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On the Economics of Crisis Contracts*

Elias Aptus† Volker Britz‡ Hans Gersbach§

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Abstract

We examine the impact of so-called "Crisis Contracts" on bank managers' risk-taking incentives and on the probability of banking crises. Under a Crisis Contract, managers are required to contribute a pre-specified share of their past earnings to finance public rescue funds when a crisis occurs. This can be viewed as a retroactive tax that is levied only when a crisis occurs and that leads to a form of collective liability for bank managers. We develop a game-theoretic model of a banking sector whose shareholders have limited liability, so that society at large will suffer losses if a crisis occurs. Without Crisis Contracts, the managers' and shareholders' interests are aligned, and managers take more than the socially optimal level of risk. We investigate how the introduction of Crisis Contracts changes the equilibrium level of risk-taking and the remuneration of bank managers. We establish conditions under which the introduction of Crisis Contracts will reduce the probability of a banking crisis and improve social welfare. We explore how Crisis Contracts and capital requirements can supplement each other and we show that the efficacy of Crisis Contracts is not undermined by attempts to hedge.

Keywords: banking crises, Crisis Contracts, excessive risk taking, banker's pay, hedging, capital requirements

JEL: C79, G21, G28

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1 Introduction

Motivation and main insight

In this paper, we provide a first analysis of so-called "Crisis Contracts" as a regulatory instrument in the banking sector. Under a Crisis Contract, bank managers have to contribute a certain share of their past earnings to public rescue funds when a banking crisis occurs.[1] Using a game-theoretic modeling approach, we will show that when they are suitably combined with capital requirements some Crisis Contracts can reduce the risk of banking crises and improve social welfare.

In the context of the 2008 financial crisis, many governments have felt compelled to provide extensive bailouts to financial institutions, implying a substantial transfer of risks from the banking sector to the government and, ultimately, the taxpayer. Moreover, there is ample evidence that excessive asset risk-taking has played a central role in this crisis (see Hellwig (2009) and Chesney et al. (2012)).[2] This development has led both to a major debate about the regulation of risk in the financial sector and to a controversy about the remuneration of bank managers and the associated bonus culture. Crisis Contracts are linked to both of these issues.

The classical regulatory response to excessive risk-taking in the banking sector is to adapt capital requirements. The most recent crisis is no exception. For instance, Admati and Hellwig (2013) have proposed a drastic increase in capital requirements. In addition, there have been a number of policy proposals that would supplement capital requirements with more direct government interventions in banking activities. Some examples of such policy proposals are forced separation of retail banking from investment banking, a limit on the size of banks, or a downright ban on trading certain kinds of financial assets, such as the prohibition of short-sales. Such direct interventions in banking activities, however, may be ineffective, as they require the regulator to identify ex ante the financial instruments to be deemed risky and hence forbidden or to be linked to high capital requirements. On the one hand, financial innovation can undermine the effectiveness of such regulation, on the other hand, an overzealous regulator might also ban the trade of assets that do play a useful role in the economy. Moreover, even if each individual bank complies with such regulations and has a sufficiently sound investment portfolio, the banking sector as a whole may still be exposed to excessive risk. This problem is discussed in Adrian and Brunnermeier (2011), who propose a modification of the common "Value-at-Risk" method

[1] Crisis Contracts were first suggested in Gersbach (2011).

[2] For empirical evidence on the evolution of the compensation of bank managers before and after the crisis see Bell and Van Reenen (2014).

that would also account for covariances.

Given the delicate nature of direct government intervention in banking activities, we propose Crisis Contracts as one alternative way to supplement capital requirements. A Crisis Contract does not intervene in the business of the bank as such, but only affects taxation of bank managers in the event of a crisis. Therefore, Crisis Contracts can also be seen as an alternative to a number of policy proposals that were made regarding the remuneration and liabilities of bank managers. We summarize some of the proposals.

For instance, one possible policy would be to limit the fixed salary of bank managers directly or limit the extent of bonus payments or subject bonus payments to an exceptional bracket of income taxation. Some governments have tried to limit managerial pay at least in those banks rescued by a bailout. The relation between government bailouts and managerial pay in the banking sector has been studied by Hakenes and Schnabel (2010). They develop a theoretical model in which bailout guarantees by the government encourage the shareholders of banks to offer their managers very variable compensation with a large bonus for high returns on investment. This remuneration policy in its turn leads to excessive risk-taking by managers. Empirically, Hakenes and Schnabel argue in favor of a regulatory cap on bonus pay. In a similar vein, Thanassoulis (2012A) proposes a limit to bonus pay that would vary with the bank's balance sheet. In a follow-up paper, Thanassoulis (2012B) examines the impact of such a regulation on banks' portfolio choices and finds that the regulation would encourage asset diversification and would also create incentives to focus more on retail banking. John et al. (2000) propose a deposit insurance scheme in which the insurance premium to be paid by a bank depends on that bank's payment practices. VanHoose (2011) gives an overview of different regulations for bankers' pay in the United States and provides a survey of theoretical and empirical findings on the effects of such regulations.

