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International Price Discovery in the Presence of Microstructure Noise*

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Abstract:
This paper addresses and resolves the issue of microstructure noise when measuring the relative importance of home and U.S. market in the price discovery process of Canadian interlisted stocks. In order to avoid large bounds for information shares, previous studies applying the Cholesky decomposition within the Hasbrouck (1995) framework had to rely on high frequency data. However, due to the considerable amount of microstructure noise inherent in return data at very high frequencies, these estimators are distorted. We offer a modified approach that identifies unique information shares based on distributional assumptions and thereby enables us to control for microstructure noise. Our results indicate that the role of the U.S. market in the price discovery process of Canadian interlisted stocks has been underestimated so far. Moreover, we suggest that rather than stock specific factors, market characteristics determine information shares.

JEL Classification: F3, G15

Keywords: International Cross-Listings, Market Microstructure Noise, Price Discovery.

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1 Introduction

According to Coffee (2002), increasing globalisation and improved technology will lead to a decay in the number of securities exchanges around the world. Small national exchanges will lose their share in trading to large international exchanges, which provide a more efficient trading environment. Carpentier and Suret (2007) examine this development for the Canadian stock exchanges with respect to the U.S. markets. They report a rapidly growing share of U.S. markets in trades of Canadian interlisted stocks, up to the point where interlisted stocks are absorbed by the foreign market and delisted on the home market. These developments foreshadow small national stock exchanges as markets for illiquid stocks that failed to attract investors on the large markets (Gaa, Lumkin, Ogrodnik, and Thurlow (2002)). Thus, within the context of international cross-listed stocks, it is of paramount interest for national stock exchanges to maintain not only their share in trading, but also to remain the dominant market in regard to the price discovery process. The competition among smaller national and the giant U.S. markets for the leadership in price discovery of interlisted stocks has therefore grown immensely and has stirred up a growing field of research.

Grammig, Melvin, and Schlag (2005) and Phylaktis and Korczak (2007) apply the Hasbrouck (1995) methodology to examine the share in price discovery (information share) of home and foreign market of interlisted stocks from various countries and find that the home market evolves as the dominant trading venue. According to their results trading on the NYSE to some extent contributes to price discovery, but mainly takes place to offset arbitrage opportunities. We argue that this evidence might be misleading, since it neglects some important features of high frequency stock return data, most importantly the presence of microstructure noise.

The methodological contribution of this paper is a modification of the Hasbrouck (1995) approach that yields unique information shares and is not distorted by microstructure noise. We connect two strands of financial research, namely studies concerned with international price discovery of cross-listed stocks[1] and those concerned with market microstructure noise and its impact on financial volatility estimators.[2] As outlined by these studies, return data of financial time series sampled at very high frequencies are subject to market microstructure noise arising from different sources, such as market frictions, the discreteness of price changes, properties of the trading mechanism or bid-ask bounces. Since the Hasbrouck (1995) information shares are determined

[1] Recent studies concerned with the price discovery process of interlisted stocks include Eun and Sabherwal (2003), Grammig, Melvin, and Schlag (2005), Hupperets and Menkveld (2002), and Phylaktis and Korczak (2007).

[2] Among other studies Bandi and Russell (2008), Hansen and Lunde (2005), and Aït-Sahalia, Mykland, and Zhang (2005) examine the link between microstructure noise and volatility estimators in financial time series.

by the decomposition of the efficient price variance, i.e. the variance of the common stochastic trend in a cointegrated system, they are affected by the presence of market microstructure noise. At lower frequencies, at which the microstructure noise problem is less severe, the applicability of the Hasbrouck (1995) methodology is limited. Since the Cholesky decomposition applied within this framework is not able to fully identify structural shocks in a cointegrated vector autoregressive system, the methodology delivers merely upper and lower bounds for information shares and empirical analysts face a dilemma: on the one hand, in the case of relatively low frequencies, such as a few minutes, the bounds diverge considerably due to the increasing contemporaneous correlation inherent in innovations of the price series. Consequently, the commonly reported midpoint as a proxy for the true information share is rather unreliable. On the other hand, using high frequency data the estimated information share is very likely to be distorted by the substantial amount of microstructure noise in the data.

Yet, the modification of the Hasbrouck (1995) approach presented in this paper yields unique information shares and thus allows a reduction of the amount of microstructure noise by choosing a lower sampling frequency. Our methodology is based on a recent contribution by Lanne and Luetkepohl (2005), who propose to rely on distributional assumptions for identification of structural shocks in a VAR framework. It is particularly appealing within the context of international cross-listed stocks, as stock return data generally exhibit a leptokurtic distribution and the application of a mixture normal distribution seems appropriate to model non-normal price innovations. Moreover, rather than arbitrarily choosing a sampling frequency as done by previous studies, we conduct preliminary analysis of realized volatility to determine the appropriate sampling frequency. As a result, we deliver information shares, which are much more accurate, since they are neither approximated by midpoints of possibly extremely large bounds, nor are they based on data containing a substantial amount of microstructure noise.

We empirically examine Canadian stocks, which are traded on the Toronto Stock Exchange (TSE) and cross-listed on the New York Stock Exchange (NYSE). Our results show that the role of the NYSE within the price discovery process is underestimated by standard methods. We also find a much smaller cross-sectional variation of information shares among our sample stocks than detected by previous studies, indicating that a market's contribution to the price discovery process is determined by market characteristics rather than stock specific factors. Thus, improving market efficiency seems to be the major key for national stock exchanges to maintain their dominance in the price discovery process of interlisted stocks.

The remainder of the paper is organized as follows: section 2 outlines the main features and caveats of standard methods and discusses the issue of microstructure noise within the concept of price discovery in internationally cross-listed stocks. We also show simulation evidence on the bias in information shares induced by microstructure noise. Section three describes the data and sampling details. Section four explains the methodological details on our modified approach, which uses distributional assumptions for identification. Section five presents the empirical results and their discussion, while section six concludes the paper.

2 Price Discovery, Information Shares, and Microstructure Noise

2.1 Standard Methods and Their Drawbacks

In the current literature concerned with international price discovery there exist two prevalent methodologies. A number of studies, including Eun and Sabherwal (2003) and Phylaktis and Korczak (2007), measures a market's contribution to the price discovery process by the common factor component weight as proposed by Gonzalo and Granger (1995). Within this approach, price discovery is regarded as a pure error correction process, and a market's information share is based on its relative adjustment to deviations from the equilibrium. The Gonzalo and Granger (1995) (GG) methodology has been criticized on the grounds that within their model the efficient price does not necessarily follow a martingale and therefore questions the economic relevance of the model (Hasbrouck (2002), Baillie, Geoffrey, Tse, and Zabotina (2002), De Jong (2002)). The second methodology was developed by Hasbrouck (1995) and decomposes the variance of the common stochastic trend in a cointegrated system. Information shares are then defined as the relative contribution of an innovation in each price series to the variance of the common stochastic trend. Thereby, the dynamics of the system, which are neglected by the Gonzalo and Granger (1995) approach, are taken into account.

The Hasbrouck (1995) information shares are based on the concept of one efficient price, which implies that log prices of stocks traded simultaneously on home and foreign market are cointegrated with cointegration vector $\beta = (1, -1)'$. Deviations from the underlying efficient price are only transitory and offset by arbitrage. The evolution of home (p^h) and foreign (p^f) log price is described by a bivariate Vector Error Correction Model (VECM):

$$\Delta y_t = \alpha\beta' y_{t-1} + \Gamma_1 \Delta y_{t-1} + \ldots + \Gamma_{p-1}\Delta y_{t-p+1} + u_t, \tag{1}$$

where $y_t = (p_t^h, p_t^f)'$. Γ_1 to Γ_{p-1} are 2×2 parameter matrices. The vector $\alpha = (\alpha_h, \alpha_f)'$ contains the coefficients associated with the speed of adjustemt of each price series to deviations from the equilibrium and $\alpha\beta' y_{t-1}$ is usually referred to as the error correction term. Within the traditional framework u_t is vector white noise with zero mean and nonsingular covariance matrix Σ_u.

Within the error correction (GG) approach, the contribution of each market to price discovery is measured by the relative adjustment of the market to deviations from the equilibrium price.

$$Adj_h = \frac{\alpha_h}{\alpha_h + |\alpha_f|} \tag{2}$$

$$Adj_f = \frac{|\alpha_f|}{\alpha_h + |\alpha_f|}. \tag{3}$$

The idea is that the market that adjusts less evolves as the market leading the price discovery process.

