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Monopoly Power Limits Hedging*

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Abstract:
When a spot market monopolist participates in a derivatives market, she has an incentive to deviate from the spot market monopoly optimum to make her derivatives market position more profitable. When contracts can only be written contingent on the spot price, a risk-averse monopolist chooses to participate in the derivatives market to hedge her risk, and she reduces expected profits by doing so. However, eliminating all risk is impossible. These results are independent of the shape of the demand function, the distribution of demand shocks, the nature of preferences or the set of derivatives contracts.

JEL Classification: D24, G32

Keywords: Spot Market Power, Derivates Market, Hedging.

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1 Introduction

In this paper, we consider the impact of spot market power on a monopolist's ability to hedge spot market price risk. When a spot market monopolist participates in a derivatives market, it leads to a moral hazard problem in the spot market. In particular, she has an incentive to deviate from the monopoly optimum in order to make her derivatives position more profitable. When rational derivatives market participants observe the monopolist's position, they will take the impact of this position on subsequent spot prices into account when setting derivatives prices. This makes it expensive for a monopolist to mitigate risk.

We show that when derivatives contracts can be written contingent on the realization of demand, a risk-averse monopolist eliminates all risk. When contracts can only be written contingent on the spot price, she partially eliminates risk and reduces expected profits by doing so. The amount of hedging increases with risk-aversion. While some of these results have been shown in the context of specific examples, we prove them without any parametric assumptions about the shape of the demand function, the distribution of demand shocks, the nature of preferences, or the set of derivatives contracts.[1] The most important – and a novel – result of the paper is that eliminating all risk is impossible; doing so is not incentive compatible. Just as "no trade" theorems (e.g., Milgrom and Stokey, 1998) show that there is no price at which informed agents can trade profitably in financial markets, we prove a "no complete hedging" theorem that shows that there is no price at which agents with spot market power can eliminate all spot market price risk.

2 Model

We envision a model with one good and two periods, $t = 0, 1$. A monopolist with an increasing utility function u is the sole producer of the good, and the good is produced only in the last period. The monopolist chooses quantity Q to maximize profits. Demand is uncertain and is realized between the two periods. The residual (net of production costs and any additional price-taking suppliers) demand function faced by the monopolist is given by $P = f(Q, D)$, where D is the realization of demand. In the initial period, the monopolist can enter into a derivatives contract

[1] Anderson and Sundaresan (1984) studied a similar setting, focusing on whether futures trading can exist in a rational expectations equilibrium under spot market power. We obtain a result equivalent to theirs: a risk-averse monopolist participates in the futures market.

that pays off $g(P)$ in the second period. Given a competitive derivatives market with risk-neutral market makers, the payoff of the derivatives contract must be zero in expectation, $E[g(P)] = 0$.[2] The monopolist's profits are then

$$\pi = Cg(P) + QP, \tag{1}$$

where C is the number of derivatives contracts bought by the monopolist in the first period and is assumed to be observable in the market.[3]

In the last period, after demand is realized, the monopolist chooses an optimal quantity Q^*. The spot market first-order condition (FOC) is given by

$$\frac{\partial \pi}{\partial Q} = \left(Cg' + Q\right) f_1 + f = 0, \tag{2}$$

where we assume that the second order condition is satisfied.

In the initial period, the monopolist will set C to satisfy the derivatives market FOC,

$$\frac{\partial E[u(\pi)]}{\partial C} = E\left[\frac{\partial \pi}{\partial C} u'(\pi)\right] = 0, \tag{3}$$

where

$$\frac{\partial \pi}{\partial C} = g + C\left(\frac{dQ}{dC} f_1 g' - E\left[\frac{dQ}{dC} f_1 g'\right]\right) + \frac{dQ}{dC} f + Q \frac{dQ}{dC} f_1. \tag{4}$$

Substituting the spot market FOC (2) into (4) yields:

$$\frac{\partial \pi}{\partial C} = g - CE\left[\frac{dQ}{dC} f_1 g'\right]. \tag{5}$$

