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Jan Pieter Krahnen and Christian Wilde

Center for Financial Studies
Goethe-Universität Frankfurt ■ House of Finance
Grüneburgplatz 1 ■ 60323 Frankfurt ■ Deutschland

Telefon: +49 (0)69 798-30050
Fax: +49 (0)69 798-30077
http://www.ifk-cfs.de ■ E-Mail: info@ifk-cfs.de

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Jan

Prof. Dr. Jan Pieter Krahnen

Center for Financial Studies
Goethe-Universität ▪ House of Finance
Grüneburgplatz 1 ▪ 60323 Frankfurt am Main ▪ Deutschland

Telefon: +49 (0)69 798-30050
Fax: +49 (0)69 798-30077
http://www.ifk-cfs.de ▪ E-Mail: info@ifk-cfs.de

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CDOs and Systematic Risk: Why Bond Ratings are Inadequate*

Jan Pieter Krahnen[1] and Christian Wilde[2]

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Abstract:
This paper analyzes the risk properties of typical asset-backed securities (ABS), like CDOs or MBS, relying on a model with both macroeconomic and idiosyncratic components. The examined properties include expected loss, loss given default, and macro factor dependencies. Using a two-dimensional loss decomposition as a new metric, the risk properties of individual ABS tranches can directly be compared to those of corporate bonds, within and across rating classes. By applying Monte Carlo Simulation, we find that the risk properties of ABS differ significantly and systematically from those of straight bonds with the same rating. In particular, loss given default, the sensitivities to macroeconomic risk, and model risk differ greatly between instruments. Our findings have implications for understanding the credit crisis and for policy making. On an economic level, our analysis suggests a new explanation for the observed rating inflation in structured finance markets during the pre-crisis period 2004-2007. On a policy level, our findings call for a termination of the 'one-size-fits-all' approach to the rating methodology for fixed income instruments, requiring an own rating methodology for structured finance instruments.

JEL Classification: G21, G28

Keywords: Credit Risk, Risk Transfer, Systematic Risk

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[1] Goethe University Frankfurt and CFS, Correspondence: Goethe University, House of Finance, Grüneburgplatz 1 (PF H29), 60323 Frankfurt am Main, Germany, E-mail: krahnen@finance.uni-frankfurt.de, Phone: +49 69 798 33705.

[2] Goethe University Frankfurt, Correspondence: House of Finance, Grüneburgplatz 1 (PF H29), 60323 Frankfurt am Main, Germany, E-mail: wilde@finance.uni-frankfurt.de, Phone: +49 69 798 33706.

1 Introduction

The securitization of assets was widely seen as one of the big success stories in financial markets over the past 15 years, among them mortgage financing, corporate and retail loans, credit card debt, auto loans and many other financial assets. Aggregate market volume and bank profitability due to structured finance business grew rapidly. For the US, the share of securitized loans in all loans (including consumer loans and mortgage loans) rose from below 20% in the mid-1970 to more than 50% in 2005 (Brender and Pisani, 2009, p. 6). For example, in the market for subprime loans, volume expanded from \$100 billion in 1998 to \$1,200 billion in 2006, two thirds of which were securitized (op.cit. p. 80). Moreover, through securitization tranches, European financial institutions are reported to hold more than 30% of outstanding US subprime debt.

Since the onset of the credit crisis, in mid-2007, analysts, politicians and researchers grapple to understand why such a disaster was possible, in spite of then existing liquid and seemingly efficient markets. There were also a larger number of sophisticated investment bankers, fund managers and central bankers than ever before who were equally caught by surprise.

Due to the high level of complexity that characterizes structured finance instruments, investors are effectively barred from carrying out any serious due diligence exercise directly (see Gorton, 2008, and Brunnermeier, 2008, for a description of the complexity of structured instruments). Thus, delegated monitoring is a sine qua non in structured finance markets, and the major line of delegation in ABS markets relies on rating agencies.

Ratings are used almost universally by investors, bankers, supervisors, and regulators as the relevant risk metric. The familiarity of markets with these letter ratings has probably encouraged investors to add these instruments to their portfolios, and has helped to establish the market for various ABS products in the first place. While it was widely known that securitization notes represent tranches, rather than shares, of underlying asset portfolios, most investors were confident that letter ratings (e.g. AAA, AA, A and so on) had the same meaning for ABS notes and for corporate bonds. Consistent with this claim, we have not found a publication of any of the three leading agencies, explaining whether and how ABS ratings differ in content from those used in bond markets. Apparently, market participants, both investors and originators, tended to believe that their bond market experience and the rating methodology, known for a very long time, could safely be carried over to the markets for asset backed securities. In an analogous manner, supervisors put great emphasis on agency ratings when assessing the risk level of portfolios containing such asset backed instruments. The most prominent example is the external ratings based approach in the Basel 2 regulatory framework.

In this paper, we analyze the risk properties of ABS notes, by relying on a standard (plain vanilla) collateralized debt obligation (CDO). Do ABS instruments have specific stochastic properties, concerning expected loss, and therefore particular risk properties, distinct from those of corporate bonds? For instance, how sensitive is expected loss to changes in correlations among underlying assets? What is the effect of a downgrade in the underlying asset quality on expected loss, loss given default and default probability of a structured instrument? Furthermore, how is systematic risk allocated to the junior and senior layers of a structured transaction? Finally, how do the results obtained for structured finance products compare to the risks of bonds with the same rating?

We use a macro-factor model driving the performance of the underlying loan portfolio and consequently also the associated tranches. This allows us to trace the allocation of systematic risk to tranches. We propose a measure to estimate macro-factor dependency of the individual tranches. In particular, total portfolio losses are decomposed in a two-dimensional way to states and tranches, where states are defined as the macro factor taking values in certain ranges. Thus, using Monte Carlo simulations, this metric yields state- and tranche-dependent loss statistics. Our methodology also allows to analyze the impact of model risk, represented by differing risk properties of the underlying asset portfolio, and to delineate its effect on the risk properties of the instruments.

Our analysis generalizes to a wide class of structured finance instruments. Its results lead to a number of important insights. First, we conclude from the analysis that carrying over the rating methodology from corporate bonds to ABS instruments results in debt instruments having very distinct risk properties, despite showing the same rating. The distinctness of bond and ABS risk properties helps to explain the puzzling fact that many investors in ABS have lost large amounts of money during the current credit crisis, despite having invested in apparently low risk instruments. Second, our analysis can be used to formulate the requirements for an appropriate rating methodology for structured finance instruments.

Related papers treat the issue of risk allocation to tranches in the context of pricing studies. Coval et al. (2007), for example, find that investors in senior tranche notes are greatly underpaid for the (high) level of systematic risk inherent in these securities. They also consider junior tranches to be exposed primarily to diversifiable risk which, according to the authors, renders the common characterization of equity tranches as ”toxic waste” obsolete. Duffie (2007), on the other hand, claims that junior-tranche prices are vulnerable to macroeconomic performance, and Eckner (2007) finds the compensation per unit of expected loss for senior tranche investors to be much higher than that of junior tranche investors. Our findings are consistent with these empirical observations.

The rest of the paper is structured as follows: Section 2 describes how we model CDOs as a general class of securitization instruments. The model setup is presented and the implementation based on Monte Carlo simulation is discussed. Section 3 covers the risk allocation to the individual tranches, both with respect to the overall risk profile, tranche interdependencies, and systematic risk of tranches. In this section, we establish a base case, describe individual tranche characteristics, provide robustness checks, and compare the results for the tranches with the properties of straight bonds. Section 4 looks at various types of shocks to the reference portfolio and explores, how they impact the risk characteristics of tranches.

Section 5 concludes and discusses implications of the findings for investors, regulators, and rating agencies.

2 Modeling CDOs

This section focusses on analyzing the risk characteristics of CDOs and related financial instruments[1]. We construct a simple tool that allows us to portray the loss distribution of asset portfolios, and of any tranche that is derived from the same underlying portfolio.