In order to derive the Hasbrouck information shares, structural shocks in the system have to be identified. These structural shocks are assumed to be uncorrelated zero mean, unit variance random variables $\varepsilon_t \tilde{(} 0, I_2)$ which are related to the reduced form innovations u_t by

$$u_t = B\varepsilon_t. \tag{4}$$

The covariance matrix of the reduced form innovations Σ_u is given by $\Sigma_u = BB'$ and the long-run effects of structural shocks in a cointegrated system can be calculated as ΞB with

$$\Xi = \beta_\perp [\alpha_\perp' (I_K - \sum_{i=1}^{p-1} \Gamma_i) \beta_\perp]^{-1} \alpha_\perp, \tag{5}$$

where $\Xi = \begin{pmatrix} \xi^h & \xi^f \\ \xi^h & \xi^f \end{pmatrix}$. $\alpha_\perp$ and $\beta_\perp$ represent the orthogonal complements of α and β (see Johansen (1995)). Since Σ_u cannot be assumed to be diagonal, i.e., the reduced form innovations are contemporaneously correlated, the matrix B is underidentified. Within the traditional Hasbrouck (1995) approach this lack of identification is resolved by the Cholesky factorization of Σ_u. Defining $B = C$, where C denotes the lower triangular matrix derived from the Cholesky decomposition, structural shocks can be identified. The triangular structure of C implies that the ordering of the variables (i.e. markets) within the system is crucial. According to Hasbrouck (1995), the information share of market ordered $j^t h$ place within the Cholesky ordering is given by

$$IS_j = \frac{[\Xi C]_j^2}{[\Xi \Sigma_u \Xi]}. \tag{6}$$

Due to the arbitrary ordering of markets in the Cholesky decomposition the information shares are not unique. Commonly, the system is re-estimated with permutating order of the variables, which yields upper and lower bounds for each information share. The average between these bounds is used as a proxy for the true information share. Yet, these bounds can diverge considerably, as the contemporaneous correlation in the innovations tends to increase with a decreasing sampling frequency. [3]

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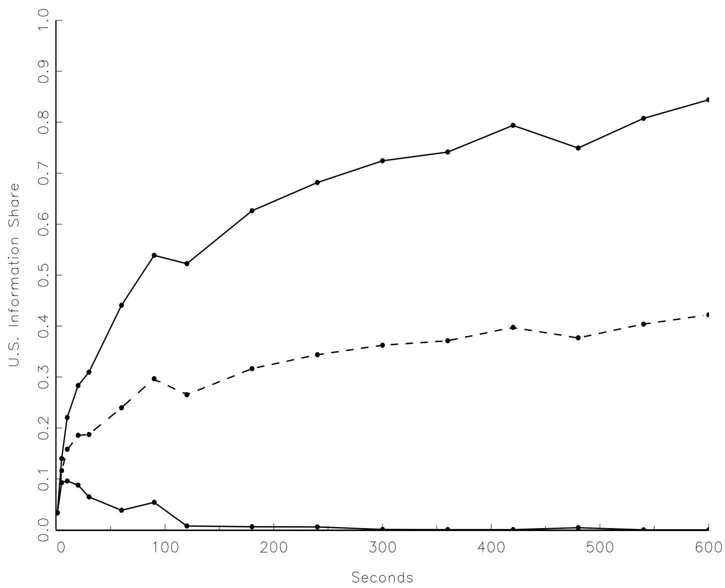

Figure 1: Estimated information shares for different frequencies
The solid lines denote the upper and lower bound for the NYSE information share of ABY as well as the midpoint (dotted line) for increasing sampling frequency.

To illustrate the dependence of the Hasbrouck (1995) information shares on the sampling frequency, Figure 2.1 displays the upper and lower bound of the U.S. market information share as well as the associated midpoint for the Canadian NYSE interlisted stock *Abitibi Consolidated Inc.* (ABY), estimated at different frequencies.[4] As clearly

[3]Using a still relatively high frequency of one minute Phylaktis and Korczak (2007) find a sample mean of 6.4% for the lower and 24.6% for the upper bound for their 64 sample stocks. However, sampling at intervals of 5 minutes as in the case Hupperets and Menkveld (2002) already leads to extremely large bounds for most of their Dutch stocks

[4]Details on the data can be found in Section 4.

depicted by the graph the bounds diverge considerably as the sampling frequency decreases. At low sampling frequencies the average over the bounds, converges to 0.5, i.e., to the point, where price discovery is divided evenly between the markets.
In order to avoid divergence of the information share bounds Grammig, Melvin, and Schlag (2005) sampled their data of British, Canadian, French and German stocks cross-listed on the NYSE at a high frequency of 10 seconds. However, they already mention the issue of potential bias arising from market microstructure noise at such fine frequencies.

2.2 Microstructure Noise in the Context of Price Discovery

As outlined by recent studies, including Bandi and Russell (2008), Hansen and Lunde (2005), and Aït-Sahalia, Mykland, and Zhang (2005) microstructure noise can induce a considerable bias of financial volatility estimator, if the data is sampled at a high frequency. Since the Hasbrouck information shares are based on the decomposition of the variance of the common stochastic trend, they are likely to be distorted as well. However, microstructure noise is mainly an issue at high frequency data. The crucial point lies in the accumulation of the noise component when the number of sampling intervals increases.

The observed log price in home and foreign market is assumed to consist of two components: the efficient underlying price and microstructure noise part that arises from different sources such as market frictions, bid-ask bounces or properties of the trading mechanism:

$$\tilde{p}_{ji} = p_{ji} + \eta_{ji}, \tag{7}$$

where p_{ji} denotes the unobservable efficient log price in period j on day i and η_{ji} gives the log microstructure noise component. Calculating continuously compounded intra-day log returns results in

$$\tilde{r}_{ji} = r_{ji} + \varepsilon_{ji}, \tag{8}$$

where $\tilde{r}_{ji}$ denotes the return in period j of day i which evolves as the sum of the efficient return r_{ji} and the microstructure noise component ε_{ji}.
The distortion of the estimated variance induced by microstructure noise at high frequencies becomes obvious when we construct the realized variance estimator that is

simply the sum of squared observed log returns:

$$\widehat{RV}_i = \sum_{j=1}^{M} \tilde{r}_{ji}^2 = \sum_{j=1}^{M} r_{ji}^2 + \sum_{j=1}^{M} \varepsilon_{ji}^2 + 2\sum_{j=1}^{M} r_{ji}^2 \varepsilon_{ji}^2. \tag{9}$$

While at very low frequencies the accumulated noise component, the second term on the right hand side in Equation (9), is still negligible, it dominates convergence of the estimator at very high frequencies, i.e. with an increasing number of sampling intervals M. The accumulation of microstructure noise and the bias it induces can be illustrated in a volatility signature plot, in which the realized volatility estimator is calculated for a range of frequencies. Figure 2.2 shows the volatility signature plot of home and U.S.

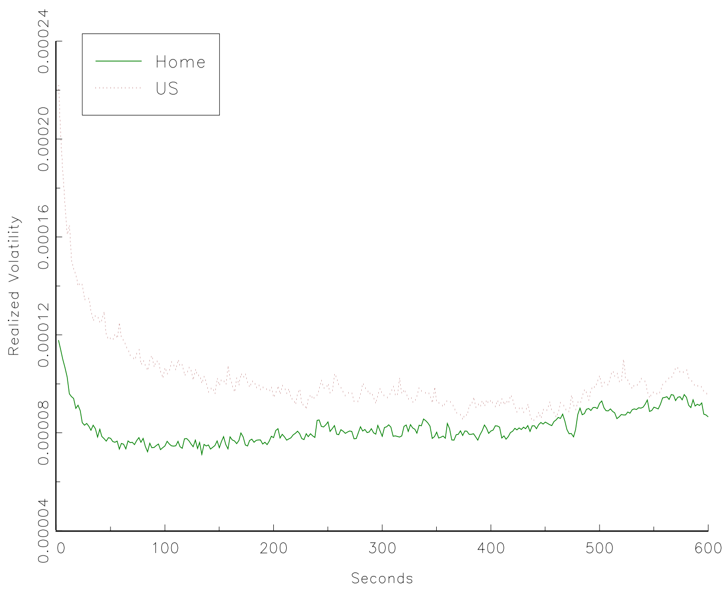

Figure 2: Volatility signature plots of TSE and NYSE log returns for ABY
The graph shows the realized variance estimator $\widehat{RV}_i = \sum_{j=1}^{M} \tilde{r}_{ji}^2 = \sum_{j=1}^{M} r_{ji}^2 + \sum_{j=1}^{M} \varepsilon_{ji}^2 + 2\sum_{j=1}^{M} r_{ji}^2 \varepsilon_{ji}^2$ for log home and foreign market returns one of our sample stocks (ABY) calculated for a range of frequencies. It clearly depicts the increasing upward bias in the estimator induced by microstructure noise.

market return for our sample stock ABY. The graph clearly depicts the increasing bias in the variance estimator at extremely fine sampling frequencies. At frequencies higher than two minutes, the realized variance estimator increases sharply, indicating that the data contains a considerable amount of microstructure noise. Therefore, in order to avoid distortion, a lower sampling frequency should be chosen, at which the noise component does not dominate the underlying efficient return variance. Since sampling at lower frequencies means a loss of observations, which increases the variance of the estimator, the appropriate sampling frequency entails a trade-off between the distortion by noise and the efficiency of the estimator. According to the volatility signature plot of ABY, a two minute sampling frequencies is a good choice to reduce the contamination by microstructure noise without discarding two much observations. We conjecture that the same arguments apply for the estimation of information shares, as we argue that these are affected by microstructure noise as well.