In addition to regulations of risk management and the tax system, the judicial system can also be used to discourage excessive risk-taking. In principle, tort law could be used to hold banks, or bank managers, responsible for any damage caused by excessive risk-taking. In reality, however, it may often be very difficult to assign individual responsibility to specific banks or managers. This is especially true in an environment where systemic risks are present at a macro level and depend among other things on the degree of interconnectedness of different banks. Moreover, tort law may only be a suitable tool if managers have taken actions explicitly or implicitly banned by the regulator. This, again, would require the regulator to discern dangerous practices ex ante. One alternative approach would leave it to the courts to determine ex post which practices can be considered excessive, but courts may find this task complex to the point of infeasibility.[3] Armour and Gordon (2013) have recently pointed out that tort law is of limited use in internalizing social costs of banking crises.

Crisis Contracts can avoid some of the aforementioned shortcomings.In particular, a Crisis Contract holds managers in the financial sector liable collectively rather than individually. To apply a Crisis Contract, it is thus not necessary to attribute individual responsibility to a specific bank or a specific manager, so it does not involve any tort action lawsuits. Furthermore, a Crisis Contract does not require the regulator to make ex ante judgments about whether a certain investment strategy is excessively risky or not. Instead, the payments stipulated in a Crisis Contract become due if and when a crisis occurs. Hence, a Crisis Contract cannot be easily undermined by financial innovation. In addition, a Crisis Contract promotes a fairer cost sharing since managers' previous earnings are "bailed in." Moreover, a Crisis Contract treats all of the manager's past earnings from the banking sector equally, rather than singling out the bonus component. Accordingly, the payments stipulated by a Crisis Contract cannot easily be circumvented by redefining a bonus as part of a fixed salary.

No Crisis Contract, however, would work well in an environment with inadequate capital requirements. Excessive risk-taking becomes extremely attractive for both shareholders and managers in such circumstances and outweighs feasible Crisis Contract disincentives. This seems to confirm the idea that in the absence of suitable capital requirements, the financial system as a whole would be highly vulnerable to small shocks that cannot be effectively dealt with by other regulatory tools (Hellwig, 2009; Gersbach, 2013). Recent conceptual contributions identifying the working and design of such requirements are Repullo (2012), Repullo and Suarez (2013), and Admati and Hellwig (2013). We show, however, that Crisis Contracts enable capital requirements to be set at lower levels than would otherwise be necessary to prevent banking crises. Thus, Crisis Contracts appear to be a useful tool for supplementing and strengthening the effects of capital requirements.

Model and formal results

In this paper we introduce a stylized two-period model of a financial sector. There are a finite number of banks. Each bank is owned by a shareholder and operated by a manager. In each of the two time periods, each manager choses between a risky and a safe investment. The shareholder cannot control the manager's choice directly but may pay the manager

[3]The authors are grateful to Roberta Romano for her helpful comments on the limitations of individual liability and tort law in restricting excessive risk-taking.

depending on the return achieved on the investment. Due to limited liability, the risky investment leads to a higher expected return for the shareholder but to social losses in the event of a crisis. Such a crisis will occur if more than a critical number of managers chose the risky investment in a given period. If a crisis occurs in the second period, then the managers lose a percentage of the income they have previously earned in the banking sector. Additionally, managers have job opportunities outside the banking sector. In this model, the optimal outcome from a social welfare point of view is that a limited subcritical number of banks invest in risky high-return assets.

Our main results are the following: In the absence of Crisis Contracts, the model admits only full risk equilibria, that is, equilibria in which all managers choose the risky investment in both periods. Such an equilibrium maximizes the risk of a crisis and minimizes social welfare. The introduction of a Crisis Contract does not change shareholders' preference for risky investments, since it directly affects the managers only. Shareholders may respond to the introduction of a Crisis Contract by offering even higher wage or bonus payments to re-align the managers' interests with their own. It is therefore not immediately clear whether a Crisis Contract will change investment choices. In short, the question of interest is: Will an appropriately designed Crisis Contract avoid crises and improve social welfare?

We show that under certain restrictions on the model parameters, the introduction of a Crisis Contract can undo the full risk equilibria we found in the benchmark case with no Crisis Contracts. Moreover, for a suitable choice of the model parameters we show that the introduction of Crisis Contracts as tools for the regulator leads to the existence of what we will call threshold equilibria. This is a type of equilibrium in which no crisis occurs and in which social welfare is maximized. Some of the parameter restrictions necessary for the effectiveness of a Crisis Contract can be suitably interpreted as bounds on bank leverage and thus as sufficient capital requirements. We conclude that Crisis Contracts can effectively lower the level of risk-taking and enhance social welfare in an environment with sufficient capital requirements. Accordingly, Crisis Contracts may be a suitable alternative to direct government interventions in banking activities. In addition, Crisis Contracts could be perceived as enhancing fairness since in case of crises bank managers are exposed to a (limited) degree of personal liability.