2.1 Model Setup

We apply a firm-value model to capture the occurrence of obligor default. More precisely, we apply a structural one-factor correlated default model as in Vasicek (1987). The driving factor is a market factor, and company value is modeled as the interplay of the market factor and a company specific, idiosyncratic risk factor. This market model approach is the model of choice in most corporate finance applications. We model company value $V_{n,t}$ of each obligor $n \in 1, 2, ..., N$ at any time t before maturity as being driven by a generalized macroeconomic factor Y_t^M that is common to all securities, and an idiosyncratic component $\epsilon_{n,t}$:

$$V_{n,t} = \sqrt{\rho_n^M} Y_t^M + \sqrt{(1 - \rho_n^M)} \epsilon_{n,t} \tag{1}$$

with $Y_t^M \sim \Phi(0,1)$ and $\epsilon_{n,t} \sim \Phi(0,1)$, where $\Phi(0,1)$ denotes the standard normal distribution. Thereby, we obtain correlated asset values of obligors. In case the sensitivities $\sqrt{\rho_n^M}$ of firm values to the macroeconomic risk factor are the same for all obligors n, then ρ_n^M corresponds to the mutual correlation coefficient for all assets.

Obligor n is assumed to default if at any time t the value $V_{n,t}$ of its assets lies below the exogenously given default boundary D_n, i.e. $V_{n,t} < D_n$. $V_{n,t}$ is assumed to be normally distributed and is standardized such that $V_{n,t} \sim \Phi(0,1)$.

There is a simple relation linking every default boundary D_n to a particular default probability p_n:

$$D_n = \Phi^{-1}(p_n). \tag{2}$$

Usually, a fraction of the notional amount can be recovered in case of default. Let ψ_n denote the recovery rate and θ_n the exposure size of security n. Portfolio loss is given as the sum of individual loan losses. We define the portfolio loss rate PLR as the final value at maturity of portfolio loss divided by the final value of all promised payments until maturity:

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[1] Collateralized Debt Obligations (CDOs) are structured financial instruments that exhibit two basic features: the pooling of underlying financial claims, and their tranching into a set of bonds, differentiated by the degree of subordination. Thus, the tranches represent claims of different seniority on a reference portfolio.

$$PLR = \frac{\sum_{n=1}^{N} 1_{\{T_n > \tau_n\}} \cdot \theta_n \cdot (F_n \cdot (1 - \psi_n) \cdot \exp^{r(T_n - \tau_n)} + C_{n,\tau_n,T_n}}{\sum_{n=1}^{N} \theta_n \cdot (F_n + C_{n,0,T_n})}, \tag{3}$$

where $1_{\{T_n > \tau_n\}}$ is an indicator function taking the value one if security n defaults during its lifetime and zero otherwise. T_n represents maturity of security n, and t_n is the time of default. F_n denotes the redemption value and C_{n,s_n,t_n} represents the present value at time t_n of all coupon payments for security n paid in the time interval $[s_n, t_n]$. All payoffs are discounted with interest rate r.

The applied firm value model (Eq. 1) is suitable for a simulation exercise.

2.2 Model Implementation

In the implementation, we do not need to apply simplifying assumptions to determine the loss distribution of the underlying portfolio. Instead, we are able to fully profit from the Monte Carlo Simulation procedure. Analytical approaches often rely on limiting assumptions, e.g. that the portfolio is composed of an infinite number of securities with identical characteristics. Thus, analytical models to some extent may be suitable for sensitivity analyses, but Monte Carlo Simulation is more appropriate for real-world applications. All individual securities in the portfolio can be accounted for by their specific exposure size, recovery rate, default probability, and maturity. Furthermore, Monte Carlo Simulation allows to differentiate between obligors and individual securities. The occurrence of joint obligor defaults is modeled by accounting for the sensitivity of each individual obligor to the common factor.

The loss distribution is simulated in 5 steps: First, a realization of the macro factor is simulated until maturity. Subsequently, default scenarios are generated for all individual obligors in the portfolio. Default occurs, if the simulated firm value of an obligor, based on realizations of the macro factor and an idiosyncratic term, falls below the default boundary which is determined by the default probability of the obligor. In the third step, individual loan losses are obtained by applying a recovery rate to loan default. Fourth, portfolio loss is given as the sum of realized individual loan losses. This corresponds to one realization in the simulation. Fifth, many simulation runs yield the loss distribution of the entire portfolio.

The loss distribution depends on various input factors that may be grouped into three categories: Individual loan components, portfolio components, and additional CDO features. Individual loan components comprise maturity, credit quality, and credit migration probability, and expected recovery rate at default. Portfolio components comprise the sensitivities of the individual loans to the common factor, portfolio diversification, and individual obligor concentration. Furthermore, in practice, CDO loan portfolios present additional complications as they are dynamic portfolios with various restrictions concerning asset replenishment over the lifetime of the issue. The implementation applied in this paper accounts for single issuer default as well as portfolio characteristics, which are the focus of the investigation.

3 Risk Allocation to Tranches

3.1 Individual Tranche Characteristics

We now investigate the nature of risk transfer from the underlying portfolio to tranches. This is at the heart of structured finance transactions, i.e. the pooling and reallocation of individual risks to investors. The transfer of risks is non-proportional, due to the principle of subordination of tranches. The resulting risk allocation is estimated by Monte Carlo simulation.

Let us consider as base case a reference portfolio with 10'000 loans. All securities have the same characteristics: They are zero bonds with identical nominal value, 10 years to maturity, 7.63% default probability, 24.15% recovery rate, and identical exposure to the macro factor, corresponding to a correlation of $\rho_n^M = 0.15$ between all securities. The applied default probability corresponds to a Baa-rating for the bonds, according to Moody's (2005), Exhibit 17. Overall, the base case represents a realistic setting for a typical CDO transaction. The high number of loans is chosen intentionally to eliminate diversifiable risk to a large extent, giving a clear picture of systematic risk in the analysis as shown later. The evolution of individual-loan credit quality over time is simulated with 500'000 simulation runs.

Panel A of Figure 1 shows the obtained loss distribution for the reference portfolio. The loss rate distribution has a typical shape for portfolios subject to credit risk, displaying a substantial positive skewness. The two main parameters determining the shape of the loss rate distribution are the default probability net of the recovery rate and the sensitivity of the individual loans to the macro factor. The higher the macro factor sensitivity, the more probability mass is shifted from the middle of the distribution to the tails, and vice versa.

Subsequently, following industry practice in the securitization market, the portfolio is split into several tranches of strict subordination. In the subsequent analysis, the number of tranches is assumed to be seven. Note that all results reported below remain essentially unchanged if the number of tranches is smaller or larger, say 5 or 9. In practice, the tranches are associated with different ratings by rating agencies. For given maturities of the tranches, the ratings in turn correspond to specific default probabilities. We define the tranches by a maximum default probability, which is fixed at the 1.01%, 2.57%, 3.22%, 7.63%, 19%, and 36.51% quantile of the loss rate distribution. These numbers correspond to the average issuer-weighted cumulative default rates by whole letter rating for the period from 1920 to 2004 as reported by Moody's (2005). Thus, the six most senior tranches are rated Aaa, Aa, A, Baa, Ba, and B according to Moody's rating scheme, while the remaining first loss piece is not rated. We number the tranches from 1 to 7, with the seventh tranche being the first loss piece, or equity piece, which covers the residual loss. Tranche no.1, at the other end of the spectrum, refers to the most senior tranche. All remaining tranches, nos. 2-6, are mezzanine tranches.

Tranching is done with the intention of maximizing the size of each tranche except the first loss piece, subject to the restrictions that the sizes of all more senior tranches are maximized and the default probability of that tranche is not greater than that required to obtain a particular credit rating. Thus, a portfolio is tranched by first determining the lower attachment points of all tranches. The lower attachment point of each tranche is given as the portfolio loss rate that is exceeded only with the default probability allowed for that tranche. Since the upper attachment point of a tranche is identical to the lower attachment point of the next senior tranche, the size of each tranche is given as the difference between the two attachment points of that tranche. Thus, the number of different layers (tranches) and their required maximum default probabilities, determined by the rating a tranche is supposed to have, determine the attachment points and, correspondingly, the sizes of the tranches.