In order to study this effect of microstructure noise on information share estimates we conduct a Monte Carlo study. The idea is to simulate the true price discovery process in home and foreign market using a parameterized version of the bivariate error correction system in Equation (1) and then distort the true prices with microstructure noise. In the basic experimental design we assume symmetry of home and foreign market. This means that the adjustment coefficients are given by $\alpha_h = -\alpha_f$ and that the short run parameter matrices Γ_j (j=1,2,..., p-1) are symmetric. Parameter values are chosen to match typical numbers found in our sample. The fundamental innovations ϵ_t^h and ϵ_t^f are normally distributed with zero mean and identical standard deviation σ_ϵ and contemporaneously uncorrelated. We choose $\sigma_\epsilon = 0.0002$ which implies an annualized log return standard deviation of 20% and a sampling frequency of 10 seconds.[5] This data generating process ensures a 50% Hasbrouck information share of home and foreign market. We also study two other experimental setups. The *asymmetric* design assumes a 30:70 distribution of home and foreign market information share. The *monopolistic* setup implies that 100% of price discovery takes place in the foreign market. In each design we generate true home and foreign market price series with 100,000 observations and then add independent microstructure noise as suggested by Equation (7). Denote by p_t^h the simulated true home market log price and by p_t^f the simulated true foreign market log price. "Observed" log prices are then generated by

$$\begin{aligned} \tilde{p}_t^h &= p_t^h + \eta_t^h \\ \tilde{p}_t^f &= p_t^f + \eta_t^f . \end{aligned} \qquad (10)$$

The microstructure noise components η_t^h and η_t^f are drawn from independent zero mean normal distributions with variances $\sigma_{\eta^h}^2$ and $\sigma_{\eta^f}^2$, respectively. Along with the noise-free reference case, we consider seven scenarios in which we vary the variances of the microstructure noise components. In scenarios one, two and three, only the foreign market is subject to microstructure noise. In scenarios four to seven, microstructure noise affects prices in both markets. While the variances of home and foreign noise

[5] With 265 trading days per year and 10 trading hours per day this implies a sampling frequency of 10 seconds since $\sqrt{265 \cdot 10 \cdot 360} \cdot \sigma_\epsilon \approx 0.2$

component are identical in scenario four, $\sigma^2_{\eta^f}$ exceeds $\sigma^2_{\eta^h}$ in scenarios five, six and seven[6]. Standard deviations of foreign and home microstructure noise components are multiples of the fundamental innovation standard deviation. In scenario six, for instance, we assume $\sigma_{\eta^h} = \sigma_\epsilon$ and $\sigma_{\eta^f} = 4\sigma_\epsilon$.

The simulation is replicated 500 times. In each replication the model parameters are estimated based on the true and noisy price series, and information shares are computed for base and noise scenarios. After each replication, the information shares are stored along with the estimates of adjustment coefficients, long run impact coefficients, and residual variances and correlations. Table 1 reports means and standard deviations of these statistics computed over 500 replications.

The first row in Table 1 reports the midpoint of upper and lower bound of the Hasbrouck information shares resulting from permuting home and foreign market in the Cholesky decomposition of the residual covariance matrix. In the base scenario, upper and lower bound are identical, since home and foreign market innovations are contemporaneously uncorrelated. However, in the microstructure noise scenarios, upper and lower bound of the information shares diverge. This is due to the fact that microstructure noise introduces spurious residual correlation (see last row of Table 1).

The conclusive evidence from Table 1 is that microstructure noise severely biases information share estimates. Consider, for instance, scenario 3 in which the home market is noise-free and the foreign market microstructure noise standard deviation is two times that of the fundamental innovation's standard deviation. The upper bound estimate of the foreign market information share is 24 %, less than half of its true value. Furthermore, the small standard errors indicate that microstructure noise does not increase the variance of the estimates but substantially biases them.

When both markets are subject to microstructure noise (scenarios four to seven), biased estimation shares result when the amount of microstructure noise differs between markets. In scenario 6, in which the difference between noise variances is biggest ($\sigma_{\eta^f} = 4\sigma_{\eta^h}$), the downward bias of the foreign market's information share is most pronounced. The estimated information share is only 12 %, less than one quarter of its true value.

The results show that microstructure noise biases all components of the Hasbrouck information share. In particular, the adjustment coefficient estimate of the noise-affected foreign market is always upward biased. As a consequence, the ratio $\alpha^f/(\alpha^f + |\alpha^h|)$ becomes severely biased away from its true value of 50 %. In scenario 6, we obtain a value equal to 89.2 %, indicating falsely that the foreign market adjusts much more to

[6]This setup is suggested by the volatility signature plot in Figure 2. The volatility signature plots for our sample of Canadian stocks suggest that the amount of microstructure noise differs between US market and TSE, and that it is typically higher in the US.

scenario	base	1	2	3	4	5	6	7
$\sigma_{\eta^h}/\sigma_{\eta^f}$	0/0	$0/0.5\sigma_\epsilon$	$0/\sigma_\epsilon$	$0/2\sigma_\epsilon$	$\sigma_\epsilon/\sigma_\epsilon$	$\sigma_\epsilon/2\sigma_\epsilon$	$\sigma_\epsilon/4\sigma_\epsilon$	$2\sigma_\epsilon/4\sigma_\epsilon$
IS^f (midpoint)	**50.0**	**45.7**	**36.3**	**19.9**	**50.0**	**30.4**	**12.0**	**23.7**
IS^f (lower)	50.0	43.8	32.7	16.2	44.3	24.9	8.9	18.6
	(0.91)	(0.86)	(0.75)	(0.51)	(0.76)	(0.53)	(0.32)	(0.41)
IS^f (upper)	50.0	47.6	39.9	23.6	55.7	36.0	15.1	28.9
	(0.92)	(0.87)	(0.82)	(0.63)	(0.75)	(0.64)	(0.44)	(0.49)
a^h	-0.20	-0.19	-0.17	-0.11	-0.29	-0.20	-0.09	-0.20
	(0.003)	(0.002)	(0.002)	(0.002)	(0.003)	(0.003)	(0.002)	(0.002)
a^f	0.20	0.24	0.33	0.55	0.29	0.50	0.77	0.68
	(0.002)	(0.003)	(0.003)	(0.005)	(0.003)	(0.004)	(0.005)	(0.005)
$a^f/(a^f+\|a^h\|)\times 100$	50.0	55.9	66.9	83.3	50.0	71.1	89.2	77.1
	(0.44)	(0.43)	(0.39)	(0.27)	(0.41)	(0.31)	(0.20)	(0.25)
ξ^h	0.53	0.53	0.57	0.66	0.34	0.45	0.55	0.42
	(0.005)	(0.005)	(0.004)	(0.004)	(0.003)	(0.002)	(0.002)	(0.002)
ξ^f	0.53	0.42	0.28	0.13	0.34	0.18	0.07	0.12
	(0.005)	(0.004)	(0.003)	(0.002)	(0.003)	(0.002)	(0.001)	(0.001)
$\sigma_\epsilon^h\times 1000$	0.20	0.20	0.20	0.21	0.31	0.31	0.32	0.50
	(0.000)	(0.000)	(0.000)	(0.000)	(0.001)	(0.001)	(0.001)	(0.001)
$\sigma_\epsilon^f\times 1000$	0.20	0.23	0.31	0.48	0.31	0.48	0.86	0.87
	(0.000)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.002)	(0.002)
$\rho_{\epsilon^f\epsilon^h}$	0.00	0.04	0.08	0.09	0.11	0.12	0.10	0.12
	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)

Table 1: Effects of microstructure noise on information share estimates. Symmetric design.