Given that the spot market quantity Q^* is chosen optimally, the derivatives market FOC is found by substituting (5) into (3):

$$\frac{\partial E[u(\pi)]}{\partial C} = E\left[\left(g - CE\left[\frac{dQ}{dC} f_1 g'\right]\right) \cdot u'(\pi)\right] = 0. \tag{6}$$

Market power introduces the term $E\left[\frac{dQ}{dC} f_1 g'\right]$ into this FOC. When setting C, the number of derivatives contracts, the monopolist takes into account that: a) hedging induces her to deviate

[2] For the sake of parsimony, demand risk is assumed to be idiosyncratic and therefore not priced.
[3] See Muermann and Shore (2008) for a treatment of strategic trading when C is not perfectly observable.

from the spot market optimum quantity ($\frac{dQ}{dC}$); b) this deviation changes spot prices (f_1); and c) this change in spot prices changes the payoff of the derivatives contract (g').

2.1 Risk-Averse Monopolists with Commitment

Proposition 1 *If a derivatives contract can be made contingent on realized demand (i.e., $g = g(D)$) and if any such contract can be written, then a risk-averse monopolist will eliminate all risk and maintain full monopoly profits in the spot market.*

Proof. See Appendix A.1. ■

A demand-contingent contract – where $g = g(D)$ or $g = g(P, D)$ instead of $g = g(P)$ – allows the monopolist to completely eliminate risk while maintaining monopoly profits. The optimal demand-contingent contract will pay off most in states where demand is lowest. These are the states with the lowest equilibrium prices. Since contract payoffs are contingent on realized demand and not prices, the monopolist has no incentive to change prices from the monopoly optimum since doing so has no impact on the derivative contract's payoff. Note that the monopolist's ability to eliminate all risk while maintaining market power depends critically on the verifiability of demand – on the existence of a contractible proxy for risk that the monopolist cannot change. However, in the real world, demand – or equivalently, quantity – may not be observable or verifiable.

2.2 Risk Averse Monopolists without Commitment

Buying a derivatives contract allows the monopolist to transfer wealth from high-price states to low-price states. In a world with demand shocks – when high profits coincide with high prices – this is exactly what a risk-averse monopolist would like to do. However, doing so comes at the cost of reduced monopoly profits. In the rest of this section, we assume for compactness that the monopolist faces demand shocks. Put another way, good states have both high prices and high profits, while bad states have both low prices and low profits.[4]

Proposition 2

[4] Note that this assumption rules out supply shocks where increased prices correspond to lower quantity and lower monopoly profits. Parallel results obtain when the monopolist faces supply shocks.

1. *A risk-averse monopolist participates in the derivatives market but gives up profits to do so. If the payoff of the derivatives contract is increasing (decreasing) in the spot price, the monopolist goes short (long), i.e.* $C^* < 0$ *(i.e.* $C^* > 0$*). She sells a larger total quantity in the spot market, receives a lower price, and makes lower expected profits than if she did not participate in the derivatives market.*

2. *The more risk-averse a monopolist is, the more derivatives contracts she goes short (long) when the payoff of the derivatives contract is increasing (decreasing) in the spot price.*

Proof. See Appendix A.2. ■

When the monopolist takes a position in the derivatives market, she has an incentive to deviate from the spot market optimum to make that position more profitable, but at the expense of her expected spot market profits. If the derivatives market is competitive and if the monopolist's derivatives market position is perfectly observable, then market makers will set prices so that the monopolist earns zero expected profit in the derivatives market. Using the derivatives market to reduce risk is costly since it reduces expected profits in the spot market. The monopolist faces a trade-off between reducing risk and maintaining monopoly profits. As the monopolist becomes more risk averse, risk reduction becomes relatively more important.

2.3 Complete Hedging is Impossible

While hedging increases with risk-aversion, an infinitely risk-averse monopolist does not eliminate all risk. Complete hedging is impossible even when the monopolist can choose among all price-contingent derivatives contracts.

