earlier version of this paper was presented at the conference on "John Taylor's Contributions to Monetary Theory and Policy" at the Federal Reserve Bank of Dallas, October 2007. We have also benefited from seminar presentations at U.C. Santa Cruz, U.C. Davis, FU Berlin, Birkbeck College, the European Central Bank, the Kiel Institute as well conferences at the Bundesbank, the University of Cambridge and the Center for Financial Studies. We would also like to thank Ignazio Angeloni, Joshua Aizenman, John Driffill, Tim Cogley, Alex Cukierman, Mark Gertler, Stefan Gerlach, Robert Hall, Otmar Issing, Robert Lucas, Ronald McKinnon, John B. Taylor, Peter Tinsley, Carl Walsh, John C. Williams and Michael Woodford for useful comments. The usual disclaimer applies.

1 Goethe University Frankfurt and CFS, E-mail: gbeck@wiwi.uni-frankfurt.de

2 Goethe University Frankfurt, CEPR, CFS and SCID

Corresponding author. Contact: Professur für Geldtheorie und -politik, Johann-Wolfgang-Goethe Universität, Mertonstrasse 17, D-60325 Frankfurt am Main, Germany, Tel.: +49 69 798-33810, Fax: +49 69 798-33907, E-mail: wieland@wiwi.uni-frankfurt.de, vwieland@stanford.edu, Homepage: <http://www.volkerwieland.com>.

# 1 Introduction

John Taylor's research on monetary policy rules changed the economics profession's focus from monetary aggregates to the interest rate as the appropriate instrument for monetary policy.<sup>1</sup> Even the late Milton Friedman, in his last published writing, studied Taylor's rule for interest rate policy, though he tried to reclaim a role for money on its right-hand side.<sup>2</sup> Recent theoretical advances in New-Keynesian macroeconomics building on microeconomic foundations with monopolistic competition and price rigidity have further de-emphasized the role of money in monetary policy. As shown by Kerr and King (1996), Svensson (1997) and Clarida et al. (1999) optimal interest rate policy in models with price rigidities is conducted with reference to inflation forecasts and output gaps but without direct concern for monetary aggregates—not unlike Taylor's rule.<sup>3</sup> Some macroeconomists, however, have expressed concern about the disappearance of money from monetary theory and policy. Lucas (2007), for example, writes:

*“New-Keynesian models define monetary policy in terms of a choice of money market rate and so make direct contact with central banking practice. Money supply measures play no role in the estimation, testing or policy simulation of these models. A role for money in the long run is sometimes verbally acknowledged, but the models themselves are formulated in terms of deviations from trends that are themselves determined somewhere off stage.”*

---

<sup>1</sup>Taylor (2006) writes on his progression from money to interest rates: “Taylor (1979) showed that a fixed money growth rule - a Friedman rule - would have led to better performance than actual policy in the post World War II period ... (but) a money growth rule which responded to economic developments could do even better. Since then I have found that policy rules in terms of interest rates have worked better as practical guidelines for central banks.”

<sup>2</sup>Friedman (2006) notes at first that he always preferred a monetary aggregate for a policy instrument but then takes the perspective of Taylor's rule with the federal funds rate as instrument: “The Taylor rule is an attempt to specify the federal funds rate that will come closest to achieving the theoretically appropriate rate of monetary growth to achieve a constant price level or a constant rate of inflation. Suppose the federal funds target rate is equal to a Taylor rule that gives 100 percent weight to inflation deviations. That may not be the right rate to achieve the desired inflation target because other variables such as output or monetary growth are not at their equilibrium levels. On this view, additional terms in the Taylor rule would reflect variables relevant to choosing the right target funds rate to achieve the desired inflation target.”

<sup>3</sup>The New-Keynesian model as laid out by Rotemberg and Woodford (1997) and Goodfriend and King (1997) and developed in detail in Woodford (2003) and Walsh (2003) has quickly become the principal workhorse model in monetary economics. The case against money is perhaps made most vigorously by Woodford (2006).

*It seems likely that these models could be reformulated to give a unified account of trends, including trends in monetary aggregates, and deviations about trend but so far they have not been. This remains an unresolved issue on the frontier of macroeconomic theory. Until it is resolved, monetary information should continue to be used as a kind of add-on or cross-check.”*

We address Lucas’s request for a unified account of trends and deviations, including monetary aggregates, and provide a formal analysis of his proposal to use monetary information as a cross-check for policy. The central bank’s beliefs regarding trends and deviations play a central role in the analysis, specifically its estimates of the economy’s potential output and the implied output gap that drives inflation forecasts in Keynesian-style models.

Research on optimal monetary policy design under uncertainty usually has to rely on a-priori modeling assumptions regarding unobservable variables such as potential output (cf. Svensson and Woodford (2003) and Wieland (2006)). These assumptions are needed to determine the optimal, model-based estimates of potential output, on which policy is then conditioned. Orphanides (2003) has provided an alternative approach for evaluating policies under uncertainty that avoids these particular a-priori assumptions by using instead historical, real-time estimates of potential output. The true value of potential output at any point in time is assumed to be equal to the central bank’s final estimate on the basis of information available many years later. We use historical series of central banks’ output gap estimates for the United States and Germany from Orphanides (2003) and Gerberding et al. (2005) respectively. Both series indicate very persistent misperceptions regarding potential output.

Model simulations indicate that historical output gap misperceptions induce an inflationary bias in interest rate policies that the central bank considered optimal conditional on its model and associated forecasts. As a result, the central bank induces trends in money growth and inflation even though it pursues a constant inflation target. Thus, as requested by Lucas, Keynesian-style models built to explain inflation deviations from trend are able to provide an account of money growth and inflation trends. This finding complements recent empirical studies that have identified proportional movements in money growth and inflation at low frequencies

using a variety of filters<sup>4</sup> and provides a structural explanation.

Next, a general definition of a policy with cross-checking that formalizes Lucas (2007) proposal is presented. The cross-check is characterized by a first-order condition that incorporates expected trend inflation, which is estimated from a simple monetary model. The cross-check is triggered in a nonlinear-fashion whenever a statistical test on the basis of the monetary model signals a trend shift. An earlier note, Beck and Wieland (2007), presented an interest rate rule that incorporates such a shift<sup>5</sup> and simulated a counterfactual example in the traditional Keynesian-style model with backward-looking dynamics of Svensson (1997), Orphanides and Wieland (2000) and Orphanides (2003). The present paper shows how to derive an interest rate rule with cross-checking from an optimization problem and proceeds to implement cross-checking in the benchmark New-Keynesian model.<sup>6</sup>

The advantage of the Keynesian model with backward-looking dynamics is that it fits the historical persistence in output and inflation and arguably embodies central bankers' beliefs on policy tradeoffs and monetary policy transmission in the 1970s and 1980s quite well. It may be the better candidate for modeling central bank perceptions and describing historical outcomes and was used for this purpose by Orphanides (2003). While the New-Keynesian model is an unlikely description of central bank perceptions in the 1970s and 1980s, it has the advantage of microeconomic foundations in optimal decision-making of households and firms. Thus, it accounts for forward-looking, optimizing decision-making by market participants and constitutes an important testing ground for policy strategies currently recommended to central banks. For this reason, the subsequent analysis is carried out in both models in parallel.

The policy with cross-checking against money-based estimates of trend inflation is found to substantially improve inflation control in the event of persistent policy mistakes due to historical output gap misperceptions. Furthermore, monetary cross-checking remains effective in

---

<sup>4</sup>See Gerlach (2004), Benati (2005), Pill and Rautananen (2006) and Assenmacher-Wesche and Gerlach (2007).

<sup>5</sup>Beck and Wieland (2007) point out that such an interest rate rule captures key elements of the ECB's description of its two-pillar policy strategy. However, the ECB has never published a formal, mathematical exposition of its strategy.

<sup>6</sup>Our definition of monetary cross-checking is different from another interesting strategy proposed by Christiano and Rostagno (2001) and Christiano et al. (2006) that combines monetary targeting with Taylor-style interest rate rules.

the event of sustained velocity shifts—the Achilles heel of traditional monetary targeting—if standard recursive money demand estimation is applied. The nonlinear nature of interest-rate adjustments due to cross-checking turns out to be essential. Linear policies with money-based estimates of trend inflation perform substantially worse than cross-checking, whether central bank estimates of the output gap are correct, on average, or not. Finally, cross-checking can also be implemented successfully using inflation-based estimates of trend inflation but money-based estimates would dominate if money leads inflation as indicated by recent empirical studies.

The remainder of this paper is structured as follows. Section 2 presents the optimal interest rate policy under uncertainty and explains how we introduce historical central bank misperceptions into the analysis. Section 3 describes the relationship between trend money growth and trend inflation and shows that central bank misperceptions represent an important source of such trends in Keynesian-style models. Section 4 introduces the general definition of cross-checking. Section 5 applies cross-checking in the event of central bank misperceptions. Section 6 subjects the policy with cross-checking to further sensitivity analysis and section 7 concludes. An appendix provides further details on the models and their solution under cross-checking.<sup>7</sup>

## 2 Output gap misperceptions and optimal policy

Keynesian-style models of inflation determination assign a central role to the output gap, that is the difference between actual output and the economy’s potential. For example, the model used by Svensson (1997), Orphanides and Wieland (2000) and Orphanides (2003) to study monetary policy incorporates an accelerationist Phillips curve that relates current inflation,  $\pi_t$ , to the gap between current and potential output (in logs),  $y_t - z_t$ , lagged inflation,  $\pi_{t-1}$ , and a cost-push shock,  $u_t$ :

$$\pi_t = \lambda(y_t - z_t) + \pi_{t-1} + u_t \tag{1}$$

The slope parameter  $\lambda$  determines the trade-off between output and inflation.

Similarly, the New-Keynesian model of Rotemberg and Woodford (1997) and Goodfriend

---

<sup>7</sup>Software for replicating the quantitative analysis in this paper is available from the authors upon request.

and King (1997) that was used by Clarida et al. (1999) to study the design of monetary policy assigns center stage to the output gap in determining deviations from steady-state inflation,  $\bar{\pi}$ , next to expected future inflation,  $\pi_{t+1}^e$ , and cost-push shocks:

$$\pi_t - \bar{\pi} = \lambda(y_t - z_t) + \beta(\pi_{t+1}^e - \bar{\pi}) + u_t, \quad \text{where } \pi_{t+1}^e = E_t[\pi_{t+1}] \quad (2)$$

Since this Phillips curve is derived from microeconomic foundations, the parameters have a clear economic interpretation.  $\beta$  refers to the discount factor of optimizing households and firms.  $\lambda$  is a function of the probability that firms are allowed to adjust prices according to Calvo (1983). Furthermore, expectations regarding future inflation are formed in a rational, forward-looking manner. Potential output,  $z_t$ , which is an unobservable and model-dependent variable, corresponds to the level of output that would be realized if prices were completely flexible.<sup>8</sup> In the following, the Keynesian-style model associated with equation (1) is referred to as the *K-Model* and the New-Keynesian model associated with (2) as the *NK-Model*.

## 2.1 Optimal interest rate policy under uncertainty

Optimal policy in the above-mentioned models prescribes that the central bank conditions its policy decisions on its best estimate of potential output. This recommendation applies even if the central bank's objective focuses exclusively on stabilizing inflation without any explicit concern for output fluctuations. The objective function of such a strictly inflation targeting central bank is given by:<sup>9</sup>

$$-\frac{1}{2}E_t \left\{ \sum_{i=0}^{\infty} \beta^i [(\pi_{t+i} - \pi^*)^2] \right\}. \quad (3)$$

$\pi^*$  denotes the central bank's inflation target. In the following, it is normalized at zero along with steady-state inflation,  $\bar{\pi}$ . The rational expectation  $E_t[\cdot]$  of the objective function is conditional

---

<sup>8</sup>Traditional Keynesian models have typically related the measure of potential in the accelerationist Phillips curve, equation (1), more loosely to the output implied by a standard model of long-run growth.