We simulate home and foreign market log prices p_t^h and p_t^f from a bivariate error correction model

$$\begin{pmatrix}\Delta p_t^h\\ \Delta p_t^f\end{pmatrix} = \begin{pmatrix}\alpha^h\\ \alpha^f\end{pmatrix}(p_{t-1}^h - p_{t-1}^f) + \Gamma_1\begin{pmatrix}\Delta p_{t-1}^h\\ \Delta p_{t-1}^f\end{pmatrix} + \Gamma_2\begin{pmatrix}\Delta p_{t-2}^h\\ \Delta p_{t-2}^f\end{pmatrix} + \begin{pmatrix}\epsilon_t^h\\ \epsilon_t^f\\ .\end{pmatrix}$$

Symmetry of home and foreign market is imposed by setting $\alpha_f = -\alpha_h = 0.2$, $\Gamma_1 = \begin{pmatrix}-0.05 & 0.1\\ 0.1 & -0.05\end{pmatrix}$ and $\Gamma_2 = \begin{pmatrix}-0.05 & 0.05\\ 0.05 & -0.05\end{pmatrix}$. The innovations ϵ^f and ϵ^f are contemporaneously and serially uncorrelated mean zero normally distributed random variables with standard deviation $\sigma_\epsilon = \sigma_\epsilon^f = \sigma_\epsilon^h = 0.0002$. According to Equation (5), the true long run impact of home and foreign market innovations is $\xi^h = \xi^f = 0.53$ and the true information share of the foreign market (IS^f) is 50 %. The simulated true prices are disturbed by additive independent microstructure noise, $\tilde{p}_t^h = p_t^h + \eta_t^h$ and $\tilde{p}_t^h = p_t^h + \eta_t^h$. The noise components η_t^h and η_t^h are mean zero uncorrelated random variables with standard deviations σ_{η^h} and σ_{η^h}. The second row shows how microstructure noise standard deviations σ_{η^h} and σ_{η^h} are varied as multiples of the fundamental innovation standard deviation σ_ϵ. The simulation is replicated 500 times with $n = 100,000$. In each replication the model parameters are estimated based on the true and noised price series. Foreign market information shares (IS^f) and price impact coefficients ξ^h and ξ^f are computed as outlined in Equations (2) and (6). The table reports mean and standard deviation (in parentheses) of the estimates computed over the 500 Monte Carlo replications. IS^f *(midpoint)* denotes the average of the upper and lower bound of the foreign market information share which result from permuting the order of home and foreign market in the Cholesky decomposition of the residual variance covariance matrix. The estimate of the correlation of ϵ^f and ϵ^f is reported in the last row.

a violation of the law of one price than the home market. The estimates of the long run impacts of the market innovations (ξ^h and ξ^f) also become biased by microstructure noise. The true long run impact of a one unit innovation in either market amounts to 0.53. However, in scenario 6 the estimate of the foreign market impact is reduced to 0.07.

The results for the alternative experimental designs reported in Tables A-1 and A-2 (see Appendix) confirm these findings. The market with limited or even zero contribution can well be falsely attributed a considerable amount of price discovery if the market which truly contributes to price discovery is subject to microstructure noise.

Estimating information shares of interlisted stocks using high frequency data thus can lead to wrong conclusions, especially if the amount of microstructure noise differs between home and foreign market. Given the different designs of international stock markets, such a scenario seems to be the rule rather than the exception. Consider a home market stock exchange which, by lowering operational costs and providing easy market access, succeeded in attracting liquidity suppliers who provide small spreads and ample depth. Microstructure noise in such a liquid market tends to be small. One may think of an European limit order book market like Euronext. Now consider a foreign market with an arcane trading technology involving specialists, order books hidden to the public, a trading crowd, huge quoted spreads, delays in reporting prices et cetera., in short, a market structure like NYSE in which we expect the observed prices to reflect a significant amount of microstructure noise. The contributions to price discovery of interlisted stocks in such an environment may be considerable. However, as a result of microstructure noise, the foreign market information share would be underestimated.[7]

The estimation of information shares within the traditional Hasbrouck (1995) approach at high frequencies is therefore misleading. As outlined in section 2.1 estimation at lower frequencies, which might reduce the bias due microstructure noise, also yields inaccurate results brought about by the non-uniqueness of the estimators as the bounds for information shares diverge considerably.

[7]The scenario of an operationally efficient and liquid home market, and a microstructure noised foreign market describes to some extend the situation faced by Grammig, Melvin, and Schlag (2005). They report - using data sampled at 10 second frequency - that the vast majority of price discovery of three German stocks takes place at the home market which was (in 1999) efficiently organized as an electronic limit order book. The NYSE, where the three stocks were interlisted, had only a marginal information share. The simulation results, however, suggest that the conclusion that the home market leads price discovery should be taken with a grain of salt.

3 Identifying and Estimating Unique Information Shares

We suggest a solution to the dilemma one faces within the traditional Hasbrouck approach by proposing an alternative identification method that yields unique information shares. As a result, it is applicable to data sampled at lower frequencies, at which the microstructure noise bias is less prevalent. Following Lanne and Luetkepohl (2005), we identify structural shocks in a cointegrated system by modelling the structural innovations as a mixture of two normal distributions. The commonly fat tailed and peaked distributions found in stock returns renders this assumption particularly plausible. Within this framework outliers in return distributions could be generated by a distribution which is associated with a higher volatility than those of the remaining observations, so that there are different regimes operating in the same sample period (Lanne and Luetkepohl (2005)).

Starting from the reduced form VAR of order p that can be derived from the VECM given in Equation (1)

$$A(L)y_t = u_t \tag{11}$$

with

$$A(L) = I_K - \alpha\beta' L - \Gamma_1 \Delta L - \ldots - \Gamma_{p-1}\Delta L^{p-1}. \tag{12}$$

Rather than modelling the structural shocks u_t as a linear combination of idiosyncratic standard normal innovations as in Equation (4) we now define

$$u_t = W w_t, \tag{13}$$

where W denotes a (2×2) non-singular parameter matrix. And as outlined above w_t is a mixture of two normal random vectors:

$$w_t = \begin{cases} e_{1t} \sim N(0, I_2) & \text{with probability } \gamma \\ e_{2t} \sim N(0, \Psi) & \text{with probability } 1-\gamma. \end{cases} \tag{14}$$

Ψ is a diagonal matrix, with positive elements ψ_1 and ψ_2. If $\psi_1 = 1$, the innovations in the home market price series would follow a normal distribution and if $\psi_1 = \psi_2 = 1$, innovations in both price series are normal. So the approach outlined in the previous section is actually a special case of the mixture model setup. γ denotes the mixture probability, so $0 < \gamma < 1$. The covariance matrix Σ_w is then given by $\gamma I_2 + (1-\gamma)\Psi$.

Since $u_t = B\varepsilon_t = Ww_t$ we have

$$\Sigma_u = W\Sigma_w W' = BB'. \tag{15}$$

The matrix B is then given by $B = W\Sigma_w^{-0.5}$. As Σ_w and W can be estimated, the matrix B containing the contemporaneous correlations can be derived and the system is locally identified by the distributional assumptions concerning w_t if and only if all elements ψ_j of Ψ are distinct (For a proof we refer to Lanne and Luetkepohl (2005)). Consequently, no prior restrictions concerning the ordering of the variables as with the Cholesky decomposition are necessary.
After identification of the matrix B unique information shares result as

$$IS_{ij} = \frac{[\Xi B]_{ij}^2}{\Xi BB'\Xi_{ii}}. \tag{16}$$

Parameter estimation can be conducted by Maximum Likelihood, since a mixture normal distribution is assumed for the innovations w_t for which the density is

$$\begin{aligned}\phi_w &= \gamma(2\pi)^{\frac{n}{2}} \exp(-\frac{1}{2}w_t w_t') \\ &\quad + (1-\gamma)(2\pi)^{\frac{n}{2}} \det(\Psi)^{-\frac{1}{2}} \exp(-\frac{1}{2}w_t'\Psi^{-1}w_t).\end{aligned}$$

For convenience constant terms are neglected. The conditional distribution of y_t is

$$\begin{aligned}f_{t-1}(y_t) &= \gamma \det W^{-1} \\ &\quad \times \exp(-\frac{1}{2}(A(L)y_t)'(WW')^{-1}(A(L)y_t)) \\ &\quad + (1-\gamma)\det \Psi^{-\frac{1}{2}} \det(W)^{-1} \\ &\quad \times \exp(-\frac{1}{2}(A(L)y_t)'(W\Psi W')^{-1}A(L)y_t).\end{aligned}$$

The log likelihood given by

$$\mathscr{L} = \sum_{t=1}^{T} log f_{t-1}(y_t) \tag{17}$$

can be maximized by standard nonlinear optimization algorithms.