Applying the presented loss distribution of the total portfolio leads to the following tranche sizes, represented as fraction of the total portfolio, starting from the most senior tranche: 0.7853, 0.0385, 0.0092, 0.0371, 0.0397, 0.0295, and 0.0607 for the equity piece. The detailed summary statistics for the tranches are provided in Panel A of Table 1. Graphical representations of the loss distributions for different tranches (senior tranche, mezzanine tranche, and first loss piece) are given in Figure 1.

As can be seen in Panel A of Table 1, the senior tranche is by far the largest part of the entire transaction, making up 78.53% of the transaction. The expected loss rate is only 5 basis points, while expected loss given a default event is 495 basis points. The mean loss rate is monotonic increasing in the degree of subordination. Its maximum value is 69.01% for the equity piece. The default probability of the equity piece is 100%, as none out of 500'000 runs in the simulation came out with a zero loss rate for the entire portfolio.

The senior tranche has the highest quality in all categories. The probability of default is lowest, with no loss in 98.99% of all cases in this example. In addition, mean loss, loss standard deviation, and loss given default are lowest among all tranches. Furthermore, the senior tranche is by far the largest of all tranches, with a claim on 78.53% of the volume of the underlying portfolio in the base case. In contrast to the senior tranche, the first loss piece suffers a loss rate of 100% with a large probability of 36.51%. Furthermore, while low losses occur at low frequency, higher losses occur with an increasing likelihood, peaking at a loss of 36% in the base case. Overall, the first loss piece (FLP) has the highest expected loss of all tranches.

The numbers for the senior tranche are particularly striking, as they show a very low mean loss given default, despite its large size. Panel D in Figure 1 explains why this is the case. Realized portfolio losses that surpass the capacities of the more subordinate tranches cluster at the low end of possible loss rates, without any observation exceeding a 40% loss rate in the simulation runs. Clearly, the shape of the loss distribution for this tranche (as for the other tranches as well) depends on the shape of the total portfolio loss distribution and the applied tranching scheme. However, the differences are typically not very pronounced and affect characteristics such as the steepness of the distribution.

As can be seen from Figure 1, the most senior mezzanine tranche, no. 2, displays a broad tendency of a downward sloping distribution function throughout its domain. The loss rate distributions of all mezzanine tranches have a similar shape. They are slightly downward sloping as long as the lower cut-off value is larger than the mode of the loss rate distribution of the total portfolio. This is the case in essentially all practically relevant cases since the mode of the loss rate distribution typically lies in the domain of the first loss piece.

The distribution of the first loss piece, depicted in Figure 1, is single peaked in the interior of its domain, abstracting from the spike at its upper boundary. This follows from the fact that the lowest tranche comprises about two thirds of the cumulative loss rate distribution, comprising the peak of the aggregate loss rate distribution.

Overall, the obtained results for the base case are typical for real-world securitization transactions. To check the stability of the results, the base case is altered with respect to three selected key characteristics of the reference portfolio. In particular, the correlation coefficient is increased to 0.3, the default probability is increased to 0.19, and the number of loans in the portfolios is reduced to 100. The resulting tranche statistics obtained when tranching these portfolios with the same tranching scheme as applied for the base case are given in Panels B, C, and D of Table 1.

Higher correlation (Panel B) than in the base case leads to a more fat-tailed portfolio loss distribution while mean loss is not affected. More extreme loss realizations lead to smaller sizes of extreme tranches (tranche no.1 and tranche no.7) and larger sizes of mezzanine tranches. This can be seen in Panel B. In Panel C, the default probability is increased from 0.0763 to 0.19, corresponding to a one notch downgrade, from Baa to Ba, according to Moody's tables. Increased default probability directly affects the mean loss of the reference portfolio and it also leads to increased mean loss of all the tranches compared to the base case. Decreasing the number of loans in the reference portfolio, as shown in Panel D, does not systematically change the results compared to the base case.

Overall, we find the results in the base case to be robust with respect to changes of the portfolio characteristics. While, of course, the numbers do change, the qualitative findings are unchanged.

From the simulation exercise, we obtain a couple of insights. By tranching, the risks of the underlying portfolio are allocated in a non-proportional way to the tranches. The loan portfolio is transformed into several securities with entirely different risk characteristics. The presented statistics illustrate that reference portfolios of average quality (a 7.63% default probability over 10 years for all loans, conforming to a Baa rating, according to Moody's) can be divided into one large tranche of the highest quality, a couple of mezzanine tranches, and a relatively small first loss piece in which the major proportion of credit risk is concentrated. The tranches or only a selection of them, as is often intended, can subsequently be sold to investors.

3.2 Tranche Interdependencies

In this section, we use the data generated in the simulation exercise in order to investigate the correlation between tranche cash flows. The correlation between tranches of different issues is analyzed, e.g. the correlation between two first loss pieces, or two senior tranches with distinct underlying asset portfolios. Since we control the data generating process, we can trace the effect of changes in the underlying asset correlations to the resulting tranche correlations.

Table 2 displays the bilateral correlations of all tranches (ranging from senior tranche to the first loss piece) from two different CDOs with identical characteristics. Note that for large portfolios, the correlation pattern converges in the limit to that of same-issue tranches. The results indicate that tranches of similar credit quality, or seniority, have higher correlation values than tranches with different credit quality. Tranche correlations decrease monotonically with increasing distance of quality between two tranches. The highest correlation values are obtained for tranches with the same credit quality. These values are close to one. Correspondingly, the lowest bilateral correlation value (0.0729) is obtained for tranche 1, the most senior tranche, and tranche 7, the most junior tranche. This shows that senior tranches are almost orthogonal to junior tranches, in particular to the equity piece. Note that even lower correlations can be attained by increasing the distance of tranches, e.g. by decreasing the maximum default probability allowed for the senior tranche.

To determine the robustness of the obtained correlation pattern, the input parameters of the base case are altered. Panel B of Table 2 displays the bilateral tranche correlations of two different CDOs where the correlation between individual obligors in the reference portfolios is increased to 0.3. The values of all individual correlations are larger than in the base case. However, the correlation pattern is very similar to that of Panel A.

In Panel C of Table 2, the default probability is increased from 7.63% to 19%, corresponding to a one-notch downgrade. While individual correlations rise for tranches with similar credit quality, they decrease for tranches with different credit quality. In addition, lower rated tranches have a higher correlation with the total reference portfolio and vice versa. However, the overall correlation pattern is similar to that in Panel A and Panel B.

In an additional robustness check, the number of individual securities in the reference portfolio is altered. In particular, the reference portfolio is assumed to consist of 100 loans from different obligors, and the correlation between them is 0.15 as in the base case. The results are presented in Panel D of Table 2. While the individual correlations are lower than those in Panel A, again, the correlation pattern is very similar to that of the base case.

According to these simulations, the results confirm that portfolio risk is transferred to tranches in a non-linear way. In particular, the risk associated with senior tranches is only to a minor extent correlated with the risk of the first loss piece. Thus, a bank not selling all tranches but retaining certain tranches will consequently be exposed to certain types of risk, different from the original risk exposure.

3.3 Estimating the systematic risk of tranches

The objective of the analysis is to trace the macroeconomic-risk exposure of individual tranches that are structured according to the principle of subordination.

With tranches given by the original tranching process and with simulated realizations of the macroeconomic risk factor, the dependency of both the underlying portfolio and the individual tranches to the macro factor can be extracted from the simulation results. The simulation results allow us to estimate directly the relationship between the macro risk factor and the realizations of particular tranches of an underlying loan portfolio. The aggregated numbers are presented in Table 3.

Systematic risk, or macroeconomic-factor-dependent risk, of a security can be represented by the average loss the security suffers in a particular state, determined by the macroeconomic risk factor taking a certain value. Since the macro factor can take any value, there is an unlimited number of states. For the exposition of the results obtained, all possible states are aggregated to several broader states, determined by the macro factor taking values in certain ranges. In order to provide a clear picture, the ranges are determined by the default probabilities of the tranches. Thus, e.g. range no.1, covering the bad states, represents all cases with the 1.01% lowest macro factor realizations, range no. 2 represents all cases with macro factor realizations in the 1.01%-2.57% range, and range no. 7 stands for the highest 63.49% macro factor realizations, corresponding to the good states.