<sup>9</sup>Our analysis can be extended to an objective function that includes output deviations from potential and has welfare-theoretic foundations in the New-Keynesian model. Here, we focus on strict inflation targeting to emphasize that our findings regarding the consequences of output gap misperceptions do not rely on including the gap in the objective function. Thus, we are more likely to understate than overstate their negative implications.



on the particular model of inflation determination preferred by the central bank.

The optimal monetary policy that maximizes the above objective must satisfy the following first-order condition:<sup>10</sup>

$$E[\pi_{t+i}|t, K/NK] = \pi^* = 0 \quad \forall i = \{0, 1, 2, \dots, \infty\}. \quad (4)$$

The output level that would achieve this optimum at time  $t$  is given by

$$\text{K-Model: } y_t = z_t - \lambda^{-1}(\pi_{t-1} + u_t) \quad (5)$$

$$\text{NK-Model: } y_t = z_t - \lambda^{-1}u_t \quad (6)$$

Thus, the central bank aims for an output level above (or below) potential to the extent necessary to offset inflationary pressures from cost-push shocks and—in the K-Model—inherited inflation.

In practice, however, the central bank cannot observe potential output or particular shocks with any certainty and needs to rely on estimates. We use the superscript  $e$  to refer to the central bank's estimates or perceptions of such unobservable variables. Thus,  $z_{t|t}^e$  refers to the central bank's estimate of potential output in period  $t$  given the information available at that point in time and  $u_{t|t}^e$  to the central bank's estimate of the cost-push shock. We assume that these perceptions represent the best available estimates of the unobservable variables from the perspective of the central bank. Similarly, we use the superscript  $e$  as a short-hand for the rational expectations of output and inflation. For example,  $\pi_{t|t}^e = E[\pi_t|t]$  represents the central bank's best forecast of inflation at the point in period  $t$  when it decides on its policy, i.e. before it can observe the joint consequences of potential output, the cost-push shock and its policy choice on inflation.

Fortunately, the optimal policy under uncertainty can be determined quite easily if the fol-

---

<sup>10</sup>See Svensson (1997) for the K-Model and Clarida et al. (1999) for the NK-Model. In the NK model the question arises whether to consider the optimal policy under discretion or commitment. Note, however, that for strict inflation targeting the optimal policies under discretion and commitment are identical. If output were to be included in the loss function we would analyze optimal policy under discretion.

lowing conditions are fulfilled: the model is linear, the parameters are known and uncertainty is additive. In this case, certainty-equivalence applies, i.e. the optimal policy must satisfy the first-order condition, equation (4), in expectation (see, for example, Svensson and Woodford (2003) for the NK-Model and Wieland (2006) for the K-Model).<sup>11</sup> Then, the expected optimal output level corresponds to:

$$\text{K-Model: } y_{t|t}^e = z_{t|t}^e - \lambda^{-1}(\pi_{t-1} + u_{t|t}^e) \quad (7)$$

$$\text{NK-Model: } y_{t|t}^e = z_{t|t}^e - \lambda^{-1}u_{t|t}^e \quad (8)$$

The conditions for certainty-equivalence—linearity, known parameters and additive uncertainty—require making important *a-priori assumptions* regarding the processes that determine unobservable variables. Svensson and Woodford (2003), for example, assume that potential output,  $z_t$ , in the NK-model follows an auto-regressive process,

$$z_t = \nu z_{t-1} + \varepsilon_t^z, \quad (9)$$

with known persistence parameter,  $\nu$ , and known variance,  $\sigma_{\varepsilon_z}$ .<sup>12</sup> Wieland (2006) makes a similar assumption regarding the natural rate in a version of the K-Model. Under these assumptions the central bank can solve the estimation problem separately from the optimal policy problem. Svensson and Woodford (2003) and Wieland (2006) show how to derive the optimal estimate of potential output,  $z_{t|t}^e$ , using the Kalman filter. Conditional on this estimate the optimal policy implies setting the nominal interest rate,  $i_t$ , so as to achieve the expected output level defined by equations (7) or (8), respectively. This value of the interest rate may be inferred from the IS

---

<sup>11</sup>Certainty-equivalence fails if multiplicative parameters such as  $\lambda$  are unknown. Then, the central bank faces a complex control and estimation problem. Examples are studied by Wieland (2000), Beck and Wieland (2002) and Wieland (2006).

<sup>12</sup>We are referring to equation (43) in Svensson and Woodford (2003). In addition, the authors assume the central bank observes a signal regarding potential output that is correct up to an i.i.d. normal noise term.

equations:

$$\text{K-Model: } y_t = y_{t-1} - \varphi(i_t - \pi_{t-1}) + g_t \quad (10)$$

$$\text{NK-Model: } y_t = y_{t+1}^e - \varphi(i_t - \pi_{t+1}^e) + g_t \quad (11)$$

Thus, the optimal interest rate setting is given by:

$$\text{K-Model: } i_t = \pi_{t-1} + (\varphi\lambda)^{-1}(\pi_{t-1} + u_{t|t}^e) + (\varphi)^{-1}(y_{t-1} - z_{t|t}^e + g_{t|t}^e) \quad (12)$$

$$\text{NK-Model: } i_t = (\varphi\lambda)^{-1}(u_{t|t}^e) + (\varphi)^{-1}(z_{t+1|t}^e - z_{t|t}^e + g_{t|t}^e) \quad (13)$$

This characterization of optimal interest rate policies can be simplified further by exploiting modeling assumptions regarding the economic shocks. In particular, we assume that  $(g_t, u_t)$  are i.i.d. normal with zero-mean and known variances  $(\sigma_g, \sigma_u)$ . With regard to the K-Model we follow common practice (cf. Svensson (1997), Orphanides and Wieland (2000)) and assume that the central bank has no information on period  $t$  shocks when setting  $i_t$ , i.e.  $g_{t|t,K}^e = 0$  and  $u_{t|t,K}^e = 0$ . With regard to the NK-Model we follow Clarida et al. (1999) and assume that the central bank has some information on current shocks. Specifically, we assume that the central bank receives a signal  $(g_t^e, u_t^e)$  that is correct up to an additive noise term  $(\varepsilon_t^g, \varepsilon_t^u)$  with zero mean. Thus,  $g_{t|t,NK}^e = g_t^e$ , and  $u_{t|t,NK}^e = u_t^e$ .<sup>13</sup>

Consequently, the optimal interest rate policies correspond to:

$$\text{K-Model: } i_t^K = \pi_{t-1} + (\varphi\lambda)^{-1}(\pi_{t-1}) + (\varphi)^{-1}(y_{t-1} - z_{t|t}^e) \quad (14)$$

$$\text{NK-Model: } i_t^{NK} = (\varphi\lambda)^{-1}(u_t^e) + (\varphi)^{-1}(z_{t+1|t}^e - z_{t|t}^e + g_t^e) \quad (15)$$

The optimal policy in the K-Model is a version of the famous Taylor rule, yet its coefficients on inflation and the output gap need not coincide with the values of 0.5 that Taylor (1993) used to match federal funds rate choices by the FOMC from 1988 to 1993. As to the optimal

---

<sup>13</sup>More specifically, we assume that the true value of the demand and cost-push shocks, denoted by  $g_t$  and  $u_t$  respectively, are given by  $g_t = g_t^e + \varepsilon_t^g$  where  $\varepsilon_t^g \sim \text{i.i.d. } N(0, \sigma_{\varepsilon_g})$  and  $u_t = u_t^e + \varepsilon_t^u$  where  $\varepsilon_t^u \sim \text{i.i.d. } N(0, \sigma_{\varepsilon_u})$ . Shocks to money demand are modeled in a similar fashion.

policy under the NK model, Clarida et al. (1999) already pointed out that it can be interpreted as a forward-looking Taylor rule that responds to expected inflation and a measure of aggregate demand disturbance.

It is important to note that we have followed Svensson and Woodford (2003) in adopting the assumption of symmetric information in the NK-Model. Thus, the central bank, price-setting firms and households share the same information regarding potential output and economic shocks and form identical expectations regarding future output and inflation.<sup>14</sup>

## 2.2 The irrelevance of monetary aggregates

So far, we have not discussed monetary aggregates because they are not needed to characterize the transmission of interest rate changes to inflation in Keynesian-style models. In these models changes in the nominal interest rate influence the real interest rate due to the presence of price rigidities; the real interest rate determines the level of output; and the gap between actual and potential output drives inflation. Of course, the models may be extended to include a standard money demand equation<sup>15</sup> such as:

$$m_t - p_t = \gamma_y y_t - \gamma_i i_t + s_t. \quad (16)$$

where  $\gamma_y$  denotes the income elasticity of money demand,  $\gamma_i$  the semi-interest rate elasticity and  $s_t$  an i.i.d. normal money demand shock. While the central bank controls interest rates via open-market operations that also affect the money supply, the equilibrium level of money balances is determined recursively from the money demand equation. For this reason, money does not

---

<sup>14</sup>Svensson and Woodford (2004) also provide certainty-equivalence results under the assumption of asymmetric information. In this case, which arises under a consistent application of the representative agent assumption, households and firms know the true value of potential output. In our view, however, the assumption of symmetric information is more appropriate for the policy problem at hand. In practice, individual private agents are unlikely to know more about aggregate potential output than the central bank. If some individuals are particularly good at estimating aggregate potential output, central banks will be eager to hire them or to buy their inflation forecasts. One might even argue that it is more realistic to assume that the private sector is less knowledgeable about macroeconomic aggregates. Interestingly, however, the Bundesbank's potential output estimates that we use later on were made public in the 1970s and 1980s consistent with our assumption of symmetry (cf. Bundesbank (1973, 1981)).

<sup>15</sup>This specification can be derived from the optimization problem of a household that values money holdings according to a utility function that is separable in real balances and consumption goods (see Walsh (2003)).

appear in the optimal interest rate policies defined by equations (14) and (15). The quantity of money adjusts so as to achieve the interest rate prescribed by the optimal policy. Technically,  $m_t$  is determined by equation (16) conditional on the desired interest rate and the current values of real income and the price level.

What about the information value of monetary aggregates? Since we treat actual output and inflation as observable variables, monetary aggregates have no additional information value. The estimate of potential output,  $z_t|t$ , discussed by Svensson and Woodford (2003) and Wieland (2006) is obtained by means of the Kalman filter from past observations of output and inflation. Monetary aggregates provide no relevant information as long as money balances do not appear directly in the Phillips curve or the IS equation.<sup>16</sup> In practice, initial values of GDP, the GDP deflator and monetary aggregates are revised for a few quarters. While GDP is only available on a quarterly basis, monetary aggregates are available on a monthly frequency and tend to be revised less. Thus, monetary aggregates may provide information that helps improve initial estimates of actual output. This information role of monetary aggregates is investigated by Coenen et al. (2005). They show that initial GDP estimates for the euro area are revised more substantially than monetary aggregates. Using an estimated model of the euro area with rational expectations they find that optimal estimates of current GDP assign some weight to monetary aggregates, but this weight is very small.<sup>17</sup>

### **2.3 Evaluating policy performance with historical central bank misperceptions**

We have already pointed out that the optimal policy depends importantly on the central bank's estimate of potential output,  $z_t|t$ . A possible route for further analysis would be to follow Svensson and Woodford (2003) and Wieland (2006) in studying policy performance using cal-

---

<sup>16</sup>Ireland (2004) and Andres et al. (2006) investigate the direct role of money balances in output and inflation determination. They suggest that such direct effects are of minor importance.