4 Data and Sampling

The data includes bid and ask quotes for 56 Canadian stocks, which are traded on the Toronto Stock Exchange and on the New York Stock Exchange simulatneously. The sampling period ranges from 1st January to 31 of March 2004. The New York data are from the TAQ data set available at the NYSE. Toronto quote data were obtained from the Equity Trades and Quotes data set from the Toronto Stock Exchange. The data also includes indicative quotes for the intradaily exchange rate as posted by Reuters from Olsen Data in Zurich .

Although the trading times of the TSE and the NYSE coincidence, only the first two hours of trading (9.30 am to 11.30 am) are used for estimation in order to avoid diurnal effects (see Madhavan, Richardson, and Roomans (1997)). Further NYSE prices were converted to Canadian dollars. Results do not change substantially if the estimation is done with US dollar converted Canadian prices. We conducted preliminary analysis using volatility signature plots to determine the appropriate sampling frequency. A two minute sampling frequency seems to be the optimal trade-off between the bias induced by microstructure noise and efficiency of the volatility estimator. We also took care to exclude overnight returns from the data set.

In order underline the plausibility of the mixture normal assumption outlined in the previous section we examine Kernel density plots of our sample stocks. Figure 4 shows the Kernel density plots for an exemplary stock (ABY). It clearly depicts the leptokurtic distribution of the return data in both markets. We also conducted Jarque-Bera tests for all our sample stocks and find that the null of normal distributed returns is rejected at any common level of significance.

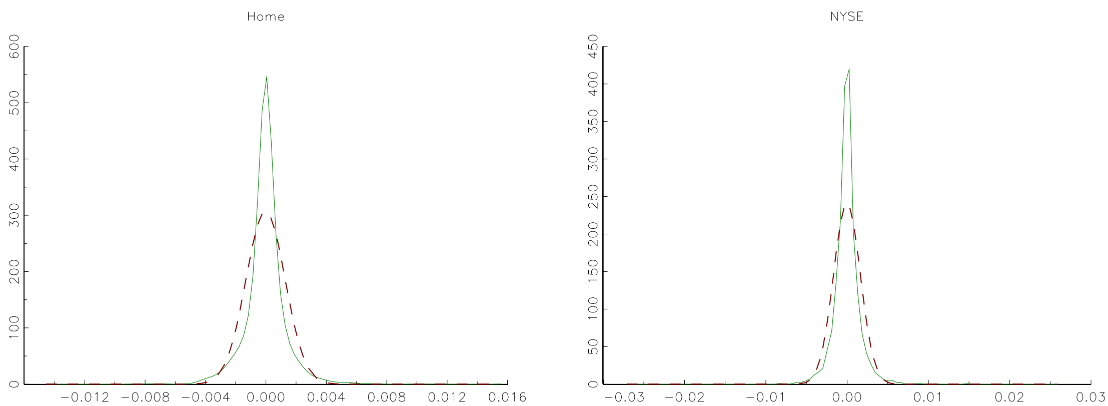

Figure 3: Kernel density plots of TSE and NYSE log returns of ABY

5 Results and Discussion

Within the bivariate VECM, we a priori estimate the vector of adjustment coefficients α with linear least squares. The optimal number of lags was chosen according to the Schwartz Criterion and ranges from one to five lags. We also test for the cointegration rank of the system by application of Johansen Trace and Maximum Eigenvalue statistics. Results clearly support the existence of one cointegration relation for all stocks. The normalized cointegration vector is fixed to $\beta = (1, -1)$. Yet, including the estimation of the cointegration parameters does not change our results. Conditioning on the pre-estimated parameter vectors α and β, the log-likelihood procedure given in the previous section evolves as quasi ML and is maximized for the remaining parameters.

Table 2 displays summary results for the modified Hasbrouck approach (more detailed results can be found in the Appendix Table A-3). Local identification of the model requires that the diagonal elements of Ψ are different. A Wald test statistic supports local identification for all of our sample stocks at any common level of significance. Moreover, the estimated ψ's are generally smaller than one, indicating that the price process evolves as a mixture of two regimes, where the second regime is characterized by a smaller variance. The estimates for γ vary considerably across the sample stocks, but overall tend to be smaller than 0.5, which leaves the regime associated with a lower variance as the less likely one. On average, the absolute value of the adjustment coefficient α_h is lower than the corresponding value on the foreign market indicating that a greater amount of error correction occurs on the U.S. market.

Turning to the estimated information shares, results concerning the modified Hasbrouck approach are displayed in the first two columns of Table 3. The last two columns show the results for the error correction approach as proposed by Gonzalo and Granger (1995) (stock specific information shares can be found in the Appendix Table A-4). Note that the interpretation of the GG coefficients is opposite to the Hasbrouck information shares: The more a market adjusts to deviations from the equilibrium price, the less it contributes to the price discovery process. The sample average of the relative adjustment that takes place on the NYSE amounts to 71% which indicates a clear leadership of the TSE with respect to price discovery.
These results correspond to previous studies by Eun and Sabherwal (2003) and Grammig, Melvin, and Schlag (2005), who also examine a set of Canadian interlisted stocks. Eun and Sabherwal (2003) apply the GG approach and report a sample average of 38% for the relative adjustment taking place on the TSE. With 29% for the relative home market adjustment, our results on the GG approach indicate that the leadership of the

	ψ^h	ψ^f	γ	α^h	α^f	ξ^h	ξ^f
5th Perc.	0.00	0.05	0.12	-0.40	0.09	0.29	-0.02
25th Perc.	0.02	0.15	0.24	-0.23	0.28	0.56	0.10
Median	0.03	0.22	0.33	-0.12	0.37	0.85	0.26
Mean	0.05	0.24	0.34	-0.15	0.38	0.80	0.32
75th Perc.	0.06	0.33	0.46	-0.04	0.48	1.03	0.55
95th Perc.	0.15	0.45	0.56	0.01	0.68	1.21	0.80

Table 2: Descriptive Statistics on Parameter Estimates of the Modified Hasbrouck Approach

ψ^h and ψ^f denote the diagonal elements matrix Ψ corresponding to home and foreign market innovations, γ gives the probability associated with the first regime. α^h and α^f give the adjustment coefficients of the home and foreign market return series and ξ^h and ξ^f denote the permanent impact of shocks on the home market and foreign market returns series, respectively.

	Modified Hasbrouck Approach		**Error Correction (GG) Approach**	
	TSE	*NYSE*	Adj_{TSE}	Adj_{NYSE}
5th Perc.	50.0	35.9	1.5	28.8
25th Perc.	52.6	42.4	7.3	51.6
Median	55.5	44.5	22.6	77.4
Mean	56.2	43.8	29.1	70.9
St. Dev.	7.1	7.1	24.7	24.7
75th Perc.	57.6	47.4	48.4	92.7
95th Perc.	64.1	50.0	71.2	98.5

Table 3: Descriptive Statistics on Information Shares

The modified Hasbrouck approach denotes the unique information shares identified through distributional assumptions. The results on the error correction approach (GG) are the relative adjustment coefficients based on the Gonzalo-Granger methodology of the home (TSE) and the foreign (NYSE) market: $Adj_{TSE} = \frac{\alpha_{TSE}}{\alpha_{TSE}+|\alpha_{NYSE}|}$ and $Adj_{NYSE} = \frac{|\alpha_{NYSE}|}{\alpha_{TSE}+|\alpha_{NYSE}|}$.