This structural-form representation of systematic risk avoids the linearity assumption implicit in the more standard covariance, or beta, statistic of systematic risk. The measure applied here to capture systematic risk is essentially a two-dimensional decomposition of total reference-portfolio losses by states and tranches.

Note that the ranges, or states, are non-overlapping and add to the total range, consisting of all macro factor realizations. With seven tranches given, we obtain by definition seven states of the economy. This particular choice of seven states provides the clearest picture. This is due to the fact that for each tranche, losses generally only tend to occur if the macro factor is in a poor state, relative to the attachment point. Thus, the values in the bottom left of each Panel are zero. This assumes, however, a minor role for idiosyncratic risk involved, as is the case for large asset portfolios, such as the one in the base case with 10'000 loans. With more idiosyncratic risk present, as is the case in Panel D with a portfolio of 100 loans, the picture becomes more blurry and losses increasingly occur if the macro factor is in a good state, relative to the attachment point.

In Table 3, the average loss for each tranche and the reference portfolio are shown for the individual states. The average losses are shown both for all states (all realizations of the macro factor) and for the seven states as described earlier. The entries in Table 3, therefore, indicate the mean loss of a particular tranche in a particular state. Note that the numbers add up column-wise for each tranche, line-wise for each state, and altogether to average total loss of the portfolio (bottom right in each Panel). Thus, Table 3 shows the allocation of portfolio risk to the tranches, indicating in which state of the macro factor these losses are occurring.

Panel A of Table 3 presents the allocation of macro-factor-dependent risk for the base case, introduced earlier. The last row of Panel A shows that the average total portfolio loss of 5.79% is mainly included in the first loss piece (4.19%), while the senior tranche only accounts for 0.04%, amounting to 0.68% of total portfolio risk. The last column shows how total portfolio losses relate to macro-factor realizations.

In particular, the total portfolio (5.79% average loss) consists of 0.26% state-1 loss and 1.97% state-7 loss. As can be seen in the second last row of Panel A, all portfolio losses in state 7 are borne by the first loss piece. State-1 losses, on the other hand, are shared almost equally across all tranches, as tranche no. 1 bears 0.0004 and tranche no. 7 bears 0.0006 of the total state-1 losses (0.0026). Thus, while the first loss piece bears losses attributable to all states, tranche 1 only bears state-1 losses.

Panels B-D of Table 3 provide robustness checks of the results. Again, the portfolio characteristics are altered with respect to correlation (Panel B), default probability (Panel C), and number of loans in the portfolio (Panel D). Note that the states are the same as in the base case. While the process of tranching leads to different tranche sizes than in the base case, the applied tranching scheme and the default probabilities of the tranches are the same. Overall, the numbers demonstrate that the basic pattern as discussed for the base case remains unchanged.

In particular, we find that, for all variations of the base case, state-7 losses are entirely covered by the first loss piece, and the most senior tranche only suffers in the bad state 1. Or, to put it differently: the senior tranche covers only a certain share of the overall macro tail risk, and at the same time it does not cover much else besides macro tail risk.

This section has an important result: under quite general assumptions about tranching, the most subordinate tranche bears most macrofactor-related losses. Furthermore, the results show that the impact of macro risk on the default rates of tranches varies systematically with the rating quality of the tranche. According to the results, the more senior a tranche is, the more likely is its default accompanied by a negative realization of the macro risk factor.

3.4 Comparing systematic tranche risk to the systematic risk of straight bonds

The last section has analyzed how total systematic portfolio risk is allocated to individual tranches in a standard stuctured finance transaction. For comparison, we now look at the macro-factor dependence of non-tranched portfolios consisting of straight bonds with identical default probabilities and ratings to that of the tranches discussed in the last section.

Panel A in Table 4 relies on Panel A of Table 3. The difference is that now, the numbers are presented as fraction of total security loss over all states, i.e. column-wise they add up to 1, while row-wise they are weighted by number of observations. Panel B has seven portfolios, each consisting of 10’000 bonds with a given rating, ranging from Aaa to B. Note that the underlying portfolio in Panel A corresponds to the Baa-rated portfolio in Panel B.

The results indicate that while the Aaa-rated senior tranche no. 1 suffers almost all of its losses in state 1, the untranched Aaa-rated straight-bond portfolio only suffers 8.25% of its losses in state 1. In fact, this bond portfolio suffers losses in all states, even when the macro factor has intermediate or good realizations. Moreover, all tranches exhibit higher state-1 losses than the bond portfolios with the same rating and lower state-7 losses. Note that the results for the bonds portfolios are independent of the number of bonds in the portfolio. While the results just presented are obtained for bond portfolios with 10'000 individual loans each, the results do not change in the case of portfolios consisting of only one bond each.

These results demonstrate that tranches have completely different risk characteristics compared to straight-bond portfolios with the same default probability or rating. Tranches, especially the senior ones, are much more exposed to tail risks.

4 Shocks to portfolio and tranche risk

4.1 Risk characteristics of tranches

Having discussed the economics of tranching and the risk properties of tranches, we now examine how changes in the original setting impact the risks of existing tranches, both in absolute terms and relative to each other. In particular, we take tranching as given, i.e. the attachment points of the tranches are fixed and the tranches are established as defined securities, and investigate subsequent shocks to the quality of the reference portfolio. In the model framework for CDOs applied in this paper, shocks affecting the loss rate distribution of the underlying portfolio can be represented by changes of the default probability and by changes of correlation.

These shocks may stem from several sources. The default probability may differ from the time of tranching because of adverse realizations of the macroeconomic factor, but also simply because it was misspecified, e.g. the original rating of the portfolio was false. Furthermore, the quality of the portfolio, represented by the default probability, may suffer if the loans are suddenly screened and monitored less well than before. This may happen if the first loss piece is sold and the institution responsible for monitoring loses its incentives to ensure timely repayment of loans. Correlation may differ from the time of tranching because the commonality of obligors' asset values increases, possibly due to business reasons.

Table 5 presents the summary statistics of existing tranches for the base case without shock (Panel A), the case of a correlation shock (Panel B), and the case of a sudden increase of the default probability (Panel C). Since tranching is taken as given, tranche size remains constant by definition. A correlation increase has a different effect on mean tranche losses. While mean loss increases for the senior tranches, it decreases for the junior tranches. The same applies to the default probability and to mean loss given default. In Table 5, the reversal occurs from tranche no.5 to tranche no.6 for mean loss and default probability, but note that the exact turning point depends on the applied tranching scheme. The new default probabilities in Panel B can be applied to determine the new tranche ratings by applying the Table provided by Moody's (2005), Exhibit 17. The new ratings are Baa (tranche nos.1-3), Ba (tranche no.4), B (tranche no.5 and no.6), and tranche no.7 is not rated. The results also show that the loss standard deviation of all tranches rises. This comes from the fact that increasing correlation moves probability mass from the middle to the tails of the portfolio loss rate distribution which naturally also affects the individual tranches. Panel C reports the tranche statistics for increased default probability. The numbers demonstrate that not only mean loss, default probability, and mean loss given default of the reference portfolio, but also of all tranches increase. Loss standard deviation, in contrast, increases for senior tranches, but decreases for junior tranches. Again, the new tranche ratings can be determined as Ba (tranche no.1), B (tranche no.2 and no.3), while tranche nos.4-7 do not receive a rating.

4.2 Systematic risk of tranches

Having examined the general risk characteristics of tranches after shocks, we now discuss how the allocation of systematic risk is affected by these shocks. Again, we look at a correlation increase from $\rho = 0.15$ to $\rho = 0.3$ and an increase of the default probability to from 0.0763 to 0.19. The results are given in Table 6. Panel A in this Table is identical to Panel A of Table 3, to facilitate comparison.