<sup>17</sup>Coenen et al. (2005) assume that the central bank and the private sector have symmetric information and apply the same filtering techniques as in Svensson and Woodford (2003). An interesting paper by Dotsey and Hornstein (2003) investigates this question in a calibrated model of the U.S. economy under the assumption of asymmetric information as in Svensson and Woodford (2004). Their findings regarding the information value of money are even more negative.

culations that make use of *a-priori* assumptions concerning the unobservable process determining potential output, i.e. equation (9). Instead, we choose a different research strategy following the influential study of Orphanides (2003). Orphanides used historical output gap estimates to argue that the Taylor rule would not have been able to prevent the “Great Inflation” of the 1970s.

Orphanides (2003) collected real-time data on U.S. inflation and output including real-time estimates of potential output obtained by the Council of Economic Advisers (1966-1980) and the Federal Reserve (1980-1994).<sup>18</sup> On this basis, we denote the difference between the real-time estimate of potential output and the final estimate as of 1994 as  $e_t$ :

$$e_t = z_{t|t}^e - z_{t|1994}^e \quad (17)$$

$e_t$  provides a lower-bound on the extent of the central bank’s misperception of potential output since estimates may still have been revised after 1994. Thus, Orphanides (2003) proposed to analyze policy treating the final estimate in 1994,  $z_{t|1994}^e$ , as the true value of potential output,  $z_t$ :

$$z_{t|t}^e = z_t + e_t \quad (18)$$

The resulting series of real-time U.S. output gap misperceptions,  $E_t[y_t - z_t] - (y_t - z_t)$ , is shown by the solid line in Figure 1.

Critics have argued that the potential GDP measures constructed by the CEA were politicized maximum measures not taken seriously by Federal Reserve decision makers. Therefore, we contrast the U.S. CEA-FRB output gap misperceptions provided by Orphanides (2003) with a similar series from Gerberding et al. (2005) for Germany from 1974 to 1999. This series is shown by the dashed line in Figure 1. In this case, the underlying production potentials are the Bundesbank staff’s estimates.<sup>19</sup> The Bundesbank started to produce its own estimates of poten-

---

<sup>18</sup>The output gap data for the 1980s and 1990s of Orphanides (2003) was constructed from the Greenbook, the Federal Reserve document summarizing the Board staff’s analysis of economic developments distributed to the FOMC members a few days before each FOMC meeting. For the 1960s and 1970s Orphanides could not recover a complete time series for potential output estimates from Federal Reserve sources but notes that discussion of output gap measures appeared in the FOMC Memorandum of Discussion throughout this period. Thus, he uses real-time estimates of potential output that were produced by the Council of Economic Advisers (CEA) in those years and available at FOMC meetings.

<sup>19</sup>The data were reconstructed from official Bundesbank publications and from internal documents such as the

tial output in the early 1970s. Its methods are described in detail in Bundesbank (1973). The Bundesbank made clear that it aimed to construct a measure consistent with price stability—not a maximum measure.

Both series of historical output gap revisions indicate very persistent misperceptions by the respective central bank. This persistence arises primarily from the estimates of unobservable potential output, since revisions to actual output decline much more rapidly than those to the output gap. Both series indicate that the production potential of these economies was overestimated through the 1970s and well into the 1980s. U.S. and German policy makers taking into account these estimates were led to believe that their respective economies suffered a very deep recession from 1974 to 1976. In retrospect, however, this period appears as a mild recession in the United States and as a decline from excessive levels towards potential in Germany.

To the extent the Federal Reserve or the Bundesbank based their inflation forecasts on the output gap estimates available at the time, they must have concluded that inflation would soon decline. In retrospect, such a forecast would have been wrong. Using an estimated variant of the K-Model for the United States Orphanides (2003) showed that if interest rates had been set according to Taylor’s rule with historical output gap estimates, inflation would have risen dramatically. Thus, he concluded that Taylor’s rule would not have helped the FOMC avoid the “Great Inflation” of the 1970s, as long as it believed the output gap estimates.

### **3 Money and inflation trends due to historical output gap misperceptions**

In the following, Orphanides (2003)’ findings regarding the effect of historical central bank misperceptions on inflation under Taylor’s rule are shown to extend to the optimal interest rate policies in the K- and NK-Model. Furthermore, it is shown that central bank misperceptions constitute an important source of common trends in money growth and inflation similar to the low-frequency co-movements identified by recent empirical studies. Thus, Keynesian-style models with central bank misperceptions can provide a unified account of short-run deviations

---

briefing material for the Council’s discussions on the monetary target for the year to come.

from trend as well as long-run movements in money growth and inflation as requested by Lucas (2007).

In a first step, the long-run equilibrium values of money growth and inflation are derived. To this end, the money demand equation (16) is re-arranged and first-differences are taken to obtain a short-run relationship between money growth and inflation:

$$\pi_t = \Delta p_t = \Delta m_t - \gamma_y \Delta y_t + \gamma_i \Delta i_t - \Delta s_t. \quad (19)$$

Here,  $\Delta$  denotes the first-difference operator. In the long run, money demand shocks will average to zero; the nominal interest rate will converge to its constant steady-state level and its first-difference to zero;<sup>20</sup> and output growth will converge to the steady-state growth rate of potential,  $\Delta \bar{y} = \Delta \bar{z}$ . Thus, long-run inflation is proportional to long-run money growth adjusted for trend output growth and trend velocity<sup>21</sup> :

$$\bar{\pi} = \Delta \bar{m} - \gamma_y \Delta \bar{y} = \bar{\mu}. \quad (20)$$

Recent empirical studies confirm the long-run proportional relationship between inflation and money growth (cf. Gerlach (2004), Benati (2005), Pill and Rautanen (2006) and Assenmacher-Wesche and Gerlach (2007)). These studies use various types of filters. Gerlach (2004), for example, defines filtered money growth as

$$\Delta m_t^f = \Delta m_{t-1}^f + \omega \left( \Delta m_t - \Delta m_{t-1}^f \right). \quad (21)$$

Accordingly, we obtain a filtered measure of adjusted money growth from equation (20):

$$\mu_t^f = \Delta m_t^f - \gamma_y \Delta y_t^f. \quad (22)$$

---

<sup>20</sup>The steady state level of the nominal interest rate corresponds to the sum of the equilibrium real interest rate and the inflation target.

<sup>21</sup>Specifically, with velocity defined as  $v_t \equiv -m_t + p_t + y_t$  and money demand determined by equation (16) the long-run trend in velocity corresponds to  $\Delta \bar{v} = (1 - \gamma_y) \Delta \bar{y}$ .



Interestingly, the empirical studies cited above report that filtered measures of money growth tend to lead filtered inflation by several quarters. This property would render  $\mu_t^f$  a particularly useful forecast of impending movements in trend inflation. However, the timing assumptions of the Keynesian-style models with money demand presented in section 2, preclude such a leading indicator role of filtered money growth. This question is discussed further in section 6.

The next step is to introduce central bank misperceptions. Thus, the perceived potential output,  $z_{t|t}^e$ , in the optimal interest rate policies (equations (14) and (15)) is replaced with the historical, real-time estimates for the U.S.A. and Germany, respectively. Similarly, the true value,  $z_t$ , in the Phillips curves (equations (1) and (2)) is replaced with the final estimates (U.S.A.: 1994, Germany: 1999). The difference between real-time and final estimates constitutes the output gap misperception,  $e_t$ . Then, the models are simulated by drawing from the shock distributions and parameter values posited in Table 1. Thus, a-priori assumptions regarding the true structural process driving unobservable potential (cf. equation (9)) are avoided and policy performance is evaluated with data on historical misperceptions. It is straightforward to show that inflation will inherit the persistence properties of historical output gap misperceptions:

$$\text{K-Model: } \pi_t = \lambda e_t + \lambda g_t + u_t \quad (23)$$

$$\text{NK-Model: } \pi_t = \lambda e_t + \lambda \varepsilon_t^g + \varepsilon_t^u \quad (24)$$

Thus, actual inflation will persistently deviate from the zero inflation target even though the central bank aims to offset all forecasted deviations conditional on its preferred Keynesian-style model and associated gap estimate.

Figure 2 reports simulations with U.S. and German output gap misperceptions in the K-model for a given draw of exogenous shocks and noise terms.<sup>22</sup> The first row of two panels shows the rate of inflation,  $\pi_t$ , and the filtered measure of adjusted money growth,  $\mu_t^f$ , with U.S. output gap misperceptions. From period 15 onwards till period 135 the difference between the true and the perceived output gap corresponds to the difference between real-time and 1994

---

<sup>22</sup>The sequence of shocks is arbitrary but we obtain similar results for many alternative draws and will discuss averages later on in this section.

estimates from Orphanides (2003). The persistent over-estimate of potential output induces the central bank to set interest rates too low to maintain price stability. Thus, money growth and inflation increase and inherit the serial correlation of the central bank's misperceptions regarding potential output. Over time, also the filtered measure of adjusted money growth,  $\mu_t^f$ , increases as shown in the second panel of the first row. The lower two panels of Figure 2 report a simulation with the Bundesbank misperceptions from Gerberding et al (2005). Again, the misperceptions start in period 15. From then on, policy is too accommodative and (adjusted) money growth and inflation increase up to a peak of around 5 percent in spite of the central bank's constant target of zero inflation. This peak is somewhat smaller than in the case of the U.S. misperceptions that trigger an increase up to an inflation rate of 6 percent. In both cases, filtered money growth provides a good mirror image of the trend movement in inflation.

Why does the central bank accept this sustained increase in inflation? The reason is that it conducts a policy that is believed to be very effective in stabilizing inflation. Its forecast of inflation that is based on its preferred estimate of the output gap indicates a recession. Consequently, the central bank continuously predicts an imminent decline in the rate of inflation. If it were to raise interest rates further its forecast would signal a worsening of the recession and an undershooting of its inflation target. Ex-post, the estimation procedure that is employed by the central bank to obtain its potential estimate,  $z_{t|t}^e$ , attributes the persistent forecast misses to a sequence of unfavorable shocks. Such a reconciliation of potential output estimates and observed inflation performance is not without historical parallel. Many accounts of the 1970s attribute the stagflation in the United States and Germany primarily to inflationary and recessionary consequences of oil price shocks.<sup>23</sup>

We obtain similar results with the New-Keynesian model (not shown). Rather than reporting more individual simulations, we turn to a summary overview in Figure 3 on the basis of averages over 1000 simulations with U.S. and German output gap misperceptions in the K- and NK-Model, respectively. For each of the four possible combinations, we show two panels that

---

<sup>23</sup>Orphanides (2003) describes how potential output estimates for the U.S.A. were eventually revised downwards following the sustained increase of inflation. Similarly, the Bundesbank learned from its mistakes. However, these revisions occurred in several steps and after a substantial period of time.

report the cross-simulation averages of inflation,  $\pi_t$ , and the filtered measure of adjusted money growth,  $\mu_t^f$ . A comparison of these panels indicates that filtered money growth matches low-frequency movements in inflation very well. Thus, the monetary model derived from the quantity theory explains trend inflation very well. Money and inflation trends are due to the same source, namely persistent central bank misperceptions with regard to potential output. These misperceptions provide a structural explanation of the trends identified in recent empirical studies by Gerlach (2004), Benati (2005), Pill and Rautanen (2006) and Assenmacher-Wesche and Gerlach (2007). In other words, the introduction of imperfect knowledge and persistent central bank misperceptions in Keynesian-style models is sufficient to provide a unified account of trends and deviations, including monetary trends—the unresolved issue on the frontier of macroeconomic theory emphasized by Lucas (2007). An alternative explanation of common trends in money growth and inflation would be an on-going shift in the central bank’s inflation target, i.e. upwards in the 1970s and downwards in the 1980s. However, there exists no direct evidence of such a change in central bank objectives. Our explanation with a constant inflation target but persistent policy mistakes offers an alternative that is grounded in empirical observation in terms of historical output gap revisions.