TSE became even more pronounced in recent years.
However, the GG approach remains questionable, neglecting the dynamics of the system. And considering the information shares derived by our modified approach, the picture changes: we find an average information share of 44% of the NYSE, which is well above the contribution detected by Eun and Sabherwal (2003). It also exceeds the average information share of 35% detected by Grammig, Melvin, and Schlag (2005), who apply the traditional Hasbrouck approach. Yet, they sampled their data at a very high frequency of 10 seconds. Their information shares are likely to be affected by microstructure noise.
Overall, our results show that the contribution of the NYSE to the price discovery process of Canadian interlisted stocks has been severly underestimated so far and the leadership of the home market is much less pronounced than indicated by previous studies.
Table 4 gives summary statistics for information shares derived with the traditional Hasbrouck approach (detailed results can be found in Table A-5 in the Appendix). It contains the lower and upper bounds as well as the midpoint for the estimated information shares. Although the effect of microstructure noise is alleviated at our sampling frequency of 2 minutes, results from the traditional Hasbrouck approach are rather unreliable: Considering the large deviation of the bounds - on average the difference between lower and upper bounds amounts to 65% - it becomes clear that midpoint as proxy for the true information share is a very inaccurate measure.

Another interesting results lies in the differences of estimated information shares among the sample stocks. We find that the cross-sectional variation of information shares using the modified Hasbrouck approach proposed in this paper, is much smaller than those detected by the traditional Hasbrouck and the GG approach. The cross-sectional standard deviation of information shares within the sample amounts to 24.7% for the GG information shares, 14.3% for the Hasbrouck approach. Compared to Grammig, Melvin, and Schlag (2005) who find a standard deviation of about 28% for their sample stocks, this finding once again illustrates the crucial effect of the sampling frequency within this framework, since averaging over large bounds acts as a smoothing factor.
Within our modified approach, we find a standard deviation of merely 7.1% for the unique information shares obtained by our modified Hasbrouck approach. This result is of paramount interest if considering possible determinants of a market's contribution to the price discovery process. So far only stock specific factors have been analysed. Yet, due to the small variation of information shares among the sample stocks, our results indicate that stock specific factors might be less important within the price discovery process as previously considered, but rather hint at a market's share in the

	TSE			NYSE		
	Low. Bound	Upp. Bound	Midpoint	Low. Bound	Upp. Bound	Midpoint
5th Perc.	4.2	66.6	36.6	0.0	43.9	22.2
25th Perc.	11.2	89.7	50.6	0.5	56.9	28.9
Median	23.1	98.1	58.3	1.9	76.9	41.7
Mean	27.3	92.2	59.8	7.8	72.7	40.2
St. Dev.	19.0	14.3	14.3	14.3	19.0	14.3
75th Perc.	43.1	99.5	71.1	10.3	88.8	49.4
95th Perc.	56.1	100.0	77.8	33.4	95.8	63.4

Table 4: Descriptive Statistics on Traditional Hasbrouck Information Shares
The table shows descriptive statistics on the lower and upper bounds of Hasbrouck information shares as well as on the associated midpoints.

price discovery process of interlisted stocks actually being a *market's* share. In other words market design seems to be a major determinant for information shares, implying that the trading venue providing the most efficient trading environment will take over the leadership in the price discovery process in the long run.

6 Concluding Remarks

This paper proposes a modification of the traditional Hasbrouck (1995) approach to measure a market's contribution to price discovery within the context of internationally cross-listed stocks. Our modified information shares are unique and in principle can be estimated on data sampled at any frequency. As a result the bias induced by market microstructure noise is reduced by sampling at lower frequencies. We use our modified approach and contrast it with the error correction approach proposed by Gonzalo and Granger (1995) and the traditional Hasbrouck approach. The error correction approach reveals the home market as the trading venue that clearly dominates price discovery. Within the traditional Hasbrouck approach, a clear detection of a market's contribution to price discovery is hampered due to the large bounds induced by our relatively low sampling frequency of two minutes.

The unique information shares derived from the modified approach proposed in this paper also detect a slight leadership of the home market, however, it clearly reveals that the contribution of the U.S. market price discovery process of Canadian interlisted stocks has been underestimated so far. According to our results price discovery is more evenly divided between home and foreign market. Moreover, we reveal that the variation of information shares among different stocks is much less pronounced than previously thought and detected by the traditional approach. This result is particularly interesting, since previous studies concerned with the factors that determine a market's contribution to price discovery, always considered stock specific factors such as the number of U.S. analysts following the stock, foreign sales or foreign investment as the most important determinants due to the large cross-sectional variation in information shares. In contrast, our results rather hint at factors common to all stocks, i.e. market specific factors, as the major determinant. As a result, it should be a prior incentive of the stock exchanges to improve market efficiency in order to ensure their share in the price discovery process of cross-listed stocks and attract further listings.

The applicability of the methodology presented in this paper is not limited to international cross-listed stocks. Figuerola-Ferrett and Gonzalo (2007) measure price discovery in commodity markets and Chakravarty, Gulen, and Mayhew (2004) use the Hasbrouck (1995) methodology to examine the relative contribution to price discovery of stock and options markets. Their results also suffer from the non-uniqueness of the traditional information shares and are also prone to microstructure noise so that our modified approach presents an appealing alternative.

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APPENDIX

scenario	base	1	2	3	4	5	6	7
$\sigma_{\eta^h}/\sigma_{\eta^f}$	0/0	$0/0.5\sigma_\epsilon$	$0/\sigma_\epsilon$	$0/2\sigma_\epsilon$	$\sigma_\epsilon/\sigma_\epsilon$	$\sigma_\epsilon/2\sigma_\epsilon$	$\sigma_\epsilon/4\sigma_\epsilon$	$2\sigma_\epsilon/4\sigma_\epsilon$
IS^f (midpoint)	**69.2**	**63.7**	**51.2**	**28.3**	**63.4**	**38.9**	**15.3**	**26.9**
IS^f (lower)	69.2	61.3	46.1	22.8	57.3	32.0	11.3	21.1
	(0.76)	(0.76)	(0.73)	(0.54)	(0.71)	(0.55)	(0.36)	(0.43)
IS^f (upper)	69.2	66.1	56.2	33.8	69.5	45.8	19.4	32.7
	(0.75)	(0.73)	(0.74)	(0.65)	(0.63)	(0.62)	(0.47)	(0.50)
a^h	-0.30	-0.29	-0.25	-0.16	-0.39	-0.26	-0.11	-0.23
	(0.003)	(0.003)	(0.003)	(0.002)	(0.003)	(0.003)	(0.002)	(0.003)
a^f	0.20	0.24	0.35	0.59	0.28	0.52	0.79	0.69
	(0.003)	(0.003)	(0.004)	(0.005)	(0.004)	(0.005)	(0.005)	(0.005)
$a^f/(a^f + \|a^h\|) \times 100$	40.0	46.1	58.8	79.1	42.4	66.6	87.6	75.5
	(0.41)	(0.40)	(0.38)	(0.27)	(0.39)	(0.31)	(0.20)	(0.25)
ξ^h	0.42	0.43	0.49	0.61	0.29	0.41	0.53	0.41
	(0.005)	(0.004)	(0.004)	(0.003)	(0.003)	(0.002)	(0.002)	(0.002)
ξ^f	0.63	0.51	0.34	0.16	0.39	0.21	0.08	0.13
	(0.005)	(0.004)	(0.003)	(0.002)	(0.003)	(0.002)	(0.001)	(0.001)
$\sigma_\epsilon^h \times 1000$	0.20	0.20	0.21	0.22	0.31	0.32	0.32	0.51
	(0.000)	(0.000)	(0.000)	(0.000)	(0.001)	(0.001)	(0.001)	(0.001)
$\sigma_\epsilon^f \times 1000$	0.20	0.23	0.30	0.48	0.31	0.48	0.85	0.87
	(0.000)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.002)	(0.002)
$\rho_{\epsilon^f \epsilon^h}$	0.00	0.05	0.10	0.12	0.13	0.14	0.11	0.13
	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)

Table A-1: Effects of microstructure noise on information share estimates. Asymmetric case.