We discuss the shock to asset correlation first. As can be seen from the last column in Panel B, an increase in correlation boosts the losses of the reference portfolio in states 1 to 5, and it lowers them in states 6 and 7. Since the overall mean-loss change is zero, the discussed correlation shock leads to a redistribution of risk. The change of state-1 losses (losses in the bad state, i.e. macro tail risk) is entirely borne by the senior tranche (tranche no.1), thereby substantially increasing its risk position (from 0.0004 to 0.0018 for state 1).

Furthermore, in contrast to the base case (Panel A of Table 3), tranche 1 now also suffers losses in the states 2, 3, and 4. As a result, total tranche-1 losses for all states rise from 0,0004 to 0.0031 in absolute numbers. The last row in Panel B shows that, with constant total losses for the reference portfolio over all states, senior tranches (tranches 1 to 5 in this case) bear more losses than in the base case, while the remaining more junior tranches have less losses than before. Junior tranches apparently benefit from a decrease in high-state losses (high realizations of the macro factor) while the senior tranches are hit disproportionately by increased low-state losses (bad macroeconomic environment). Also, senior tranches are generally more affected in good states than before.

Panel C of Table 6 presents the results for increased default probability. The numbers are all equal or higher than in Panel A and show that all tranches are negatively affected. However, the losses are split disproportionately to the tranches. In general, the higher the seniority of the trance, the higher is the relative increase of losses. While tranche-7 losses increase by 39% for all states, tranche-1 losses increase by 2850%. The senior tranches bear a higher share of portfolio losses. This applies both to all states combined and individually. In lower states, all tranches suffer higher losses than before.

The general picture that emerges from these results is as follows: if the initial loss distribution of the underlying portfolio is changing then, holding the original transaction fixed, there is a strong incremental effect on the systematic risk of senior tranches in particular. For both cases, increases in default probability and increases in correlation of the reference portfolio, senior tranches experience by far the largest relative change in default expectation. For junior tranches, in contrast, the relative change in mean loss due to an increase of reference portfolio mean loss is relatively small, and they can actually benefit from an increase in correlation.

Naturally, the results are reversed in the case of a reduction in default probability or correlation. Then, junior tranches will only have a slight benefit, or they will even suffer (if correlation is falling), while senior tranches will quite generally benefit greatly.

5 Conclusion

In this paper, we examine the risk properties of multi-layer CDO transactions and investigate the adequacy of ratings as a measure for the quality of structured finance products. In the numerical analysis, asset values are generated by a model with both macro economic and idiosyncratic risk. This setup allows tracing the allocation of asset risk from the underlying asset portfolio to junior and senior securitization tranches of structured instruments in general. To capture the macro-factor dependency of individual tranches, a simple metric, the state-dependent expected loss, is proposed. To this end, total portfolio losses are decomposed by states and by tranches, where states are defined by a particular range of values of the macro factor.

The major risk properties of tranches can be summarized as follows. We find junior and senior tranches to have very different exposure to the macro factor. The common presumption that tail risk, i.e. extreme systematic risk, is largely held by senior tranches is shown to be false. While it is true that senior tranches are bearing but macro tail risk, the reverse is not true, i.e. the share of macro tail risk borne by senior tranches is actually quite limited.

Corporate bonds, in contrast, irrespective of their rating quality, have substantial expected loss in all states of the world, not only in extreme tail events. As a consequence, and holding the rating category constant, tranches and bonds will respond differently to a given change in the underlying macroeconomic risk factor. As a corollary, loss given default is highly tranche-dependent. LGD is typically large for mezzanine and junior tranches, but small for senior tranches. In standard rating agency models, however, LGD is assumed to be constant across rating categories.

Furthermore, we find the response of tranche risk to a given change in risk of the underlying asset portfolio to be seniority-dependent. E.g., if the default probability of the underlying asset portfolio rises unexpectedly, the relative change of expected loss is higher for senior tranches than for junior tranches. In contrast, if the correlations in the underlying asset pool change, then the expected loss of junior and senior tranches move in opposite directions.

These basic results on the allocation of asset risk to securitization tranches may be used, e.g., to interpret the dramatic fall in value of senior ABS tranches observed during the 2007/2009 financial crisis. According to the presented model, tranche losses are the consequence of a rise in the underlying portfolio risk, i.e. a negative first-order shift of its loss distribution. The rise in asset risk, in turn, may be the consequence of misaligned lending incentives, as they were experienced in the US subprime lending industry (Gorton 2008). Note that the existant rating methodology, as applied by Moody's or S&P for instance, did not account for these incentive problems. Thus, if the risk transfer implied by the securitization of relationship-specific financial assets (loans) had altered the loss distribution of the underlying loan portfolio, and this effect was not anticipated at the time of issue, then effective tranche risk exceeds tranche risk according to its initial ratings.

Consider commercial or retail mortgage loans as a class of assets underlying a large share of all securitizations. As is well recognized in the theoretical literature, the unconditional sale of the equity piece, the most subordinate tranche, may destroy the long-term monitoring incentives of the originating bank, thus triggering a process of value degradation as outlined in the last paragraph (see Ashcraft and Schuermann, 2008).

Our findings show that the ultimate effect on tranche risk depends on the layer, i.e. senior or junior, and the source of the shock, i.e. whether asset correlations or default probabilities are changing. We observe the strongest increase in systematic risk at the level of the most senior tranche, typically an AAA tranche. This statement is true for both, increases of correlations and default probabilities.

What lessons can we draw from these results?

On a policy level, our findings are relevant to issuers, investors and regulators in the banking industry. Let us discuss these issues in turn.

First, while we do not intend to augment our analysis by a pricing model, we can make a prediction concerning CDO-tranche price changes. This is also related to movements in credit spreads, observed on housing markets during real estate recessions and other occasions of deteriorating collateral quality. A shift of the underlying default expectation offers a possible explanation for observed spread changes of bonds and tranche notes.

Second, the analysis of risk shifting can be linked to the literature on lending standards in credit markets. For example, there is an extensive literature on the role of first loss piece retention for maintaining proper lending standards (deMarzo 2005; Franke and Krahnen 2007). Thus, if an originator sells the equity piece of a given transaction, he essentially provokes the risk distribution to shift, with consequences for tranche spreads, in particular for the senior tranche.

Third, a rating methodology avoiding the pitfalls described in this paper will have to signal the exposure to systematic risk, along with default risk.

Finally, the findings are relevant for regulators and risk management aiming at monitoring risk transfer from balance sheets to capital markets. This study has demonstrated that risk management based on traditional bond ratings applied to structured finance transactions is not reliable. Corporate ratings, as they are currently in use, exclusively reflect the default probability of a given security, and are agnostic about loss given default, and about systematic risk. An improved measure of rating quality would capture the loss given default of each tranche, the macro factor sensitivity of each tranche, and the equity retention covenant at the level of the issue. A suitable industry standard for reporting these features still needs to be developed, e.g. by relying on an easy-to-understand traffic light system or a letter-rating format added to the traditional rating notch.

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Table 1: Summary statistics for tranches

This table presents summary statistics for the seven tranches, representing claims of strict subordination on the underlying portfolio. The statistics indicate the allocation of losses of the underlying portfolio to the individual tranches. The cut-off values for a particular tranche is determined by the default probability allowed for that tranche as indicated in the fifth column and the default probability allowed for the next senior tranche. The most junior tranche (tranche number 7) corresponds to the first loss piece. It bears the first losses occurring in the portfolio, and only when its capacity is exhausted does the next senior tranche take on losses. The columns present, from left to right, tranche number, tranche size, mean loss, loss standard deviation, default probability, and mean loss given default (LGD). The last row of each panel displays the statistics for the underlying portfolio. In Panel A (base case), the reference portfolio consists of 10'000 zero bonds, and all of them are assumed to have a default probability of 7.63%, 10 years maturity, 24.15% recovery rate, and a default correlation of 0.15. The loss distribution is calculated with 500'000 simulations. In Panel B, the base case is altered and the default correlation is increased to 0.3. Panel C applies the portfolio characteristics of the base case, except the default probability, which is increased to 19%. In Panel D, the settings of the base case are applied, with the exception that the number of loans in the reference portfolio is 100.