## 4 A general definition of cross-checking

In light of the empirical evidence on concurring trends in money growth and inflation, Lucas (2007) proposed to use monetary information as an add-on or cross-check in interest rate policy. In this section, we provide a formal interpretation of his proposal. We start by reiterating the first-order condition that describes the optimal policy derived under certainty-equivalence:

$$E[\pi_{t+i} - \pi^* | t] = 0 \quad \forall i = \{0, 1, 2, \dots, \infty\} \quad (25)$$

It implies that trend inflation equals the inflation target in expectation. Specifically,  $E[\pi_{t+N} | t] \rightarrow E[\bar{\pi}]$  as  $N \rightarrow \infty$ , and consequently:

$$E[\bar{\pi}] = \pi^* \quad (26)$$

Thus, a policy maker who trusts that the Keynesian-style model correctly describes the economy, expects that trend inflation will turn out to match the target as long as policy is set to stabilize expected inflation in every period.<sup>24</sup>

However, such confidence in model-based forecasts and estimates of unobservable variables may be misplaced. The simulations of historical output gap misperceptions conducted in the preceding section provide an example that sustained trend deviations from target may occur even under policies that aim to stabilize inflation as close to target as seems feasible on the basis of model forecasts. Following Lucas's recommendation, a sceptical policy maker may instead prefer a simpler model of trend inflation based on monetary information. In fact, the preceding section offers a simple candidate model derived from the quantity equation:

$$E[\bar{\pi}] = E[\mu^f] \quad (27)$$

This relationship holds in the Keynesian-style models of section 2, but would also remain valid if the true structure of the economy were to correspond to a real business cycle model without any price rigidities. Thus, a policy maker in the monetarist tradition, who distrusts short-run inflation forecasts, may instead focus on controlling trend inflation. Such a monetarist policy maker would conduct open-market operations in period  $t$  so that trend inflation as estimated by the most recent observation on filtered adjusted money growth is expected to equal the inflation target:

$$E[\bar{\pi}|\mu_t^f] = \pi^* = 0 \quad (28)$$

Since  $\mu_t^f$  is constructed from money growth and actual output growth observations, it may be monitored without relying on model-based estimates of potential output. As a result, the monetary strategy can succeed in stabilizing trend inflation in spite of output gap misperceptions.<sup>25</sup>

Clearly, such a monetarist approach may appeal to a central banker who gives priority to managing first-order risks. In our view, however, it goes too far in abandoning any attempt at

---

<sup>24</sup>Similarly, a flexibly inflation targeting central bank that aims to stabilize inflation and output gap would expect that trend inflation equals the target since the trend output gap equals zero in expectation.

<sup>25</sup>Of course, a natural question concerns the implications of sustained velocity shifts for this strategy of stabilizing trend inflation. We return to this question in section 6.

short-run inflation stabilization. After all, Keynesian-style models may not be that far off the mark and potential output estimates need not always be utterly wrong. Instead, we follow Lucas (2007) and investigate how to use monetary information as a cross-check rather than as a policy prescription that is applied in every period.

We formalize the idea of cross-checking in the following manner. In every period, the central bank checks whether filtered money growth is still consistent with attaining the inflation target, or whether money growth trends have shifted, by monitoring the test statistic,

$$\kappa_t = \frac{\mu_t^f - \pi^*}{\sigma_{\mu^f}}, \quad (29)$$

and comparing it to a critical value  $\kappa^{crit}$ . Here,  $\sigma_{\mu^f}$  denotes the standard deviation of the filtered money growth measure. It can be determined under the null hypothesis that the central bank's preferred Keynesian-style model is correct.

As long as the test statistic does not signal a sustained shift in filtered money growth, the central bank implements the optimal policy under the preferred Keynesian-style model, i.e. the policy that satisfies the first-order condition (4). Thus, in the absence of persistent output gap misperceptions it can stabilize short-run inflation variations very effectively.

Once the central bank receives successive signals of a shift in trend inflation as estimated by filtered money growth, i.e. ( $\kappa_t > \kappa^{crit}$  for  $N$  periods) or ( $\kappa_t < -\kappa^{crit}$  for  $N$  periods), policy is adjusted so as to control trend inflation.<sup>26</sup> The policy with cross-checking may be characterized with a first-order condition that includes trend inflation:

$$E[\pi_t | z_t^e, K/NK] = -E[\bar{\pi} | \mu_k^f] \quad (30)$$

This condition guarantees that the central bank acts to offset any significant shift in trend inflation as estimated on the basis of monetary information.  $\mu_k^f$  denotes the most recent significant

---

<sup>26</sup>The two parameters  $\kappa^{crit}$  and  $N$  play different roles.  $\kappa^{crit}$  reflects the probability that an observed deviation of  $\mu^f$  from  $\pi^*$  is purely accidental (for example a 5% or 1% significance level).  $N$  defines the number of successive deviations in excess of this critical value. Thus, the greater  $N$  the longer the central bank waits to accumulate evidence of a sustained policy bias.

estimate of a trend shift in period  $k$ , i.e.  $(\kappa_k > \kappa^{crit}, \dots, \kappa_{k-N} > \kappa^{crit})$  or  $(-\kappa_k < -\kappa^{crit}, \dots, -\kappa_{k-N} < -\kappa^{crit})$ . Following a significant cross-check, the interest rate is set according to:

$$\text{K-Model: } i_t = (1 + (\varphi\lambda)^{-1})\pi_{t-1} + (\varphi)^{-1}(y_{t-1} - z_{t|t}^e) + (\varphi\lambda)^{-1}\mu_k^f \quad (31)$$

$$\text{NK-Model: } i_t = (\varphi\lambda)^{-1}(u_t^e) + (\varphi)^{-1}(z_{t+1|t}^e - z_{t|t}^e + g_t^e) + (\varphi\lambda)^{-1}\mu_k^f \quad (32)$$

This policy implies that the expectation of period  $t$  inflation on the basis of the respective Keynesian-style model corresponds to  $-\mu_k^f$  as prescribed by the first-order condition, equation (30).<sup>27</sup>

The interest rate policy with cross-checking consists of two components,

$$i_t = i_t^{K/NK} + i_t^{CC}, \text{ where } i_t^{CC} = (\varphi\lambda)^{-1}\mu_k^f, \quad (33)$$

the optimal interest rate policy conditional on the preferred Keynesian-style model and gap estimates denoted by  $i_t^{K/NK}$  and the (occasional) adjustment in interest rate levels due to cross-checking,  $i_t^{CC}$ .  $i_t^{CC}$  changes in a non-linear fashion whenever a new significant trend shift in money growth is detected.

## 5 Monetary cross-checking succeeds in stabilizing inflation trends due to historical output gap misperceptions

We now turn to exploring the performance of interest rate policy with monetary cross-checking as defined by equations (31) and (32). The cross-checking parameters,  $N$  and  $\kappa^{crit}$ , are calibrated such that the central bank considers the likelihood that cross-checking will come into play in the foreseeable future as negligible conditional on its preferred Keynesian-style model.<sup>28</sup>

Figures 4 and 5 report stochastic simulations of monetary cross-checking in the K- and

---

<sup>27</sup>The derivation for the NK model is presented in more detail in the appendix that is made available on the ScienceDirect website.

<sup>28</sup>Given  $\kappa^{crit} = 2.575$ , the 1% critical value for the normal distribution, and  $N = 4$ , the probability of a cross-check under the null hypothesis is less than  $10^{-8}$ .

NK-models. Comparing inflation in the top left panel of Figure 4, i.e. for the case of US misperceptions in the K-Model, to the outcome without cross-checking given the same draw shocks shown previously in Figure 2, we find that it evolves quite differently. The sustained upward trend observed previously is broken. The policy with cross-checking responds to an increase in filtered money growth,  $\mu_t^f$ , fairly quickly after the output gap misperceptions have induced an inflationary policy bias, and succeeds in reversing the inflation surge. The policy response is not driven by an improved gap estimate. Rather, the central bank responds to monetary trends over and above what is prescribed by the model-specific output-gap based inflation forecast.

To illustrate the interest rate adjustment from cross-checking we have added a panel that compares the interest rate effect of the current output gap misperception,  $(\phi)^{-1}e_t$ , to the cross-checking adjustment,  $i_t^{CC} = (\phi\lambda)^{-1}\mu_k^f$ . Once the monetary test statistic  $\kappa$  signals a trend shift in period  $k$ , it triggers an adjustment in the overall level of interest rates. This new level is maintained until the next significant trend shift is detected. Since output gap misperceptions increase further and continue to induce an inflationary policy bias the test statistic triggers two more upward adjustments in the level of interest rates. Cross-checking also works “on the way down” as output gap misperceptions subside and inflation and adjusted money growth decline below target. Consequently, monetary cross-checking triggers three successive downward adjustments. In sum, the cross-checks, on average, offset the inflationary or disinflationary consequences of sustained output gap misperceptions.

The lower set of three panels in Figure 4 reports the simulation with German output gap misperceptions. Again, monetary cross-checking serves to offset the inflationary trends arising from mistaken beliefs and policies. However, one large upward shift and two smaller downward shifts turn out to broadly match the contours of the inflationary bias arising from central bank misperceptions. Figure 5 confirms that cross-checking also works in the New-Keynesian model with interest rate setting defined by equation (32).

It is confirmed that the above findings hold true on average by simulating 1000 draws of shocks of 150 periods length from the respective normal distributions. The results (not shown) indicate that cross-checking effectively reduces the duration of the policy bias arising from

persistent output gap misperceptions. Of course, the simulations are still characterized by inflationary or disinflationary trends lasting for shorter periods. These movements serve to signal a trend shift and trigger the interest rate adjustment due to cross-checking.

## 6 Three questions concerning cross-checking

In this section, the effectiveness of cross-checking is investigated in further detail. Three questions are considered, namely how to account for velocity shifts, how important is the nonlinear nature of cross-checking and what alternative estimates of trend inflation may be considered.

### 6.1 How would you account for velocity shifts in monetary cross-checking?

It is well-known that a strategy of strict monetary targeting would transmit variations in the velocity of money to output and inflation fluctuations. For example, the money-demand equation (16) implies that short-run changes in money demand arise from three sources: shocks,  $s_t$ , changes in interest rates,  $\gamma_i \Delta i_t$  and changes in real income,  $\gamma_y \Delta y_t$ . While such fluctuations render a strict monetary targeting strategy undesirable, they do not inhibit monetary cross-checking as shown in the preceding section. A more interesting question concerns the performance of monetary cross-checking in the event of sustained changes in trend velocity, for example due to financial innovations. Two interesting examples regarding U.S. money demand have been documented by Orphanides and Porter (2000, 2001), and Reynard (2004).<sup>29</sup> Orphanides and Porter point out that M2 velocity increased substantially in the mid 1990s. They report a sustained intercept shift in their estimated velocity equation occurring over a period of several years. They also show that this shift was identified in real time by recursive estimation techniques allowing for intercept shifts. Such a shifting intercept may be included in the money demand equation:

$$m_t - p_t = \gamma_{0,t} + \gamma_y y_t - \gamma_i i_t + s_t. \quad (34)$$

---

<sup>29</sup>Reynard (2004) notes an apparent increase in the interest-rate elasticity of money demand, i.e.  $\gamma_i$ , in the early 1970s from the perspective of time-series analysis. He emphasizes the usefulness of cross-sectional analysis for obtaining improved estimates of the true structural parameters of money demand.