Proposition 3 *There exists no price-contingent derivatives contract that can eliminate all risk.*

Proof. See Appendix A.3. ■

Any price-contingent contract that eliminates all risk must pay out relatively more when prices indicate a "bad" state, exactly offsetting any reduced spot market profits in these states. As a result, the monopolist will always have an incentive to set prices as if a "bad" state had occurred, even in "good" states. Since spot market profits are higher in better states – holding prices fixed – and derivatives contract payoffs must be the same in all states with the same price, the monopolist will earn higher profits in "good" states. There is no incentive compatible contract that eliminates all risk.

Illustrative Example. Consider a negative exponential demand function with a binomial demand shock. Since there are only two states of the world, a futures contract is sufficient to span the set of price-contingent contracts. The demand function is given by

$$P = f(Q, D) = De^{-bQ}, \tag{7}$$

where the realization of demand, D, takes values $D^H > D^L$ with probabilities p^H and $p^L = 1 - p^H$, respectively. The payoff of each futures contract is $g(P) = P - E[P|C]$. Substituting equation (7) into equation (1) yields

$$\pi = C\left(De^{-bQ} - E\left[De^{-bQ}|C\right]\right) + DQe^{-bQ}, \tag{8}$$

which implies

$$Q^* = \frac{1}{b} - C;\; P^* = De^{-1+bC}. \tag{9}$$

Therefore, profits given optimal spot market production can be found by substituting equation (9) into equation (8):

$$\pi = \left(\frac{D}{b} - CE[D]\right)e^{-1+bC}. \tag{10}$$

Note that profits are increasing in D regardless of the value of C so that risk cannot be eliminated completely. A quantity C can be chosen to maximize profits in a given state of the world. An infinitely risk-averse monopolist will maximize profits in the worst state and thus set

$$C = -\frac{1}{b}\left(1 - \frac{D^L}{E[D]}\right).$$

An infinitely risk-seeking monopolist will maximize profits in the best state and therefore choose

$$C = -\frac{1}{b}\left(1 - \frac{D^H}{E[D]}\right),$$

while a risk-neutral monopolist sets $C = 0$. Note that any $C \notin \left[-\frac{1}{b}\left(1 - \frac{D^L}{E[D]}\right), -\frac{1}{b}\left(1 - \frac{D^H}{E[D]}\right)\right]$ is state-wise dominated. Outside of this range the monopolist can increase profits in both states of the world by bringing C closer to zero.