Panel A: Base case

Tranche	size	mean loss	loss std	default prob	mean LGD
(Aaa) 1	78.53%	0.05%	0.69%	1.01%	4.95%
2	3.85%	1.68%	11.82%	2.57%	65.47%
3	0.92%	2.88%	16.41%	3.22%	89.54%
4	3.71%	5.12%	20.32%	7.63%	67.04%
5	3.97%	12.47%	30.07%	19.00%	65.61%
6	2.95%	26.79%	40.89%	36.51%	73.37%
(FLP) 7	6.07%	69.01%	31.01%	100.00%	69.01%
Total PF	100.00%	5.79%	4.55%	100.00%	5.79%

Panel B: Different correlation ($\rho = 0.3$)

Tranche	size	mean loss	loss std	default prob	mean LGD
(Aaa) 1	67.72%	0.10%	1.31%	1.01%	9.80%
2	7.02%	1.69%	11.84%	2.57%	65.66%
3	1.71%	2.88%	16.40%	3.22%	89.45%
4	6.85%	5.11%	20.30%	7.63%	66.93%
5	6.93%	12.39%	29.99%	19.00%	65.19%
6	4.52%	26.57%	40.79%	36.51%	72.76%
(FLP) 7	5.25%	60.11%	37.51%	99.87%	60.19%
Total PF	100.00%	5.80%	6.89%	99.87%	5.81%

Panel C: Different default probability (p=0.19)

Tranche	size	mean loss	loss std	default prob	mean LGD
(Aaa) 1	61.32%	0.07%	0.93%	1.01%	6.93%
2	4.72%	1.69%	11.86%	2.57%	65.86%
3	1.23%	2.88%	16.40%	3.22%	89.48%
4	5.18%	5.15%	20.39%	7.63%	67.53%
5	6.28%	12.60%	30.22%	19.00%	66.33%
6	5.34%	27.02%	41.00%	36.51%	74.01%
(FLP) 7	15.93%	73.83%	27.27%	100.00%	73.83%
Total PF	100.00%	14.42%	8.22%	100.00%	14.42%

Panel D: Different number of loans (100 loans)

Tranche	size	mean loss	loss std	default prob	mean LGD
(Aaa) 1	77.24%	0.05%	0.72%	1.01%	5.12%
2	4.55%	1.73%	11.88%	2.57%	67.22%
3	0.76%	3.02%	17.12%	3.22%	93.92%
4	3.79%	5.16%	20.54%	7.63%	67.60%
5	4.55%	12.77%	30.28%	19.00%	67.22%
6	3.04%	27.66%	41.64%	36.51%	75.75%
(FLP) 7	6.07%	66.45%	34.21%	95.18%	69.81%
Total PF	100.00%	5.79%	4.95%	95.18%	6.09%

Table 2: Bilateral correlations of tranches from two different CDO issues

This table displays the bilateral correlations of loss rates for all tranches from two different CDO issues ranging from tranche number 1 (most senior tranche) to tranche number 7 (first loss piece). Both CDOs have similar characteristics. In Panel A (base case), the reference portfolio consists of 10'000 zero bonds, and all of them are assumed to have a default probability of 7.63%, 10 years maturity, 24.15% recovery rate, and a default correlation of 0.15. The loss distribution is calculated with 500'000 simulations. In Panel B, the base case is altered and the default correlation is increased to 0.3. Panel C applies the portfolio characteristics of the base case, except the default probability, which is increased to 19%. In Panel D, the settings of the base case are applied, with the exception that the number of loans in the reference portfolio is 100.

Panel A: Base case

	CDO 2							
CDO 1	1	2	3	4	5	6	7	Total PF
(Aaa) 1	0.9957	0.6061	0.4315	0.3400	0.2121	0.1305	0.0729	0.3697
2		0.9925	0.8413	0.6637	0.4141	0.2548	0.1423	0.5440
3			0.9785	0.8187	0.5113	0.3146	0.1757	0.5936
4				0.9935	0.7324	0.4508	0.2517	0.7143
5					0.9948	0.7421	0.4144	0.8341
6						0.9943	0.6549	0.8552
(FLP) 7							0.9961	0.7647

Panel B: Different correlation ($\rho = 0.3$)

	CDO 2							
CDO 1	1	2	3	4	5	6	7	Total PF
(Aaa) 1	0.9982	0.6262	0.4460	0.3515	0.2200	0.1356	0.0801	0.4194
2		0.9971	0.8432	0.6648	0.4161	0.2564	0.1515	0.6077
3			0.9910	0.8193	0.5130	0.3160	0.1868	0.6584
4				0.9976	0.7347	0.4526	0.2676	0.7802
5					0.9980	0.7430	0.4392	0.8737
6						0.9976	0.6927	0.8422
(FLP) 7							0.9977	0.6934

Panel C: Different default probability (p=0.19)

	CDO 2							
CDO 1	1	2	3	4	5	6	7	Total PF
(Aaa) 1	0.9957	0.6243	0.4461	0.3500	0.2179	0.1341	0.0723	0.2915
2		0.9935	0.8442	0.6630	0.4127	0.2540	0.1369	0.4520
3			0.9817	0.8157	0.5081	0.3127	0.1686	0.5068
4				0.9949	0.7306	0.4497	0.2425	0.6336
5					0.9963	0.7422	0.4002	0.7883
6						0.9963	0.6324	0.8626
(FLP) 7							0.9980	0.8375

Panel D: Different number of loans (100 loans)

	CDO 2							
CDO 1	1	2	3	4	5	6	7	Total PF
(Aaa) 1	0.7120	0.5033	0.3803	0.3227	0.2066	0.1252	0.0706	0.3153
2		0.6633	0.5973	0.5689	0.4062	0.2523	0.1426	0.4723
3			0.5925	0.6092	0.4757	0.3053	0.1733	0.5041
4				0.6883	0.6169	0.4259	0.2458	0.6077
5					0.7327	0.6317	0.4037	0.7128
6						0.7137	0.5745	0.7161
(FLP) 7							0.7294	0.6321

Table 3: Tranche loss in different states

This table displays the loss for each tranche and the underlying portfolio in different states, represented as ranges of the macroeconomic factor. The losses are shown both for all realizations of the macro factor and for the macro factor taking values in certain ranges that are given by the attachment points of the tranches. The tranches range from tranche number 1 (most senior tranche) to tranche number 7 (first loss piece). In Panel A (base case), the reference portfolio consists of 10'000 zero bonds, and all of them are assumed to have a default probability of 7.63%, 10 years maturity, 24.15% recovery rate, and a default correlation of 0.15. The loss distribution is calculated with 500'000 simulations. In Panel B, the base case is altered and the default correlation is increased to 0.3. Panel C applies the portfolio characteristics of the base case, except the default probability, which is increased to 19%. In Panel D, the settings of the base case are applied, with the exception that the number of loans in the reference portfolio is 100.

Panel A: Base case

	Tranche							
State	1	2	3	4	5	6	7	Total PF
1	0.0004	0.0004	0.0001	0.0004	0.0004	0.0003	0.0006	0.0026
2	0.0000	0.0003	0.0001	0.0006	0.0006	0.0005	0.0009	0.0030
3	0.0000	0.0000	0.0000	0.0002	0.0003	0.0002	0.0004	0.0011
4	0.0000	0.0000	0.0000	0.0007	0.0017	0.0013	0.0027	0.0064
5	0.0000	0.0000	0.0000	0.0000	0.0019	0.0033	0.0069	0.0122
6	0.0000	0.0000	0.0000	0.0000	0.0000	0.0023	0.0106	0.0129
7	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0197	0.0197
All states	0.0004	0.0006	0.0003	0.0019	0.0049	0.0079	0.0419	0.0579

Panel B: Different correlation ($\rho = 0.3$)