We use the NK model to simulate a fairly dramatic shift in the intercept,  $\gamma_{0,t}$ , that generates a velocity trend of 2 percentage points for 75 periods.<sup>30</sup> As the intercept increases, velocity declines and equilibrium money growth rises to accommodate increased demand at a given level of the interest rate. First, we assume that the central bank sticks to the original estimate of the intercept, never re-estimates and never considers the possibility of a structural shift. The resulting simulation is reported in the top two panels of Figure 6. The observed, sustained increase in money growth due to shifting velocity triggers a cross-check and thus a policy tightening. As a result, inflation declines below target by 2 percentage for the duration of the downward trend in velocity. Once velocity stabilizes, another cross-check brings inflation back to target.

Alternatively, we allow the central bank to recursively estimate money demand and consider the possibility of structural shifts.<sup>31</sup> The resulting simulation is reported in the lower two panels of Figure 6. We find that recursive estimation can detect the velocity shift considered and ensure the usefulness of monetary cross-checking. This finding underscores the importance of money demand analysis at the central bank.

## 6.2 Why does the cross-check take a nonlinear form rather than a linear feedback coefficient?

Cross-checking takes the form of an occasional adjustment of the model-dependent optimal interest policies derived from the first-order condition (4). This adjustment is triggered in period  $k$  when the test-statistic,  $\kappa$ , defined by equation (29), has exceeded the critical value  $\kappa^{crit}$  for  $N$  successive periods.<sup>32</sup> A seemingly simpler alternative would be to augment the linear interest-rate equations with an additional linear feedback on filtered money growth,  $\mu_t^f$ , that applies at

---

<sup>30</sup>Again, this is an a-priori assumption regarding the structural process determining an unobservable variable. In future work, we aim to collect real time estimates of money demand and velocity trends to be able to analyze velocity trends in the same manner as output gap estimates in sections 3 and 5.

<sup>31</sup>Orphanides and Porter (2001) propose regression tree methods as a new approach to identify such shifts in real time. Here, we consider a more traditional tool of money demand analysis in form of recursive least squares with time-varying parameters or recursive least-squares with forgetting (see Harvey (1990), Chapter 4).

<sup>32</sup>Similarly,  $N$  successive negative realizations of  $\kappa$  smaller than the negative of  $\kappa^{crit}$  trigger a symmetric downward adjustment. Note, the counter for  $N$  is reset to zero after every significant cross-check. The first-order condition, equation (30) based on  $\mu_k^f$  then applies till the next adjustment is triggered.

all times. Including such a linear feedback in the K-Model implies:

$$i_t = (1 + (\varphi\lambda)^{-1})\pi_{t-1} + (\varphi)^{-1}(y_{t-1} - z_{t|t}^e) + (\varphi\lambda)^{-1}\mu_{t-1}^f \quad (35)$$

Here, the time subscript on  $\mu_{t-1}^f$  refers to the most recent observation prior to the policy decision from period  $t - 1$ , rather than from period  $k$ , that was the most recent period with a significant cross-check.

We compare the rule with linear feedback to the optimal interest rate policy conditional on the gap estimate, equation (14), and the policy with cross-checking, equation (31). Since central bank loss is measured by squared deviations of inflation from a zero inflation target we evaluate policies by the simulation outcomes for  $E(\pi^2)$ . Table 2 reports the average central bank loss over 1000 simulations under three different scenarios: no output gap misperceptions,  $e_t = 0$ , U.S. output gap misperceptions,  $e_t^{US}$ , and German output gap misperceptions,  $e_t^{DE}$ .

By default, the model-dependent optimal policy (equation (14)) is the best performing policy when implemented under the assumption that the central bank's estimate of the output gap is correct. However, the policy with cross-checking performs just as well in that scenario. Its nonlinearity ensures that in times when the central bank's preferred Keynesian-style model and associated output gap estimates deliver reliable inflation forecasts, the model-dependent optimal policy is implemented. Thus, as long as the central bank's beliefs regarding the appropriate macroeconomic model, the implied concept of potential output, and the appropriate estimation method do not imply sustained misperceptions, monetary cross-checking will not trigger policy adjustments. Policy is only adjusted according to the first-order condition (30) when the simple monetary model of trend inflation, equation (27), strongly signals a trend shift. The policy with linear feedback, however, always responds to variations in trend money growth, and consequently causes more than twice the mean-squared error of inflation.

In the case of U.S. output gap misperceptions, the policy without cross-checking generates a mean squared error of inflation of 5.39. The rule with linear feedback to trend money growth responds more aggressively and improves inflation performance in terms of a mean squared

error of 3.36. In this horse race, however, the policy with cross-checking again performs best by generating a lower mean-squared error of 2.07. A similar ranking, but with smaller absolute values, is obtained with German output gap misperceptions. Thus, the nonlinear policy response implied by the occasional adjustments triggered via cross-checking promises to be a useful approach when the central bank is uncertain about the appropriate model of the economy and would like to revert to a simple fall-back option when outcomes persistently deviate from model-based forecasts.

### **6.3 Why cross-check with money instead of other variables?**

So far, the focus of the analysis has been on the two points raised by Lucas (2007), namely, to provide an explanation of the joint trends in money growth and inflation observed empirically, and to develop a formal approach for cross-checking Keynesian-style interest rate policies against information from monetary aggregates. However, there are other variables that may be used as trend inflation estimates in cross-checking. Since central bank policy is considered to be the source of sustained money and inflation trends, it is important to use variables that are affected by policy even in the long run, that is nominal variables. Two candidates, in addition to filtered money-growth, are long-run nominal interest rates and filtered measures of inflation itself.

Nominal interest rates are difficult to read with regard to implications for trend inflation because of the interaction of the short-run negative liquidity effect and the long-run positive Fisher effect. As to filtered inflation,  $\pi_t^f$ , our analysis in section 5 suggests that it should perform at least as well as filtered money growth, because our modeling assumptions imply that money growth and inflation move contemporaneously and that money growth is more noisy than inflation. Table 3 confirms this conjecture with simulations of German and US output gap misperceptions. In the K-Model, cross-checking with filtered inflation performs about as well as with filtered money growth. In the NK-Model it performs slightly better.

However, the timing assumptions in the K- and NK-Model that are behind this result are at odds with the recent empirical literature, which finds that money growth leads inflation at low

frequencies by several quarters.<sup>33</sup> If the timing assumptions were to be modified to replicate this leading indicator role of money at low frequencies, filtered money growth would gain a competitive advantage over filtered inflation as a forecast of trend inflation.

We also note two other reasons that would motivate a preference for cross-checking against long-run money growth measures. First, the Keynesian-style models we use preclude direct effects of money and credit beyond those captured by the interest rate on output and inflation. If these modeling assumptions fail to hold then cross-checking with filtered money growth would be preferable to filtered inflation. The second reason is concerned with the use of models in the political decision-making process at the central bank. A central bank staff that relies exclusively on a Keynesian-style model and the associated output gap estimates interprets the sustained increase in inflation induced by output gap misperceptions as a consequence of unfavorable shocks. From this perspective, the staff will recommend against an additional policy response to past filtered measures of inflation, because it would imply forecasting a deflation. Policy makers may therefore find it preferable to request policy recommendations derived from a set of competing models and may want to give special attention to monetary models of trend inflation.

## 7 Conclusions

In Keynesian-style models sustained trends in money growth and inflation can be explained by successive policy mistakes due to central bank misperceptions. Using historical measures of output gap misperceptions for the U.S. and German economies from the 1970s to the 1990s we have provided a unified treatment of money growth and inflation trends along with short-run deviations in Keynesian models as requested by Lucas (2007). This result is obtained without relying on undocumented shifts in central bank inflation targets.

Central bankers today, might argue that they would not make such mistakes in estimating potential output since they employ sophisticated filtering techniques. However, this optimism may be unfounded. Orphanides and van Norden (2002) have shown that a variety of modern

---

<sup>33</sup>See Gerlach (2004), Benati (2005), Pill and Rautananen (2006) and Assenmacher-Wesche and Gerlach (2007).

filtering techniques lead to persistent output revisions similar to the Federal Reserve's historical estimates when applied to real-time data on U.S. output and inflation without a-priori assumptions. Also, central banks in the 1970s already had very sophisticated techniques at their disposal. Federal Reserve researchers in the 1970s knew how to use the Kalman filter and solve complex models (cf. Kalchbrenner and Tinsley (1977)) and economists at the Bundesbank used fairly sophisticated production functions to calculate potential output (see the monthly reports in October 1973 and October 1981).

Furthermore, we have provided a formal implementation of Lucas's proposal to use monetary information as an add-on or cross-check to the model-based interest rate policy. An earlier note, Beck and Wieland (2007), posited an interest rate rule with monetary cross-checking. In this paper, a more general definition of cross-checking has been provided by deriving such interest rate policies from a central bank objective function and first-order condition that incorporate trend inflation. A further innovation has been to analyze central bank misperceptions and cross-checking in the New-Keynesian model. Following Orphanides (2003) alternative policy strategies under output gap uncertainty have been evaluated without a-priori assumptions regarding the structure of the process driving potential output by using instead historical real-time and final estimates of the output gap. Monetary cross-checking has been shown to substantially improve inflation performance relative to the policy that would be optimal conditional on the Keynesian model and the a-priori assumptions on potential output.

Finally, we have addressed three additional questions regarding cross-checking. Firstly, monetary cross-checking is found to remain effective in the presence of long-lasting velocity shifts if standard recursive estimation techniques allowing for such shifts are used in money demand analysis. Secondly, cross-checking has been compared with a policy that incorporates linear feedback on filtered money growth. The linear feedback rule was shown to be dominated by cross-checking whether persistent output gap misperceptions occur or not. Thirdly, filtered inflation is found to constitute a good alternative to filtered money growth for cross-checking as long as the timing assumptions of the benchmark Keynesian-style models hold up. If these assumptions were modified such that money leads inflation as indicated by the recent empirical

literature, filtered money growth would gain an advantage over filtered inflation. We aim to explore the role of timing assumptions and the optimal choice of estimate of trend inflation in future research.

For practical central bank policy we recommend to use the gap-based Keynesian models of inflation regularly for policy design, but consider the quantity-theory based model of trend inflation as a fall-back option. We suggest to implement the quantity-theory based policy recommendation in circumstances when policies based on the Keynesian models have persistently under-performed, i.e. when trend inflation is better captured by the monetary models. In future research, we aim to collect historical money demand estimates for the United States and Germany to study whether monetary cross-checking would have helped preventing double-digit inflation in the United States in the 1970s and 1980s, and whether Germany escaped double-digit inflation, because the Bundesbank gave more weight to monetary models of trend inflation.