We simulate home and foreign market log prices p_t^h and p_t^f from a bivariate error correction model

$$\begin{pmatrix} \Delta p_t^h \\ \Delta p_t^f \end{pmatrix} = \begin{pmatrix} \alpha^h \\ \alpha^f \end{pmatrix} (p_{t-1}^h - p_{t-1}^f) + \Gamma_1 \begin{pmatrix} \Delta p_{t-1}^h \\ \Delta p_{t-1}^f \end{pmatrix} + \Gamma_2 \begin{pmatrix} \Delta p_{t-2}^h \\ \Delta p_{t-2}^f \end{pmatrix} + \begin{pmatrix} \epsilon_t^h \\ \epsilon_t^f \end{pmatrix}$$

. A true information share of 70 % of foreign and 30% of home market is imposed by setting $\alpha_h = -0.3$ and $\alpha_f = 0.2$, $\Gamma_1 = \begin{pmatrix} -0.05 & 0.1 \\ 0.1 & -0.05 \end{pmatrix}$ and $\Gamma_2 = \begin{pmatrix} -0.05 & 0.05 \\ 0.05 & -0.05 \end{pmatrix}$. The innovations ϵ^f and ϵ^f are contemporaneously and serially uncorrelated mean zero normally distributed random variables with standard deviation $\sigma_\epsilon = \sigma_\epsilon^f = \sigma_\epsilon^h = 0.0002$.

scenario	base	1	2	3	4	5	6	7
$\sigma_{\eta^h}/\sigma_{\eta^f}$	0/0	$0/0.5\sigma_\epsilon$	$0/\sigma_\epsilon$	$0/2\sigma_\epsilon$	$\sigma_\epsilon/\sigma_\epsilon$	$\sigma_\epsilon/2\sigma_\epsilon$	$\sigma_\epsilon/4\sigma_\epsilon$	$2\sigma_\epsilon/4\sigma_\epsilon$
IS^f (midpoint)	**100.0**	**99.7**	**96.5**	**74.6**	**97.7**	**78.4**	**38.5**	**45.4**
IS^f (lower)	100.0	99.6	95.2	69.6	96.7	74.0	32.3	39.2
	(0.01)	(0.11)	(0.37)	(0.76)	(0.31)	(0.69)	(0.66)	(0.62)
IS^f (upper)	100.0	99.9	97.8	79.6	98.8	82.9	44.7	51.5
	(0.01)	(0.05)	(0.24)	(0.64)	(0.18)	(0.57)	(0.69)	(0.61)
a^h	-0.20	-0.20	-0.19	-0.16	-0.29	-0.24	-0.14	-0.23
	(0.002)	(0.002)	(0.002)	(0.001)	(0.002)	(0.002)	(0.001)	(0.002)
a^f	0.00	0.01	0.05	0.21	0.03	0.19	0.52	0.46
	(0.002)	(0.002)	(0.003)	(0.004)	(0.002)	(0.004)	(0.005)	(0.005)
$a^f/(a^f+\|a^h\|)\times 100$	0.7	3.6	19.4	57.2	10.5	43.7	78.4	66.5
	(0.49)	(0.93)	(0.88)	(0.51)	(0.68)	(0.52)	(0.27)	(0.31)
ξ^h	0.00	0.03	0.15	0.46	0.07	0.28	0.51	0.37
	(0.009)	(0.008)	(0.007)	(0.006)	(0.005)	(0.004)	(0.003)	(0.002)
ξ^f	1.05	0.87	0.63	0.34	0.62	0.36	0.14	0.19
	(0.009)	(0.008)	(0.006)	(0.004)	(0.005)	(0.003)	(0.002)	(0.002)
$\sigma^h_\epsilon \times 1000$	0.20	0.20	0.20	0.21	0.31	0.32	0.32	0.51
	(0.000)	(0.000)	(0.000)	(0.000)	(0.001)	(0.001)	(0.001)	(0.001)
$\sigma^f_\epsilon \times 1000$	0.20	0.24	0.32	0.51	0.32	0.51	0.90	0.91
	(0.000)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.002)	(0.002)
$\rho_{\epsilon^f\epsilon^h}$	0.00	0.03	0.07	0.12	0.07	0.11	0.13	0.12
	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)

Table A-2: Effects of microstructure noise on information share estimates. Monopolistic case.

We simulate home and foreign market log prices p_t^h and p_t^f from a bivariate error correction model

$$\begin{pmatrix}\Delta p_t^h\\ \Delta p_t^f\end{pmatrix} = \begin{pmatrix}\alpha^h\\ \alpha^f\end{pmatrix}(p_{t-1}^h - p_{t-1}^f) + \Gamma_1\begin{pmatrix}\Delta p_{t-1}^h\\ \Delta p_{t-1}^f\end{pmatrix} + \Gamma_2\begin{pmatrix}\Delta p_{t-2}^h\\ \Delta p_{t-2}^f\end{pmatrix} + \begin{pmatrix}\epsilon_t^h\\ \epsilon_t^f\end{pmatrix}$$

. A 100 % information share of the foreign market is imposed by setting $\alpha_h = -0.2$ and $\alpha_f = 0$, $\Gamma_1 = \begin{pmatrix}-0 & 0.1\\ 0 & -0.05\end{pmatrix}$ and $\Gamma_2 = \begin{pmatrix}0 & 0.05\\ 0 & -0.05\end{pmatrix}$. The innovations ϵ^f and ϵ^f are contemporaneously and serially uncorrelated mean zero normally distributed random variables with standard deviation $\sigma_\epsilon = \sigma_\epsilon^f = \sigma_\epsilon^h = 0.0002$.

	ψ^h	ψ^f	γ	ξ^h	ξ^f	α^h	α^f
ABX	0.15	0.45	0.43	0.67	0.29	-0.21	0.48
ABY	0.02	0.30	0.47	0.93	0.16	-0.08	0.49
AEM	0.08	0.36	0.48	0.57	0.58	-0.32	0.31
AGU	0.03	0.27	0.40	0.51	0.53	-0.22	0.21
AL	0.18	0.33	0.22	0.57	0.60	-0.39	0.37
BCE	0.05	0.47	0.48	1.09	0.14	-0.05	0.38
BCM	0.03	0.37	0.58	1.07	0.05	-0.03	0.67
BMO	0.08	0.17	0.22	1.04	0.08	-0.04	0.55
BNS	0.01	0.36	0.59	0.97	0.11	-0.08	0.71
BVF	0.09	0.20	0.27	0.74	0.56	-0.30	0.40
CJR	0.00	0.04	0.17	0.52	0.35	-0.06	0.10
CLS	0.11	0.08	0.12	0.91	0.44	-0.23	0.49
CNQ	0.04	0.28	0.43	1.11	0.20	-0.12	0.66
CP	0.02	0.30	0.47	1.01	0.09	-0.05	0.53
DTC	0.01	0.30	0.48	0.94	0.13	-0.05	0.32
ECA	0.10	0.39	0.33	0.85	0.41	-0.18	0.37
ERF	0.04	0.15	0.20	0.44	0.84	-0.39	0.20
FDG	0.02	0.09	0.34	0.54	0.62	-0.31	0.27
FFH	0.02	0.12	0.29	0.47	0.51	-0.27	0.24
FHR	0.04	0.26	0.30	0.43	0.59	-0.29	0.22
GG	0.13	0.49	0.39	0.53	0.56	-0.36	0.34
GLG	0.06	0.39	0.45	0.48	0.71	-0.42	0.28
HBG	0.01	0.05	0.21	0.22	0.84	-0.08	0.02
IDR	0.03	0.12	0.21	0.32	0.67	-0.32	0.16
IPS	0.00	0.15	0.34	0.79	0.15	-0.06	0.29
IQW	0.01	0.23	0.39	1.10	0.12	-0.04	0.35
KGC	0.15	0.35	0.31	0.63	0.51	-0.30	0.38
LAF	0.02	0.18	0.24	0.06	0.88	-0.15	0.01
MDG	0.08	0.32	0.30	0.41	0.62	-0.46	0.31
MDZ	0.00	0.21	0.46	1.02	0.04	-0.01	0.32
MGA	0.05	0.24	0.24	0.59	0.55	-0.23	0.25
MHM	0.00	0.21	0.33	0.85	0.12	-0.04	0.28
MIM	0.03	0.12	0.18	0.12	0.79	-0.48	0.07
N	0.12	0.22	0.19	1.02	0.18	-0.13	0.77
NCX	0.02	0.23	0.29	0.71	0.34	-0.16	0.33
NRD	0.03	0.17	0.56	1.16	0.07	-0.03	0.46
NXY	0.02	0.13	0.46	1.21	-0.04	0.01	0.47
PCZ	0.05	0.20	0.26	1.21	0.03	-0.02	0.69
PDG	0.16	0.31	0.26	0.76	0.22	-0.15	0.50
PDS	0.05	0.27	0.39	0.81	0.42	-0.16	0.31
PGH	0.04	0.03	0.12	0.71	0.48	-0.17	0.26
PKZ	0.05	0.10	0.16	0.95	0.55	-0.24	0.41
PWI	0.03	0.07	0.12	0.76	0.38	-0.21	0.43
RCN	0.01	0.13	0.27	0.54	0.54	-0.13	0.13
RY	0.02	0.38	0.60	1.08	-0.02	0.01	0.60
RYG	0.00	0.20	0.44	0.91	0.12	-0.05	0.37
SLF	0.06	0.35	0.40	1.01	-0.02	0.01	0.41
SU	0.07	0.21	0.30	1.08	0.17	-0.07	0.45
TAC	0.00	0.31	0.55	0.91	0.08	-0.03	0.37
TD	0.04	0.47	0.50	1.10	0.01	0.00	0.45
TEU	0.03	0.19	0.32	0.82	0.21	-0.09	0.34
TLM	0.02	0.44	0.50	1.01	0.29	-0.13	0.45
TOC	0.01	0.35	0.47	1.45	-0.08	0.03	0.53
TRP	0.05	0.02	0.03	1.24	0.03	-0.01	0.48
TU	0.00	0.10	0.50	1.18	-0.11	0.05	0.58
ZL	0.05	0.16	0.28	0.93	0.19	-0.08	0.38

Table A-3: Estimated Parameters of the Modified Hasbrouck Approach

ψ^h and ψ^f denote the diagonal elements matrix Ψ corresponding to home and foreign market innovations, γ gives the probability associated with the first regime. α^h and α^f give the adjustment coefficients of the home and foreign market return series and ξ^h and ξ^f denote the permanent impact of shocks on the home market and foreign market returns series, respectively.