	Tranche							
State	1	2	3	4	5	6	7	Total PF
1	0.0007	0.0007	0.0002	0.0007	0.0007	0.0005	0.0005	0.0039
2	0.0000	0.0005	0.0003	0.0011	0.0011	0.0007	0.0008	0.0044
3	0.0000	0.0000	0.0001	0.0004	0.0005	0.0003	0.0003	0.0016
4	0.0000	0.0000	0.0000	0.0013	0.0031	0.0020	0.0023	0.0087
5	0.0000	0.0000	0.0000	0.0000	0.0033	0.0051	0.0060	0.0144
6	0.0000	0.0000	0.0000	0.0000	0.0000	0.0034	0.0092	0.0126
7	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0124	0.0124
All states	0.0007	0.0012	0.0005	0.0035	0.0086	0.0120	0.0316	0.0580

Panel C: Different default probability (p=0.19)

	Tranche							
State	1	2	3	4	5	6	7	Total PF
1	0.0004	0.0005	0.0001	0.0005	0.0006	0.0005	0.0016	0.0043
2	0.0000	0.0003	0.0002	0.0008	0.0010	0.0008	0.0025	0.0056
3	0.0000	0.0000	0.0000	0.0003	0.0004	0.0003	0.0010	0.0022
4	0.0000	0.0000	0.0000	0.0010	0.0028	0.0024	0.0070	0.0131
5	0.0000	0.0000	0.0000	0.0000	0.0031	0.0061	0.0181	0.0273
6	0.0000	0.0000	0.0000	0.0000	0.0000	0.0043	0.0279	0.0322
7	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0595	0.0595
All states	0.0004	0.0008	0.0004	0.0027	0.0079	0.0144	0.1176	0.1442

Panel D: Different number of loans (100 loans)

	Tranche							
State	1	2	3	4	5	6	7	Total PF
1	0.0003	0.0004	0.0001	0.0004	0.0005	0.0003	0.0006	0.0026
2	0.0000	0.0003	0.0001	0.0005	0.0007	0.0005	0.0009	0.0030
3	0.0000	0.0001	0.0000	0.0002	0.0003	0.0002	0.0004	0.0011
4	0.0000	0.0001	0.0000	0.0006	0.0017	0.0013	0.0027	0.0064
5	0.0000	0.0000	0.0000	0.0003	0.0021	0.0029	0.0069	0.0122
6	0.0000	0.0000	0.0000	0.0000	0.0006	0.0024	0.0099	0.0129
7	0.0000	0.0000	0.0000	0.0000	0.0000	0.0008	0.0189	0.0198
All states	0.0004	0.0008	0.0002	0.0020	0.0058	0.0084	0.0403	0.0579

Table 4: Macro factor dependency of tranches versus untranched bond portfolios

This table displays the macro factor dependency of tranches with different ratings compared to macro factor dependencies of untranched bond portfolios with the same ratings. Macro-factor dependency is represented as losses occurring in different states, represented as ranges of the macroeconomic factor. The losses are shown both for all realizations of the macro factor and for the macro factor taking values in certain ranges that are given by the attachment points of the tranches. The tranches range from tranche number 1 (most senior tranche) to tranche number 7 (first loss piece). In Panel A (base case), the reference portfolio consists of 10'000 zero bonds, and all of them are assumed to have a default probability of 7.63%, 10 years maturity, 24.15% recovery rate, and a default correlation of 0.15. The loss distribution is calculated with 500'000 simulations. In Panel B, various portfolios are considered that solely consist of 10'000 bonds with a specific rating, ranging from Aaa to B and determining the default probability. The applied default probabilities are 1.01% (Aaa), 2.57% (Aa), 3.22% (A), 7.63% (Baa), 19.00% (Ba), and 36.51% (B). All losses are given as fraction of total losses of the examined tranche/bond portfolio attributable to a particular state. Securities are given in columns, states are given in rows.

Panel A: Base case

	Tranche							
State	1	2	3	4	5	6	7	Total PF
1	0.9985	0.5992	0.3503	0.1975	0.0810	0.0377	0.0146	0.0442
2	0.0017	0.3985	0.5350	0.3050	0.1251	0.0582	0.0226	0.0519
3	0.0000	0.0025	0.1079	0.1260	0.0521	0.0243	0.0094	0.0192
4	0.0000	0.0000	0.0069	0.3698	0.3530	0.1646	0.0639	0.1109
5	0.0000	0.0000	0.0000	0.0018	0.3875	0.4236	0.1648	0.2100
6	0.0000	0.0000	0.0000	0.0000	0.0012	0.2906	0.2535	0.2230
7	0.0000	0.0000	0.0000	0.0000	0.0000	0.0010	0.4711	0.3407
All states	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Panel B: Bond portfolios

	PF rating					
State	Aaa	Aa	A	Baa	Ba	B
1	0.0825	0.0636	0.0594	0.0442	0.0301	0.0213
2	0.0797	0.0671	0.0640	0.0519	0.0390	0.0298
3	0.0274	0.0238	0.0229	0.0192	0.0150	0.0118
4	0.1445	0.1306	0.1269	0.1109	0.0912	0.0749
5	0.2316	0.2250	0.2226	0.2100	0.1893	0.1680
6	0.2045	0.2151	0.2172	0.2230	0.2231	0.2164
7	0.2298	0.2747	0.2870	0.3407	0.4124	0.4778
All states	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Table 5: Summary statistics for existing tranches

This table presents summary statistics for seven existing tranches for the case that characteristics of the reference portfolio change at any time after the reference portfolio has been split into tranches. The statistics indicate the allocation of losses of the underlying portfolio to the individual tranches. The most junior tranche (tranche number 7) corresponds to the first loss piece. It bears all losses not covered by the other, more senior, tranches. The columns present, from left to right, the tranche number, the tranche size, mean loss, loss standard deviation, default probability, and mean loss given default (LGD). The last row of each panel displays the statistics for the underlying portfolio. Panel A represents the base case as presented in Table 1. The cut-off values for a particular tranche is determined by the default probability allowed for that tranche as indicated in the fifth column. Panel B displays the tranche characteristics, taking tranche size as given according to the base case and subsequently increasing the default correlation to 0.3. Panel C takes tranching as given and displays the tranche characteristics after default probability rises to 19%.

Panel A: Base case

Tranche	size	mean loss	loss std	default prob	mean LGD
(Aaa) 1	78.53%	0.05%	0.69%	1.01%	4.95%
2	3.85%	1.68%	11.82%	2.57%	65.47%
3	0.92%	2.88%	16.41%	3.22%	89.54%
4	3.71%	5.12%	20.32%	7.63%	67.04%
5	3.97%	12.47%	30.07%	19.00%	65.61%
6	2.95%	26.79%	40.89%	36.51%	73.37%
(FLP) 7	6.07%	69.01%	31.01%	100.00%	69.01%
Total PF	100.00%	5.79%	4.55%	100.00%	5.79%

Panel B: Increased correlation ($\rho = 0.3$)

Tranche	size	mean loss	loss std	default prob	mean LGD
(Aaa) 1	78.53%	0.40%	2.61%	4.19%	9.46%
2	3.85%	5.38%	21.58%	6.79%	79.16%
3	0.92%	7.20%	25.59%	7.62%	94.45%
4	3.71%	9.78%	28.37%	12.30%	79.48%
5	3.97%	16.26%	34.89%	21.03%	77.31%
6	2.95%	26.10%	41.78%	32.13%	81.25%
(FLP) 7	6.07%	56.62%	37.57%	99.87%	56.69%
Total PF	100.00%	5.80%	6.89%	99.87%	5.81%

Panel C: Increased default probability (p=0.19)

Tranche	size	mean loss	loss std	default prob	mean LGD
(Aaa) 1	78.53%	1.50%	4.40%	18.47%	8.13%
2	3.85%	23.89%	40.34%	30.00%	79.64%
3	0.92%	31.69%	45.91%	33.44%	94.76%
4	3.71%	41.27%	46.39%	49.74%	82.98%
5	3.97%	59.96%	45.32%	70.54%	84.99%
6	2.95%	78.40%	37.91%	85.90%	91.27%
(FLP) 7	6.07%	95.65%	13.40%	100.00%	95.65%
Total PF	100.00%	14.42%	8.22%	100.00%	14.42%

Table 6: Tranche loss in different states for existing tranches

This table presents the losses of the reference portfolio and the individual tranches for the case that the characteristics of the reference portfolio change at any time after the reference portfolio has been split into tranches. The loss for each tranche and the underlying portfolio is shown in different states, represented as ranges of the macroeconomic factor. The losses are shown both for all realizations of the macro factor and for the macro factor taking values in certain ranges that are given by the attachment points of the tranches. The tranches range from tranche number 1 (most senior tranche) to tranche number 7 (first loss piece). In Panel A (base case), the reference portfolio consists of 10'000 zero bonds, and all of them are assumed to have a default probability of 7.63%, 10 years maturity, 24.15% recovery rate, and a default correlation of 0.15. The loss distribution is calculated with 500'000 simulations. In Panel B, the base case is altered and the default correlation is increased to 0.3. Panel C presents the losses for a tranched portfolio as in the base case that experiences a default probability increase to 19%.