## References

- Andres, J., Lopez-Salido, J. D., and Valles, J. (2006). Money in an estimated business cycle model of the euro area. Economic Journal, 116:457–477.
- Assenmacher-Wesche, K. and Gerlach, S. (2007). Interpreting euro area inflation at high and low frequencies. Journal of the European Economic Association, 5:534–542.
- Beck, G. W. and Wieland, V. (2002). Learning and control in a changing economic environment. Journal of Economic Dynamics and Control, 26(9-10):1359–1377.
- Beck, G. W. and Wieland, V. (2007). Money in monetary policy design: A formal characterization of ECB-style cross-checking. Journal of the European Economic Association, 5:524–533.
- Benati, L. (2005). Long-run evidence on money growth and inflation. Bank of England Quarterly Bulletin.
- Bils, M. and Klenow, P. (2004). Some evidence on the importance of sticky prices. Journal of Political Economy, 112:987–985.
- Bundesbank, D. (1973). Production potential of the Federal Republic of Germany. Monthly Report, October, 27-32.
- Bundesbank, D. (1981). Recalculation of the production potential of the Federal Republic of Germany. Monthly Report, October, 30-36.
- Calvo, G. A. (1983). Staggered prices in a utility-maximizing framework. Journal of Monetary Economics, 12:383–398.
- Christiano, L. J., Motto, R., and Rostagno, M. (November 9-10, 2006). Monetary policy and stock market boom-bust cycles. paper prepared for the Fourth ECB Central Banking Conference, The Role of Money: Money and Monetary Policy in the Twenty-First Century.
- Christiano, L. J. and Rostagno, M. (2001). Money growth monitoring and the Taylor rule. NBER Working Paper No. 8539.
- Clarida, R., Gali, J., and Gertler, M. (1999). The science of monetary policy: A new keynesian perspective. NBER Working Paper Series No. 7147.
- Coenen, G., Levin, A. T., and Wieland, V. (2005). Data uncertainty and the role of money as an information variable for monetary policy. European Economic Review, 49:975–1006.
- Dotsey, M. and Hornstein, A. (2003). Should a monetary policymaker look at money? Journal of Monetary Economics, 50:547–579.
- Friedman, M. (2006). Tradeoffs in monetary policy. prepared for a Festschrift conference in honor of David Laidler.
- Gerberding, C., Seitz, F., and Worms, A. (2005). How the Bundesbank really conducted monetary policy. North American Journal of Economics and Finance, 16:277–292.

- Gerlach, S. (2004). The two pillars of the European Central Bank. Economic Policy, 40:389–439.
- Goodfriend, M. and King, R. G. (1997). The new neoclassical synthesis and the role of monetary policy. In Bernanke, B. S. and Rotemberg, J. J., editors, National Bureau of Economic Research Macroeconomics Annual 1997. MIT Press, Cambridge, MA.
- Harvey, A. C. (1990). The Econometric Analysis of Time Series. The MIT Press.
- Ireland, P. N. (2004). Money's role in the monetary business cycle. Journal of Money, Credit and Banking, 36:969–983.
- Kalchbrenner, J. H. and Tinsley, P. A. (1977). On filtering auxiliary information in short-run monetary policy. Carnegie-Rochester Conference Series on Public Policy, 7.
- Kerr, W. and King, R. G. (1996). Limits on interest rate rules in the is model. Economic Quarterly, (Spr):47–75.
- Lucas, Jr., R. E. (2007). Central banking: Is science replacing art? in Monetary Policy: A Journey From Theory to Practice, European Central Bank.
- Orphanides, A. (2003). The quest for prosperity without inflation. Journal of Monetary Economics, 50:633–663.
- Orphanides, A. and Porter, R. (2001). Money and inflation: The role of information regarding the determinants of M2 behavior. in Klöckers, Hans-Joachim and Caroline Willeke, eds., 2001, Monetary Analysis: Tools and Applications, ECB.
- Orphanides, A. and Porter, R. D. (2000). P\* revisited: Money-based inflation forecasts with a changing equilibrium velocity. Journal of Economics and Business, 52:87–100.
- Orphanides, A. and van Norden, S. (2002). The unreliability of output gap estimates in real time. The Review of Economics and Statistics, 84(4):569–583.
- Orphanides, A. and Wieland, V. (2000). Inflation zone targeting. European Economic Review, 44:1351–1387.
- Pill, H. and Rautananen, T. (2006). Monetary analysis - the ECB experience. Paper presented at the conference “The ECB and its Watchers VIII”, May 5, 2006.
- Reynard, S. (2004). Financial market participation and the apparent instability of money demand. Journal of Monetary Economics, 51:1297–1317.
- Rotemberg, J. J. and Woodford, M. (1997). An optimization-based econometric framework for the evaluation of monetary policy. in B. Bernanke and J. Rotemberg, (eds.), NBER Macroeconomics Annual, The MIT Press.
- Svensson, L. E. O. (1997). Inflation forecast targeting: Implementing and monitoring inflation targets. European Economic Review, 41:1111–1146.
- Svensson, L. E. O. and Woodford, M. (2003). Indicator variables for optimal policy. Journal of Monetary Economics, 50:691–720.



- Svensson, L. E. O. and Woodford, M. (2004). Indicator variables for optimal policy under asymmetric information. Journal of Economic Dynamics and Control, 28:661–690.
- Taylor, J. B. (1979). Estimation and control of a macroeconomic model with rational expectations. Econometrica, 47(5):1267–1286.
- Taylor, J. B. (1993). Discretion versus policy rule in practice. In Carnegie-Rochester Conference Series, pages 195–214, North Holland.
- Taylor, J. B. (2006). Comment on Friedman, Milton, “tradeoffs in monetary policy”. prepared for a Festschrift conference in honor of David Laidler.
- Walsh, C. E. (2003). Monetary theory and policy. The MIT Press, Cambridge.
- Wieland, V. (2000). Learning by doing and the value of optimal experimentation. Journal of Economic Dynamics and Control, pages 1–29.
- Wieland, V. (2006). Monetary policy and uncertainty about the natural unemployment rate: Brainard-style conservatism versus experimental action. Advances in Macroeconomics, 6(1).
- Woodford, M. (2003). Interest and prices: Foundations of a theory of monetary policy. Princeton University Press, Princeton.
- Woodford, M. (2006). How important is money in the conduct of monetary policy? paper prepared for the Fourth ECB Central Banking Conference, The Role of Money: Money and Monetary Policy in the Twenty-First Century.

Table 1: Parameter Values

Parameter	Value	Economic interpretation
$\beta$	0.99	Discount factor of the policy maker.
$-\phi$	-1	Real interest rate elasticity of aggregate demand (in line with Andres et al. (2006) and Ireland (2004)).
$\lambda$	0.5	Elasticity of Phillips curve w.r.t. output gap (broadly in line with Gerlach (2004)). <sup>34</sup>
$\gamma_y$	0.1	Income elasticity of money demand (in line with Andres et al. (2006) and Ireland (2004)).
$-\gamma_i$	-0.4	Interest rate elasticity of money demand (in line with Andres et al. (2006) and Ireland (2004)).
$\omega$	0.2	Weighting parameter of filter (broadly in line in Gerlach (2004))
$\bar{\Delta}y, \bar{\pi}, \pi^*$	0	Equilibrium real interest rate, potential output growth, steady-state inflation and inflation target
$\sigma_g, \sigma_u, \sigma_s$	0.8	Standard deviation of cost-push, demand and money demand shocks
$\sigma_{\varepsilon_g}, \sigma_{\varepsilon_u}$	0.4	Standard deviation of noise of aggregate demand and cost-push shocks
$\sigma_{\varepsilon_s}$	0.1	Standard deviation of money demand shocks
$\kappa^{crit}$	1.96	5% critical value for the cross-checking rule.
$N$	4	Number of periods required for a sustained deviation in the cross-checking rule.
$\sigma_{\mu^f}$		Standard deviation of $\mu^f$ is not exogenous but determined consistently with model and policy

Notes: Table 1 provides an overview of the parameter values that we have used in our model simulations.

Table 2: Linear Feedback vs Nonlinear Cross-Checking

Policy	Central bank loss (K-Model)		
	$e_t = 0$	$e_t^{US}$	$e_t^{DE}$
No cross-checking	0.79	5.39	2.67
Cross-checking with $\mu_t^f$	0.79	2.07	1.79
Linear feedback with $\mu_t^f$	1.73	3.36	2.60

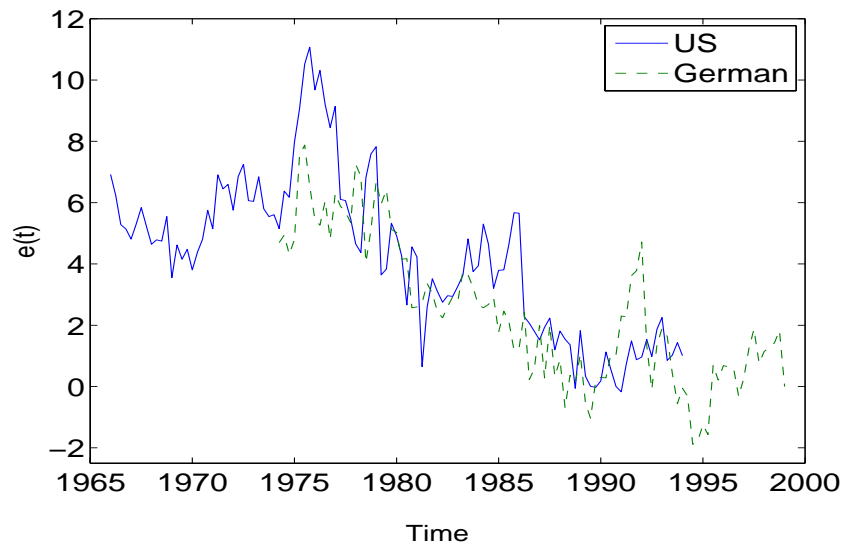
Notes: Central bank loss corresponds to the mean squared deviations,  $E[(\pi)^2]$ , and is measured by averages over 1000 simulations of 150 periods length as in Figure 3.  $e_t^{US}$  refers to U.S. output gap misperceptions and  $e_t^{DE}$  to German output gap misperceptions.

Table 3: Filtered Money Growth vs Filtered Inflation

Policy	Central bank loss			
	K-Model		NK-Model	
	$e_t^{US}$	$e_t^{DE}$	$e_t^{US}$	$e_t^{DE}$
No cross-checking	5.39	2.67	5.10	2.32
Cross-checking $\mu_t^f$	2.07	1.79	1.85	1.57
Cross-checking $\pi_t^f$	2.06	1.75	1.72	1.50

Notes: Central bank loss corresponds to the mean squared deviations,  $E[(\pi)^2]$ , and is measured by averages over 1000 simulations of 150 periods length.  $e_t^{US}$  refers to U.S. output gap misperceptions and  $e_t^{DE}$  to German output gap misperceptions.

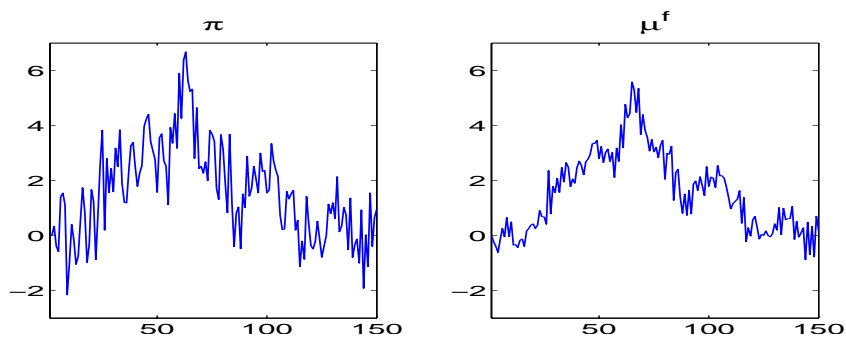
Figure 1: Output Gap Misperceptions in the United States and Germany



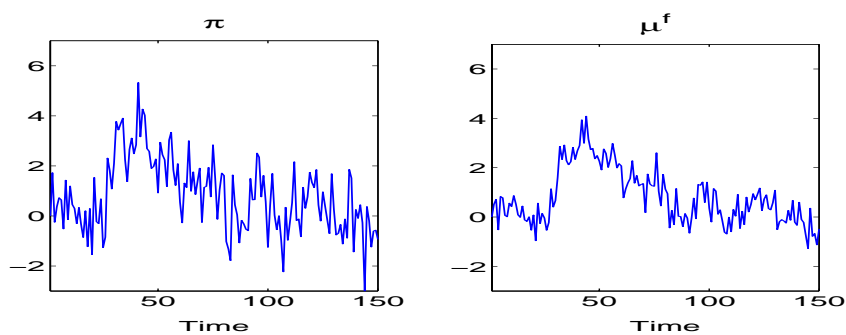
Note: Figure 1 plots historical U.S. and German output gap misperceptions. The data for the U.S. output gap misperceptions were provided by Orphanides (2003), the German output gap misperception data were provided by Gerberding et al. (2005).