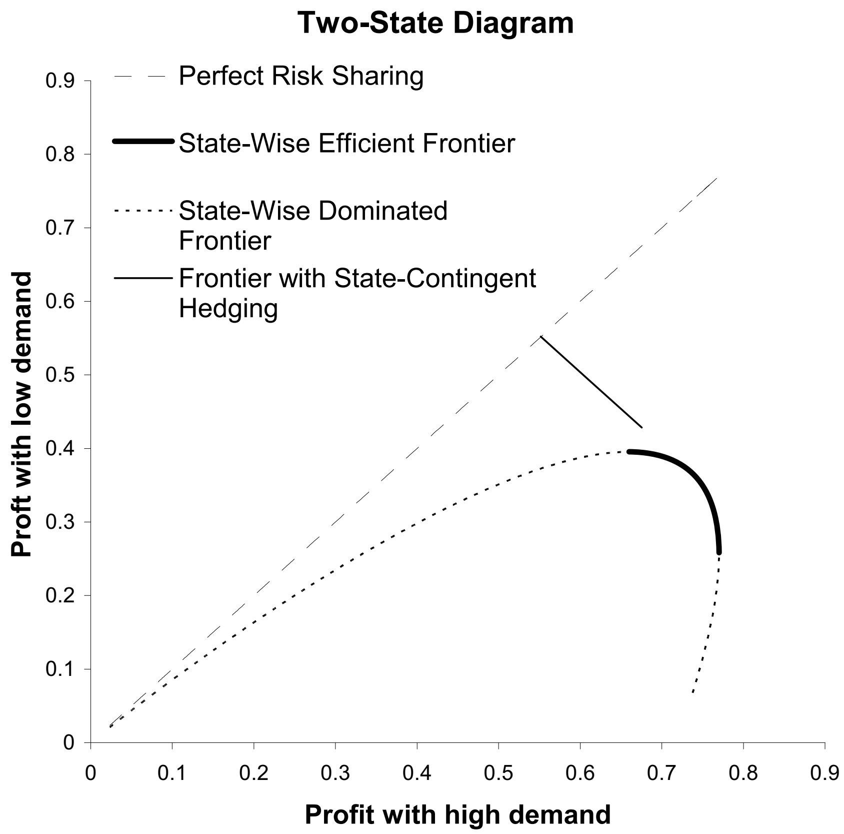

Figure 1 plots profits in the L-state with low demand on the y-axis against profits in H-state with high demand on the x-axis for the case when $b = 1$, $D^H = 2$, $D^L = 1$, and $p^H = p^L = 0.5$. Any point on the 45° dashed line represents a situation with no risk. The thin solid line (tangent to the curved line) represents the wealth levels in the two states if the monopolist can write state-contingent contracts. The slope of this line is the rate at which she can transfer wealth from the bad state to the good state; it is constant and equal to $-\frac{p^H}{1-p^H} = -1$. When only price-contingent contracts are possible, the rate at which wealth can be transferred – the slope – depends on the amount of wealth transferred between the two states. Put another way, the marginal cost of additional hedging increases with the amount being hedged. This feature is represented by the curved line. Note that any hedging quantities outside the range between $-\frac{1}{3}$ and $\frac{1}{3}$ are state-wise dominated, i.e. the cost of wealth transfer is infinite outside this range. An infinitely risk-averse monopolist would thus choose $C = -\frac{1}{3}$, an infinitely risk-seeking monopolist would choose $C = \frac{1}{3}$, and a risk-neutral monopolist would choose $C = 0$.

3 Conclusions

In this paper, we examined the trade-off a spot market monopolist faces between reducing risk and maintaining monopoly profits. Participation in the derivatives market creates a moral hazard problem in the spot market that makes mitigating risk expensive. While the reduction of risk is increasing in the degree for risk aversion, eliminating all risk is not incentive-compatible.

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[2] Milgrom, P., and N. Stokey (1998): "Information, Trade and Common Knowledge," *Journal of Economic Theory*, 26(1), 17—27.

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A Appendix: Proofs

A.1 Proof of Proposition 1

Profits are given by $\pi = Cg(D) + QP$. The spot market FOC is given by $\frac{\partial \pi}{\partial Q} = Qf_1 + f = 0$. This FOC is identical to the one without derivatives market participation, $C = 0$. The monopolist therefore maintains full market power in the spot market. The FOC determines an optimal quantity, $Q^*(D)$, and a price, $f(Q^*(D), D)$. Assume the monopolist designs a derivatives contract with the following payoff:

$$g(D) = E[Q^*(D) f(Q^*(D), D)] - Q^*(D) f(Q^*(D), D).$$

In this case for $C = 1$, profits are constant so that risk is eliminated completely. Note that any perturbation to g reduces expected utility for the risk-averse monopolist.

A.2 Proof of Proposition 2

In this proof, we assume that the payoff of the derivatives contract is increasing in the spot price, i.e. $g' > 0$. The proof for a contract whose payoff is decreasing in the spot price is equivalent and parallel results obtain.