	Modified Hasbrouck Approach		Error Correction (GG) Approach	
	TSE	*NYSE*	*TSE*	*NYSE*
ABX	49.8	50.2	30.5	69.5
ABY	55.8	44.2	17.3	82.7
AEM	49.5	50.5	52.3	47.7
AGU	54.9	45.1	50.1	49.9
AL	50.0	50.0	48.8	51.2
BCE	58.8	41.2	11.7	88.3
BCM	53.3	46.7	5.7	94.3
BMO	56.6	43.4	3.3	96.7
BNS	53.6	46.4	8.5	91.5
BVF	51.9	48.1	40.3	59.7
CJR	73.2	26.8	42.2	57.8
CLS	54.6	45.4	40.4	59.6
CNQ	52.1	47.9	33.8	66.2
CP	55.5	44.5	15.2	84.8
DTC	58.8	41.2	7.5	92.5
ECA	53.7	46.3	12.5	87.5
ERF	50.4	49.6	32.4	67.6
FDG	51.6	48.4	6.4	93.6
FFH	53.8	46.2	62.9	37.1
FHR	52.4	47.6	5.5	94.5
GG	50.0	50.0	53.8	46.2
GLG	51.0	49.0	53.1	46.9
HBG	59.3	40.7	59.9	40.1
IDR	56.7	43.3	95.5	4.5
IPS	58.1	41.9	51.5	48.5
IQW	58.2	41.8	59.1	40.9
KGC	53.0	47.0	81.4	18.6
LAF	73.0	27.0	67.7	32.3
MDG	50.2	49.8	9.8	90.2
MDZ	58.6	41.4	6.1	93.9
MGA	53.8	46.2	44.1	55.9
MHM	57.7	42.3	60.2	39.8
MIM	55.8	44.2	3.7	96.3
N	50.8	49.2	48.2	51.8
NCX	57.2	42.8	12.9	87.1
NRD	59.5	40.5	87.7	12.3
NXY	58.3	41.7	33.0	67.0
PCZ	55.1	44.9	8.9	91.1
PDG	51.2	48.8	0.3	99.7
PDS	53.1	46.9	1.2	98.8
PGH	96.9	3.1	22.6	77.4
PKZ	52.7	47.3	34.3	65.7
PWI	52.3	47.7	37.9	62.1
RCN	55.5	44.5	36.6	63.4
RY	56.0	44.0	38.0	62.0
RYG	57.1	42.9	3.1	96.9
SLF	55.6	44.4	1.6	98.4
SU	57.5	42.5	4.3	95.7
TAC	58.5	41.5	18.2	81.8
TD	57.0	43.0	8.5	91.5
TEU	54.6	45.4	0.6	99.4
TLM	52.7	47.3	19.8	80.2
TOC	55.9	44.1	22.5	77.5
TRP	55.7	44.3	5.0	95.0
TU	57.1	42.9	2.0	98.0
ZL	61.1	38.9	8.4	91.6

Table A-4: Estimated Information Shares

The modified Hasbrouck approach denotes the unique information shares identified through distributional assumptions. The results on the error correction approach (GG) are the relative adjustment coefficients based on the Gonzalo-Granger methodology.

	TSE			NYSE		
	Low. Bound	Upp. Bound	Midpoint	Low. Bound	Upp. Bound	Midpoint
ABX	11.5	97.3	54.4	2.7	88.5	45.6
ABY	43.7	98.3	71.0	1.7	56.3	29.0
AEM	6.3	91.8	49.0	8.2	93.7	51.0
AGU	13.1	86.4	49.8	13.6	86.9	50.2
AL	4.2	94.9	49.6	5.1	95.8	50.4
BCE	45.7	99.0	72.4	1.0	54.3	27.6
BCM	26.9	99.9	63.4	0.1	73.1	36.6
BMO	42.2	99.7	71.0	0.3	57.8	29.0
BNS	32.3	99.5	65.9	0.5	67.7	34.1
BVF	8.3	95.0	51.6	5.0	91.7	48.4
CJR	53.6	89.5	71.6	10.5	46.4	28.4
CLS	15.3	96.4	55.8	3.6	84.7	44.2
CNQ	18.2	99.4	58.8	0.6	81.8	41.2
CP	41.6	99.6	70.6	0.4	58.4	29.4
DTC	52.5	98.8	75.6	1.2	47.5	24.4
ECA	13.6	96.9	55.3	3.1	86.4	44.7
ERF	3.8	82.0	42.9	18.0	96.2	57.1
FDG	7.1	89.4	48.3	10.6	92.9	51.7
FFH	10.5	86.5	48.5	13.5	89.5	51.5
FHR	7.6	84.5	46.1	15.5	92.4	53.9
GG	6.1	91.7	48.9	8.3	93.9	51.1
GLG	4.2	89.6	46.9	10.4	95.8	53.1
HBG	6.4	53.6	30.0	46.4	93.6	70.0
IDR	8.5	67.1	37.8	32.9	91.5	62.2
IPS	39.7	98.6	69.1	1.4	60.3	30.9
IQW	47.8	99.4	73.6	0.6	52.2	26.4
KGC	14.3	89.7	52.0	10.3	85.7	48.0
LAF	6.9	11.8	9.3	88.2	93.1	90.7
MDG	4.4	88.1	46.2	11.9	95.6	53.8
MDZ	51.0	99.9	75.4	0.1	49.0	24.6
MGA	11.4	90.8	51.1	9.2	88.6	48.9
MHM	42.9	99.1	71.0	0.9	57.1	29.0
MIM	0.9	64.9	32.9	35.1	99.1	67.1
N	15.6	99.5	57.6	0.5	84.4	42.4
NCX	29.6	93.5	61.5	6.5	70.4	38.5
NRD	54.0	99.8	76.9	0.2	46.0	23.1
NXY	56.9	99.9	78.4	0.1	43.1	21.6
PCZ	31.8	100.0	65.9	0.0	68.2	34.1
PDG	15.5	98.4	56.9	1.6	84.5	43.1
PDS	17.8	94.8	56.3	5.2	82.2	43.7
PGH	31.4	83.6	57.5	16.4	68.6	42.5
PKZ	13.4	95.3	54.3	4.7	86.6	45.7
PWI	19.4	93.3	56.3	6.7	80.6	43.7
RCN	14.7	86.9	50.8	13.1	85.3	49.2
RY	44.6	100.0	72.3	0.0	55.4	27.7
RYG	35.1	99.4	67.3	0.6	64.9	32.7
SLF	39.2	100.0	69.6	0.0	60.8	30.4
SU	32.9	99.3	66.1	0.7	67.1	33.9
TAC	55.8	99.4	77.6	0.6	44.2	22.4
TD	45.2	100.0	72.6	0.0	54.8	27.4
TEU	27.8	98.1	62.9	1.9	72.2	37.1
TLM	17.3	98.5	57.9	1.5	82.7	42.1
TOC	42.8	99.9	71.3	0.1	57.2	28.7
TRP	71.6	99.9	85.7	0.1	28.4	14.3
TU	69.1	99.0	84.0	1.0	30.9	16.0
ZL	45.4	98.1	71.8	1.9	54.6	28.2

Table A-5: Traditional Hasbrouck Information Shares
The table shows the lower and upper bounds for Hasbrouck information shares as well as the associated midpoints.

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