Figure 2: Money Growth and Inflation Trends in the K-Model

U.S. Output Gap Misperceptions

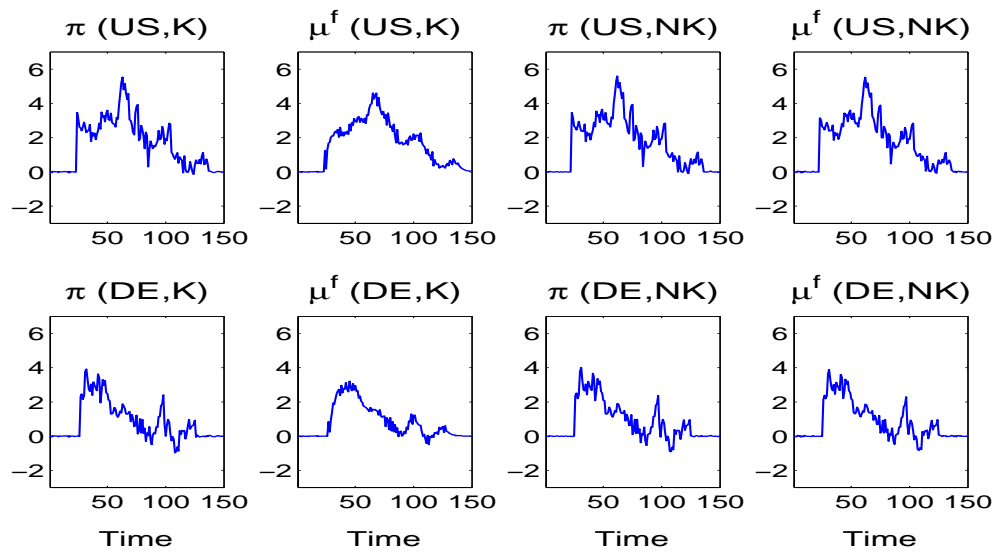


German Output Gap Misperceptions



Notes: Figure 2 reports simulations with U.S. and German output gap misperceptions in the K-model for a given draw of exogenous shocks and noise terms. The upper two panels show the inflation rate,  $\pi$ , and the filtered measure of adjusted money growth,  $\mu^f$ , with U.S. output gap misperceptions, the lower two panels show the corresponding series with German output gap misperceptions.

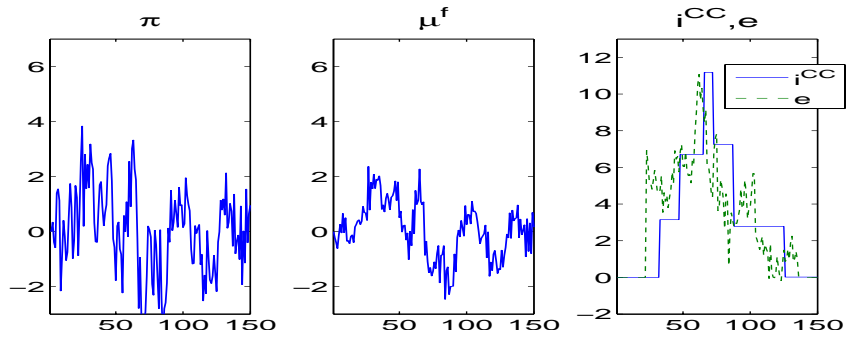
Figure 3: Money Growth and Inflation Trends - Averages of 1000 Draws of Shocks



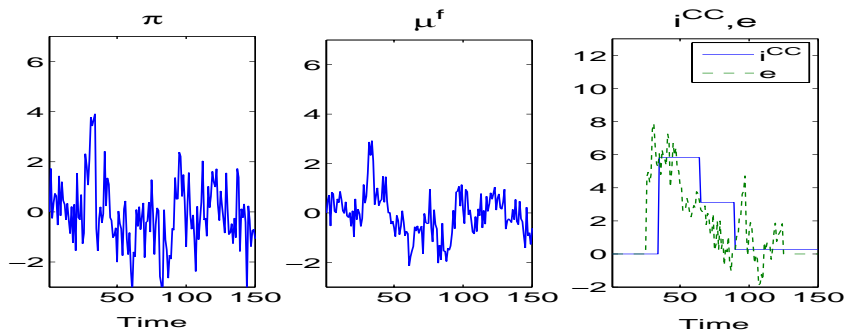
Notes: Figure 3 reports averages of 1000 simulations with U.S. and German (DE) output gap misperceptions in the K- and NK-model. For each of the four possible combinations two panels are shown that report the cross-simulation averages of the inflation rate,  $\pi$ , and the filtered adjusted money growth rate,  $\mu^f$ .

Figure 4: Monetary Cross-Checking in the K-Model

U.S. Output Gap Misperceptions



German Output Gap Misperceptions

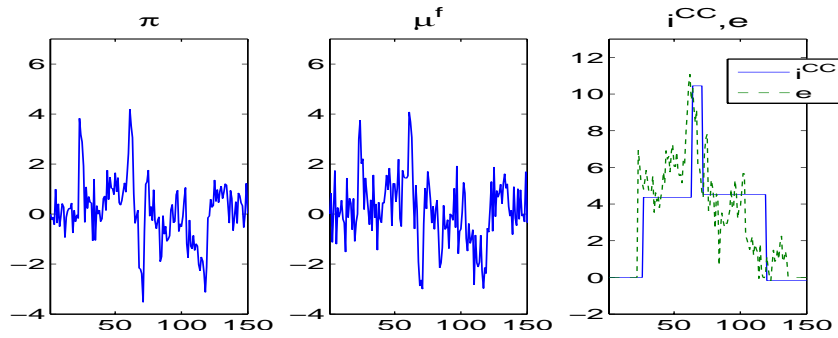


Notes: Figure 4 reports simulations of monetary cross-checking in the K-model for U.S. (upper three panels) and German (lower three panels) output gap misperceptions. In each case the inflation rate,  $\pi$ , the filtered measure of adjusted money growth,  $\mu^f$ , the output gap perception error,  $e$ , and the cross-checking adjustment,  $i^{CC}$ , are plotted.

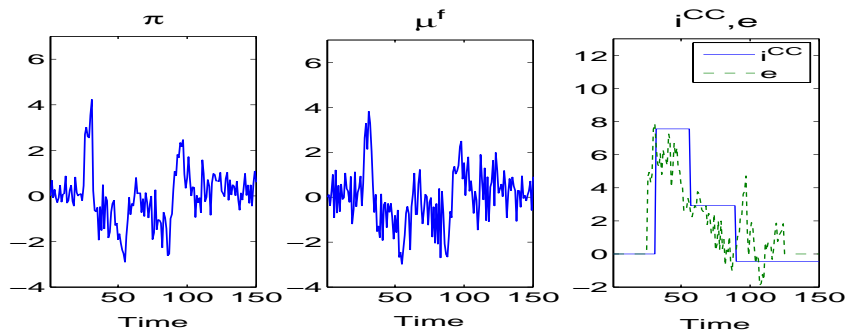


Figure 5: Monetary Cross-Checking in the NK-Model

U.S. Output Gap Misperceptions



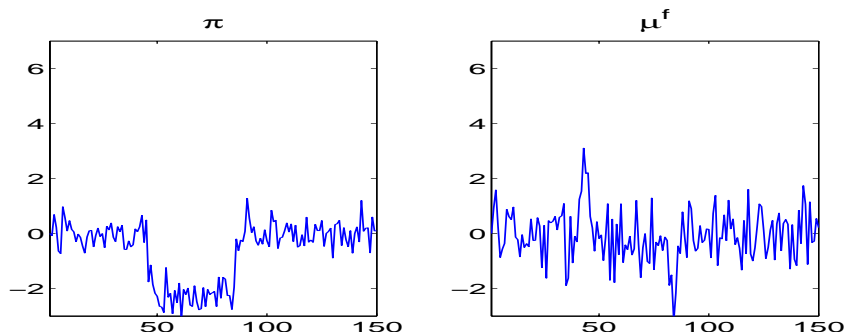
German Output Gap Misperceptions



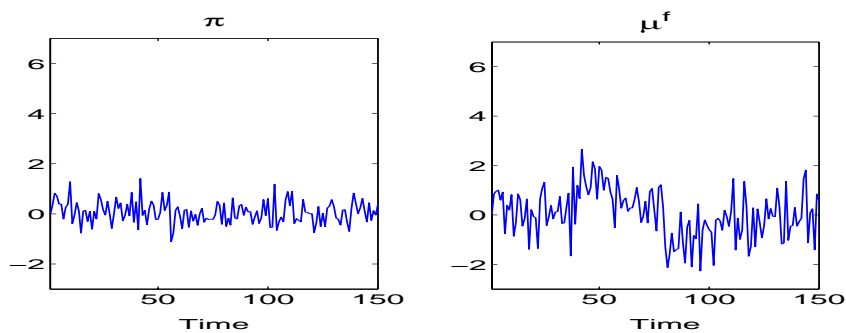
Notes: Figure 5 reports simulations of monetary cross-checking in the NK-model for U.S. (upper three panels) and German (lower three panels) output gap misperceptions. In each case the inflation rate,  $\pi$ , the filtered measure of adjusted money growth,  $\mu^f$ , the output gap perception error,  $e$ , and the cross-checking adjustment,  $i^{CC}$ , are plotted.

Figure 6: Monetary Cross-Checking and Velocity Shifts in the NK-Model

Central Bank Never Considers the Possibility of Shifts



Central Bank Uses Recursive Least Squares with Possibility of Shifts



Notes: Figure 6 reports simulations with U.S. and German output gap misperceptions in the NK-model for a given draw of exogenous shocks and noise terms when changes in trend velocity occur. The upper three panels show the inflation rate,  $\pi$ , and the filtered measure of adjusted money growth,  $\mu^f$ , for the case that the central bank sticks to the original estimate of the intercept,  $\gamma_0$ , in the money demand equation and never considers the possibility of a structural shift. The lower three panels plot the same series for the case that the central bank recursively estimates money demand and considers the possibility of structural shifts.