1. Evaluating the derivatives market FOC (6) at $C = 0$ yields

$$\frac{\partial E[u(\pi)]}{\partial C}|_{C=0} = Cov\left(g, u'(QP)\right).$$

Since the monopolist faces demand shocks high prices coincide with high profits. Formally, for any demand shock such that $\frac{\partial f(Q,D)}{\partial D} > 0$, we have $\frac{\partial \pi}{\partial D} > 0$. In this case, a shock to demand has the following impacts on marginal utility and on the payoff of the derivatives contract:

$$\begin{aligned} \frac{\partial u'(QP)}{\partial D} &= \frac{\partial \pi}{\partial D}|_{C=0} \cdot u''(QP) < 0; \\ \frac{\partial g(P)}{\partial D} &= \frac{\partial f(Q,D)}{\partial D} \cdot g'(P) > 0. \end{aligned}$$

These two effects imply $\frac{\partial E[u(\pi)]}{\partial C}|_{C=0} = Cov(g, u'(QP)) < 0$ and thus $C^* < 0$. Going short has the following impacts on spot market quantity and price:

$$\begin{aligned} \frac{\partial Q}{\partial C} &= -\frac{g' f_1}{(2 + Cf_1 g'') f_1 + (Cg' + Q) f_{11}} < 0; \\ \frac{\partial f}{\partial C} &= -\frac{g' (f_1)^2}{(2 + Cf_1 g'') f_1 + (Cg' + Q) f_{11}} > 0. \end{aligned}$$

Note that the spot market SOC implies that the denominators in both fractions are negative. Going short thus implies that the monopolist increases spot market production and receives lower spot prices. The derivative of expected profits with respect to C (see equation (5)) is $\frac{\partial}{\partial C} E[\pi] = -CE\left[\frac{dQ}{dC} f_1 g'\right] > 0$ for all $C < 0$. This implies that the monopolist's expected profits – given optimal participation in the derivatives market ($C^* < 0$) – are lower than if she did not participate.

2. We compare two monopolists with utility functions u and v where the monopolist with utility function v is more risk averse than the one with u. There exists an increasing, concave function h such that $v = h \circ u$. Let $C^{u*} < 0$ denote the optimal number of derivatives contracts for monopolist u. In this case, C^{u*} satisfies the FOC

$$\frac{\partial E\left[u\left(\pi\left(C\right)\right)\right]}{\partial C}|_{C=C^{u*}} = E\left[\frac{\partial \pi\left(C^{u*}\right)}{\partial C} u'\left(\pi\left(C^{u*}\right)\right)\right] = 0. \tag{11}$$

The first derivative of expected utility of monopolist v with respect to C evaluated at C^{u*} is

$$\begin{aligned}\frac{\partial E\left[v\left(\pi\left(C\right)\right)\right]}{\partial C}|_{C=C^{u*}} &= E\left[\frac{\partial \pi\left(C^{u*}\right)}{\partial C} v'\left(\pi\left(C^{u*}\right)\right)\right] \\ &= E\left[\frac{\partial \pi\left(C^{u*}\right)}{\partial C} h'\left(u\left(\pi\left(C^{u*}\right)\right)\right) u'\left(\pi\left(C^{u*}\right)\right)\right].\end{aligned}$$

Recall from equation (5) that

$$\frac{\partial \pi}{\partial C} = g - CE\left[\frac{dQ}{dC} f_1 g'\right], \tag{12}$$

and define

$$\bar{P} \equiv CE\left[\frac{dQ}{dC} f_1 g'\right]. \tag{13}$$

Note that $\frac{\partial \pi}{dC} > 0$ if and only if $g < \bar{P}$. Recall that for any demand shock such that $\frac{\partial f(Q,D)}{\partial D} > 0$, we have $\frac{\partial \pi}{\partial D} > 0$ which implies $\frac{\partial g(P)}{\partial D} = \frac{\partial f(Q,D)}{\partial D} g' > 0$. Let $\bar{D}$ be a level of demand for which $g\left(f\left(Q\left(\bar{D}\right), \bar{D}\right)\right) = \bar{P}$. As g is increasing in the level of demand and as FOC (11) implies that $\frac{\partial \pi}{dC}$ switches sign, there exists a unique level of demand $\bar{D}$ for which $g\left(f\left(Q\left(\bar{D}\right), \bar{D}\right)\right) = \bar{P}$. Therefore

$$\frac{\partial \pi\left(C, D\right)}{\partial C} > 0 \text{ if and only if } D > \bar{D}. \tag{14}$$