Panel A: Base case

	Tranche							
State	1	2	3	4	5	6	7	Total PF
1	0.0004	0.0004	0.0001	0.0004	0.0004	0.0003	0.0006	0.0026
2	0.0000	0.0003	0.0001	0.0006	0.0006	0.0005	0.0009	0.0030
3	0.0000	0.0000	0.0000	0.0002	0.0003	0.0002	0.0004	0.0011
4	0.0000	0.0000	0.0000	0.0007	0.0017	0.0013	0.0027	0.0064
5	0.0000	0.0000	0.0000	0.0000	0.0019	0.0033	0.0069	0.0122
6	0.0000	0.0000	0.0000	0.0000	0.0000	0.0023	0.0106	0.0129
7	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0197	0.0197
All states	0.0004	0.0006	0.0003	0.0019	0.0049	0.0079	0.0419	0.0579

Panel B: Increased correlation ($\rho = 0.3$)

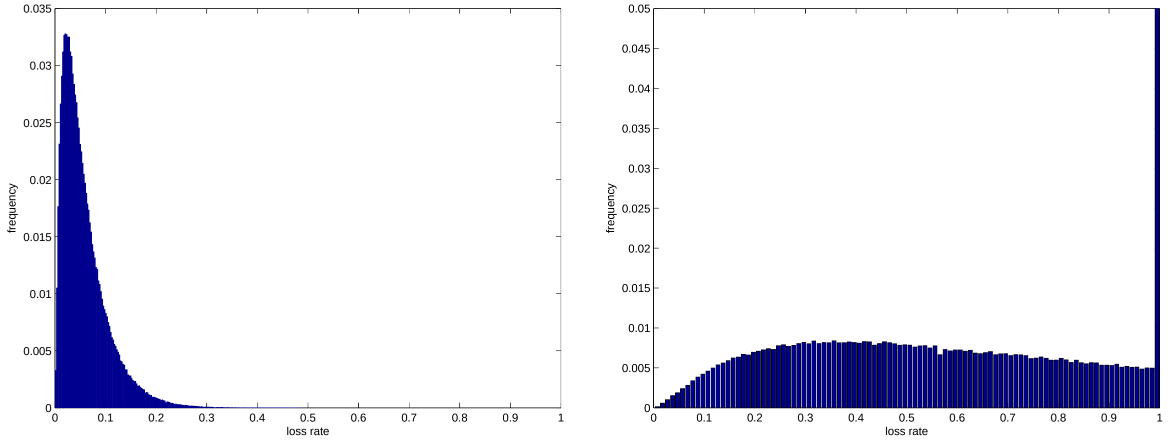
	Tranche							
State	1	2	3	4	5	6	7	Total PF
1	0.0018	0.0004	0.0001	0.0004	0.0004	0.0003	0.0006	0.0039
2	0.0011	0.0006	0.0001	0.0006	0.0006	0.0005	0.0009	0.0044
3	0.0002	0.0003	0.0001	0.0002	0.0003	0.0002	0.0004	0.0016
4	0.0001	0.0008	0.0004	0.0016	0.0018	0.0013	0.0027	0.0087
5	0.0000	0.0000	0.0000	0.0008	0.0033	0.0034	0.0069	0.0144
6	0.0000	0.0000	0.0000	0.0000	0.0001	0.0021	0.0104	0.0126
7	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0124	0.0124
All states	0.0031	0.0021	0.0007	0.0036	0.0065	0.0077	0.0344	0.0580

Panel C: Increased default probability (p=0.19)

	Tranche							
State	1	2	3	4	5	6	7	Total PF
1	0.0022	0.0004	0.0001	0.0004	0.0004	0.0003	0.0006	0.0043
2	0.0023	0.0006	0.0001	0.0006	0.0006	0.0005	0.0009	0.0056
3	0.0008	0.0003	0.0001	0.0002	0.0003	0.0002	0.0004	0.0022
4	0.0037	0.0017	0.0004	0.0016	0.0018	0.0013	0.0027	0.0131
5	0.0029	0.0044	0.0010	0.0042	0.0045	0.0034	0.0069	0.0273
6	0.0000	0.0019	0.0012	0.0064	0.0070	0.0052	0.0106	0.0322
7	0.0000	0.0000	0.0000	0.0019	0.0093	0.0124	0.0359	0.0595
All states	0.0118	0.0092	0.0029	0.0153	0.0238	0.0231	0.0581	0.1442

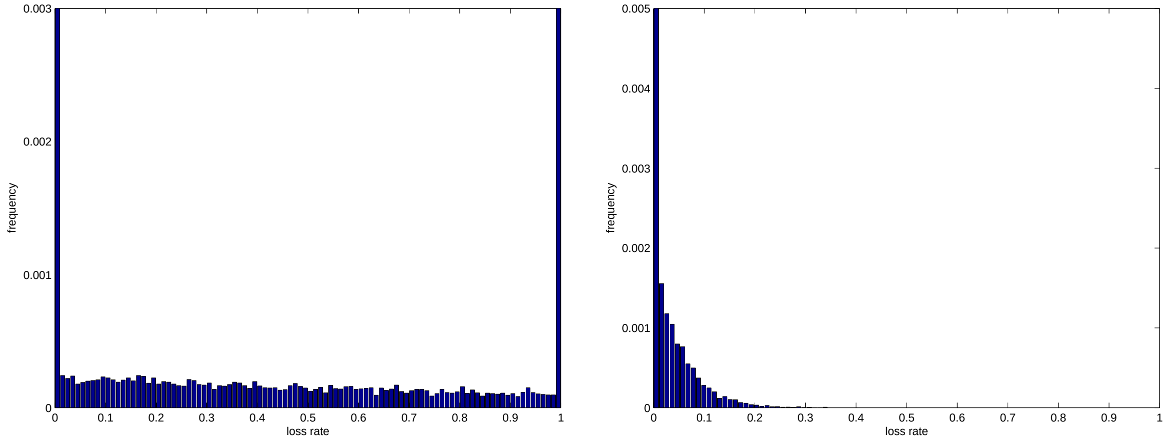
Figure 1: Loss distribution of tranches

This diagram presents the loss distribution of a loan portfolio (Panel A) and three tranches (Panel B-D) at maturity. The three tranches depicted are the first loss piece (Panel B), the most senior mezzanine tranche (Panel C), and the most senior tranche overall (Panel D). The underlying portfolio consists of 10’000 securities from different obligors. All securities have the same characteristics: They are zero bonds with 10 years to maturity, 7.63% default probability, 24.15 % recovery rate, and identical exposure to the macro factor ($\rho_n^M = 0.15$). The evolution of individual-loan credit quality over time is simulated with 500’000 simulation runs. The horizontal axis shows the loss rate; the vertical axis shows the observed frequency, truncated at 5%, 0.3%, and 0.5% for the three tranches, respectively. There are several values surpassing these thresholds: For the first loss piece, 100% loss occurs at a frequency of 36.51%. For the depicted mezzanine tranche, zero loss occurs at a frequency of 87.43%, and 100% loss occurs at a frequency of 1.01%. For the senior tranche, zero loss occurs at a frequency of 98.99%.



Panel A: Total portfolio

Panel B: First loss piece (tranche 7)



Panel C: Mezzanine tranche (tranche 2)

Panel D: Senior tranche (tranche 1)

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