## A Model Equations

**Table A1: Model Equations**

Description	Model Equation
<u>Common equations</u>	
Central bank objective	$-\frac{1}{2}E_t \left\{ \sum_{i=0}^{\infty} \beta^i [(\pi_{t+i} - \pi^*)^2] \right\}, \pi^* = 0$
Perceived potential	$z_{t t}^e = z_t + e_t$
Money demand	$m_t - p_t = \gamma_y y_t - \gamma_i i_t + v_t, v_t \sim \text{i.i.d. N}(0, \sigma_v)$
<u>K-Model</u>	
Phillips curve	$\pi_t = \pi_{t-1} + \lambda(y_t - z_t) + u_t, u_t \sim \text{i.i.d. N}(0, \sigma_u)$
IS Curve	$y_t = y_{t-1} - \phi(i_t - \pi_{t-1}) + g_t, g_t \sim \text{i.i.d. N}(0, \sigma_g)$
<u>NK-Model</u>	
Phillips curve	$\pi_t - \bar{\pi} = \beta(\pi_{t+1}^e - \bar{\pi}) + \lambda(y_t - z_t) + u_t,$ $u_t \sim \text{i.i.d. N}(0, \sigma_u), \bar{\pi} = \pi^* = 0$
IS Curve	$y_t = y_{t+1}^e - \phi(i_t - \pi_{t+1}^e) + g_t, g_t \sim \text{i.i.d. N}(0, \sigma_g)$
Demand signal/noise	$g_t = g_t^e + \varepsilon_t^g, \varepsilon_t^g \sim \text{i.i.d. N}(0, \sigma_{\varepsilon_g})$
Cost-push signal/noise	$u_t = u_t^e + \varepsilon_t^u, \varepsilon_t^u \sim \text{i.i.d. N}(0, \sigma_{\varepsilon_u})$
Money-demand signal/noise	$v_t = v_t^e + \varepsilon_t^v, \varepsilon_t^v \sim \text{i.i.d. N}(0, \sigma_{\varepsilon_v})$

**Notes:**

Table A provides an overview of the equations, parameters and assumptions regarding the traditional Keynesian and New-Keynesian models used in “Central Bank Misperceptions and the Role of Money in Interest Rates Rules”.

## B Cross-checking in the NK-Model: Detailed derivations

In the following two subsections a detailed derivation of the optimal monetary policy under uncertainty in the NK-model without and with cross-checking is provided.

### B.1. Optimal policy under uncertainty without cross-checking

We start by deriving the optimal policy under uncertainty given symmetric information between the central bank and market participants. Thus, the central bank and market participants share the same information regarding central bank objectives, potential output estimates and expectations regarding future inflation and output. In principle, the derivation is for the case of optimal policy under discretion, but given a strictly inflation targeting central bank it turns out that the policies under discretion and commitment are identical.

The policy maker's objective under strict inflation targeting is to maximize the following loss function

$$\max -\frac{1}{2}E_t \left\{ \sum_{i=0}^{\infty} \beta^i [(\pi_{t+i} - \pi^*)^2] \right\} \quad (36)$$

subject to the Phillips curve and the IS curve (see Table A1). The inflation target is normalized at zero,  $\pi^* = 0$ . The associated first-order condition is

$$E[\pi_{t+i}|t] = \pi^* = 0 \quad \forall i = \{0, 1, 2, \dots, \infty\} \quad (37)$$

where  $\pi_{t+i}$  depends on the output gap,  $y_{t+i} - z_{t+i}$ , according to the New-Keynesian Phillips curve. It follows that the central bank and market participants expect future inflation to be equal to the zero inflation target:

$$\pi_{t+1|t}^e = 0 \quad (38)$$

Furthermore, since we have assumed that the cost-push shocks  $u_t$  are serially uncorrelated, the expected future output gap is also equal to zero, consistent with the expected future inflation rate:

$$x_{t+1|t}^e = 0 \iff y_{t+1|t}^e = z_{t+1|t}^e. \quad (39)$$

Solving the Phillips curve for  $y_t$  and applying  $\pi_{t+1|t}^e = 0$  yields the level of output compatible with the expected inflation rate for period  $t$ :

$$y_{t|t}^e = z_{t|t}^e - \frac{1}{\lambda} u_{t|t}^e. \quad (40)$$

Using the IS curve, which corresponds to the log-linear version of the household Euler equation,

$$y_t = y_{t+1}^e - \varphi (i_t - \pi_{t+1}^e) + g_t \quad (41)$$

we can determine the optimal interest  $i_t$  as

$$\begin{aligned} i_t &= \pi_{t+1|t}^e - \frac{1}{\varphi} y_{t|t}^e + \frac{1}{\varphi} y_{t+1|t}^e + \frac{1}{\varphi} g_{t|t}^e \\ &= \frac{1}{\lambda\varphi} u_{t,t}^e + \frac{1}{\varphi} (z_{t+1|t}^e - z_{t|t}^e + g_{t|t}^e). \end{aligned} \quad (42)$$

In the derivation of  $i_t$  we have made use of the above-mentioned expressions for  $y_{t|t}^e$ ,  $y_{t+1|t}^e$  and  $\pi_{t+1|t}^e$ .

### B.2 Optimal policy with cross-checking using $\mu_k^f$ as estimate of trend inflation

The policy with cross-checking may be characterized by a first-order condition that includes trend inflation:

$$E[\pi_t | z_{t|t}^e] = -E[\bar{\pi} | \mu_k^f] \quad (43)$$

This condition guarantees that the central bank acts to offset any significant shift in trend inflation as estimated on the basis of monetary information.  $\mu_k^f$  denotes the most recent significant estimate of a trend shift. Thus, in period  $k$  the test statistic  $\kappa$  defined by

$$\kappa_t = \frac{\mu_t^f - \pi^*}{\sigma_{\mu^f}}, \quad (44)$$

satisfies the condition ( $\kappa_k > \kappa^{crit}, \dots, \kappa_{k-N} > \kappa^{crit}$ ) or ( $-\kappa_k < -\kappa^{crit}, \dots, -\kappa_{k-N} < -\kappa^{crit}$ ).

To determine the interest rate setting induced by a significant cross-check in the NK model it is important to consider the effect of cross-checking on market participants' expectations of future inflation. First, we note that conditional on the NK model and the associated estimate of potential output neither the central bank nor market participants expect cross-checking to kick in. Recall, that the probability that the test statistic  $\kappa$  exceeds the critical value is negligible, and even more so the probability that it exceeds  $\kappa^{crit}$  for  $N$  periods. Thus, in the absence of a significant cross-check the expectations for inflation in period  $t$  under the null hypothesis of the New-Keynesian model and the potential output estimate  $z_{t|t}^e$  are

$$\pi_{t|t}^e = 0 \quad (45)$$

Once a significant cross-check occurs, the first-order condition with the monetary estimate of

trend inflation governs policy. As a consequence:

$$\pi_{t|t}^e = -\mu_f^k \quad (46)$$

Thus, under symmetric information the central bank and market participants will expect current inflation—conditional on the New-Keynesian model and potential output estimate—to fall below the target by the extent of the trend inflation estimate provided by filtered money growth.

To solve the New-Keynesian Phillips curve for the expected output level that the central bank should aim at according to the policy with cross-checking, it is necessary to characterize market participants' expectation of inflation in period  $t + 1$ . In the baseline case it is assumed that market participants expect future inflation to return to the zero inflation target of the central bank, i.e.  $\pi_{t+1|t}^e = 0$ . This assumption is standard for the optimal policy under discretion. It implies that the central bank cannot manipulate market participants' inflation expectations by promising to commit to delivering future inflation outcomes different from the objective function with its long-run target.

Next, the Phillips curve is solved for the level of output that the central bank expects to achieve in period  $t$ ,  $y_{t|t}^e$ . Using  $\pi_{t+1|t}^e = 0$  and  $\pi_{t|t}^e = -\mu_f^k$  one obtains

$$\begin{aligned} y_{t|t}^e &= z_{t|t}^e + \frac{1}{\lambda} \pi_{t|t}^e - \frac{\beta}{\lambda} \pi_{t+1|t}^e - \frac{1}{\lambda} u_{t|t}^e \iff \\ y_{t|t}^e &= z_{t|t}^e - \frac{1}{\lambda} u_{t|t}^e - \frac{1}{\lambda} \mu_f^k. \end{aligned} \quad (47)$$

In the next step, the IS curve is solved for the interest rate  $i_t$  that achieves the expected optimal level of output, that is, the level of output consistent with the central bank's first-order condition. To this end, it is necessary to characterize market participants' expectation of output in period  $t + 1$ . Consistent with the expectation that inflation will be equal to the target in period  $t + 1$ , market participants expect output to be equal to potential output in period  $t + 1$ :

$$y_{t+1|t}^e = z_{t+1|t}^e. \quad (48)$$

Solving the IS equation for  $i_t$  given the expressions for  $y_{t|t}^e$ ,  $y_{t+1|t}^e$  and  $\pi_{t+1|t}^e$  yields the interest rate policy with cross-checking:

$$\begin{aligned} i_t &= 0 - \frac{1}{\varphi} \left[ z_{t|t}^e - \frac{1}{\lambda} u_{t|t}^e - \frac{1}{\lambda} \mu_f^k \right] + \frac{1}{\varphi} z_{t+1|t}^e + \frac{1}{\varphi} g_{t|t}^e \\ &= \frac{1}{\lambda \varphi} u_{t|t}^e + \frac{1}{\varphi} \left( z_{t+1|t}^e - z_{t|t}^e + g_{t|t}^e \right) + \frac{1}{\lambda \varphi} \mu_f^k. \end{aligned} \quad (49)$$

For the sake of completeness, we have also investigated the policy with cross-checking

under the assumption that the central bank is able to credibly commit to maintaining the disinflationary stance implied by the policy with cross-checking for a finite number of periods,  $T$ , in the future. In this case, the central bank is able to influence future inflation expectations s.t.  $\pi_{T|t} = \pi_{T-1|t} = \dots = \pi_{t|t} = -\mu_k^f$ . Thus, future inflation expectations move in a way that will help offsetting the apparent increase in trend inflation. The implied expected path of output and interest rates can be solved for recursively by starting in period  $T + 2$  and solving backwards for expected inflation, output and interest rates. The interest rate level expected for period  $T$  coincides with the expectation of equation (49):

$$i_{T|t} = \frac{1}{\varphi} \left( z_{T+1|t}^e - z_{T|t}^e \right) + \frac{1}{\lambda\varphi} \mu_f^k. \quad (50)$$

The interest rate level set in period  $t$ , however, incorporates the response of market participants expectation of future inflation to the central bank's announcement of the policy with cross-checking:

$$i_t = \frac{1}{\lambda\varphi} u_{t|t}^e + \frac{1}{\varphi} (z_{t+1|t}^e - z_{t|t}^e + g_{t|t}^e) - \mu_k^f. \quad (51)$$

We have simulated this policy under historical U.S. and German output gap misperceptions in the New-Keynesian model. The simulation results are similar to the baseline case discussed in the paper concerning the success of cross-checking in offsetting the inflationary bias due to persistent central bank misperceptions. Of course, due to the response of inflation expectations to the announcement of cross-checking, the particular interest rate path followed by the central bank differs from the baseline case.

## CFS Working Paper Series:

No.	Author(s)	Title
2008/24	<b>Christos Koulovatianos Carsten Schröder Ulrich Schmidt</b>	Confronting the Robinson Crusoe paradigm with household-size heterogeneity
2008/23	<b>Jin Cao Gerhard Illing</b>	Endogenous Systemic Liquidity Risk
2008/22	<b>Zeno Enders Gernot J. Müller Almut Scholl</b>	How do Fiscal and Technology Shocks affect Real Exchange Rates? New Evidence for the United States
2008/21	<b>Miriam Sperl</b>	Quantifying the Efficiency of the Xetra LOB Market. A Detailed Recipe
2008/20	<b>Maria Concetta Chiuri Tullio Jappelli</b>	Do the elderly reduce housing equity? An international comparison
2008/19	<b>Annamaria Lusardi</b>	Financial Literacy: An Essential Tool for Informed Consumer Choice?
2008/18	<b>Annamaria Lusardi</b>	Increasing the Effectiveness of Financial Education in the Workplace
2008/17	<b>Volker Wieland</b>	Learning, Endogenous Indexation and Disinflation in the New-Keynesian Model
2008/16	<b>Athanasios Orphanides Volker Wieland</b>	Economic Projections and Rules-of-Thumb for Monetary Policy
2008/15	<b>Jan Pieter Krahenen Christian Wilde</b>	Risk Transfer with CDOs

Copies of working papers can be downloaded at <http://www.ifk-cfs.de>