Furthermore, $\frac{\partial \pi}{\partial D} > 0$ implies that

$$\frac{\partial h'\left(u\left(\pi\left(C, D\right)\right)\right)}{\partial D} = h''\left(u\left(\pi\left(C, D\right)\right)\right) u'\left(\pi\left(C, D\right)\right) \frac{\partial \pi}{\partial D} < 0,$$

and therefore

$$h'\left(u\left(\pi\left(C, D\right)\right)\right) < h'\left(u\left(\pi\left(C, \bar{D}\right)\right)\right) \text{ if and only if } D > \bar{D}. \tag{15}$$

Equations (14) and (15) yield

$$\frac{\partial \pi\left(C, D\right)}{\partial C} h'\left(u\left(\pi\left(C, D\right)\right)\right) < \frac{\partial \pi\left(C, D\right)}{\partial C} h'\left(u\left(\pi\left(C, \bar{D}\right)\right)\right)$$

for all D. This implies

$$\frac{\partial E\left[v\left(\pi\left(C\right)\right)\right]}{\partial C}|_{C=C^{u*}} < h'\left(u\left(\pi\left(C,\bar{D}\right)\right)\right)E\left[\frac{\partial \pi\left(C^{u*}\right)}{\partial C}u'\left(\pi\left(C^{u*}\right)\right)\right].$$

FOC (11) implies that $\frac{\partial E[v(\pi(C))]}{\partial C}|_{C=C^{u*}} < 0$. If expected utility is concave in C, then $C^{v*} < C^{u*} < 0$.

A.3 Proof of Proposition 3

Let $Q\left(P,D\right)$ be the inverse demand function and suppose demand D is indexed such that $Q\left(P,D_i\right) < Q\left(P,D_j\right)$ if and only if $D_i < D_j$ for $P > 0$. For a demand realization D_i, profits are given by

$$\pi\left(D_i,P\right) = g\left(P\right) + Q\left(P,D_i\right)P.$$

We prove this proposition by contradiction. Suppose there exists a price-contingent contract $g\left(P\right)$ that eliminates all risk, which implies that

$$\pi\left(D_i,P^*\left(D_i\right)\right) = \pi\left(D_j,P^*\left(D_j\right)\right) \tag{16}$$

for any two demand realization D_i and D_j where $P^*\left(D_i\right)$ and $P^*\left(D_j\right)$ are the spot prices the monopolist sets optimally contingent on realized demand. The contract must be incentive compatible in the sense that

$$\pi\left(D_j,P^*\left(D_j\right)\right) \geq \pi\left(D_j,P^*\left(D_i\right)\right) \tag{17}$$

for all D_i and D_j. Assume that D_j is the realized level of demand but that the monopolist set prices equal to $P^*\left(D_i\right)$ for some $D_i < D_j$. Profits are then

$$\pi\left(D_j,P^*\left(D_i\right)\right) = g\left(P^*\left(D_i\right)\right) + Q\left(P^*\left(D_i\right),D_j\right)P^*\left(D_i\right).$$

Since $Q\left(P^*\left(D_i\right),D_i\right) < Q\left(P^*\left(D_i\right),D_j\right)$, we have $\pi\left(D_j,P^*\left(D_i\right)\right) > \pi\left(D_i,P^*\left(D_i\right)\right)$. Equation (16) then implies $\pi\left(D_j,P^*\left(D_i\right)\right) > \pi\left(D_j,P^*\left(D_j\right)\right)$. This violates the incentive compatibility constraint (17) as the monopolist gets a higher profit by pretending to be in demand state D_i when realized demand is D_j. Therefore, no derivatives contracts can exist that eliminate all risk.

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