We also discuss the robustness of Crisis Contracts. In particular, we show that the efficacy of Crisis Contracts is not undermined by bank managers' attempts to hedge against the crisis tax. Moreover, Crisis Contracts remain a useful regulatory tool if some shareholders are adversely affected by banking crises or if a share of smaller banks has no influence on the occurrence of banking crises.

The rest of the paper is organized as follows: In Section 2 we give a formal description of our game-theoretic model. In Section 3 we discuss subgames played in the second period of the two-period model. We analyze the equilibrium remuneration of managers in Section 4. Then we focus on two specific kinds of equilibrium, the full risk equilibrium and the threshold equilibrium, which from a social welfare point of view turn out to be the worst and best equilibria respectively. We discuss these two kinds of equilibrium in detail in Sections 5 and 6. In Section 7 we study the effects of Crisis Contracts on risk-taking behavior and on social welfare. The relation between capital regulation and Crisis Contracts is identified in Section 8. In Section 9 we explore the possibility of hedging against Crisis Contracts. Finally, in Section 10 we discuss some ramifications and conclude.

2 The Model

2.1 A Two-period Economic Environment

We model a banking sector consisting of a finite set of identical banks $N = \{1, \dots, n\}$ with $n \geq 2$, where the members of N are sometimes indexed by i or j. The economy has two periods $t = 1, 2$.[4] In each period, each bank either invests in a *risky asset* or in a *safe asset* or is *out of business.* We capture the investment decision of bank i in period t with the indicator $A^{it} \in \{R, S, O\}$. We denote the *activity profile* $(A^{11}, \dots, A^{n1}, A^{12}, \dots, A^{n2})$ by $\mathcal{A}$ and write A^{-it} for the restriction of the activity profile to round t and banks $j \in N \setminus \{i\}$. Given the activity profile $\mathcal{A}$, we use $n_t(\mathcal{A})$ to stand for the number of banks choosing the risky asset in round t. This variable reflects the overall level of risk-taking in the banking sector. We assume that there is a threshold $\bar{n} \in \{1, \dots, n-1\}$ such that investment choices $\mathcal{A}$ trigger a banking crisis in period t with positive probability if and only if $n_t(\mathcal{A})$ attains this threshold.

Assumption 2.1. *Let the activity profile be given by $\mathcal{A}$. If $n_t(\mathcal{A}) \geq \bar{n}$, then a banking crisis will occur in period t with probability $p \in (0, 1)$. If $n_t(\mathcal{A}) < \bar{n}$, then no banking crisis will occur in period t.*

The restriction $\bar{n} \leq n - 1$ implies that no single bank can prevent a crisis by going out of business or by investing in the safe asset.

Assumption 2.1 is a particular simple and stylized formalization of the idea that the risk of a banking crisis is the result of the joint behavior of banks in the system.

[4] We need at least two periods to examine Crisis Contracts, as a first period is needed to generate the wage income of bank managers for future taxation when a banking crisis occurs.

We use the pair of indicators $\mathcal{Z} = (Z_1, Z_2)$, where $Z_t = 1$ if a crisis occurs at period $t = 1, 2$ and $Z_t = 0$ otherwise.

We now turn to the asset structure of the economy. The safe asset yields a sure payoff of x_{rf}, independently of the occurrence of a crisis. The risky asset yields a payoff of x_g when there is no crisis and a payoff of x_b when there is a crisis. We assume that

$$x_g > x_{rf} > 0 > x_b. \tag{1}$$

These inequalities clearly reflect a risk-return trade-off. The risky asset outperforms the safe asset when there is no crisis, but the safe asset performs better than the risky asset when a crisis does occur. We assume that the bank could guarantee itself a sure payoff of zero in each period by staying out of business. Therefore, the payoff $x_b < 0$ from the risky investment in the case of a crisis is suitably interpreted as an (avoidable) loss. In other words, some risk-taking in the economy (by up to $\bar{n} - 1$ banks) guarantees high returns, but major losses are possible when risk-taking is excessive. A more detailed rationale for this payoff structure can be found in Section 8.

2.2 Limited Liability and Bailout

We assume in this paper that the public is to a large extent liable for the losses made by banks during a crisis. The rationale behind this assumption is that, on the one hand, a functioning banking sector is vital for the economy and thus for the public good as a whole, while on the other hand, banks are owned by shareholders whose liability is limited.[5] Government-backed deposit insurance schemes and downright bailouts as in the recent financial crisis are examples of mechanisms that eventually hold the public liable for losses in the banking sector. Such explicit or implicit liability of the public creates a distortion of the risk-taking incentives. In particular, banks may have an incentive to take risks that are harmful from the social welfare point of view. In a popular phrase, banks may have the possibility to *"privatize gains but socialize losses."* In order to restrict attention to those cases where such a conflict of interest does indeed arise, we assume henceforth that

$$px_b + (1-p)x_g < x_{rf} < (1-p)x_g. \tag{2}$$

[5] We do not consider the case of private or family-owned banks, where the owners are personally liable for losses.

The leftmost term is the expected payoff from the risky asset, given that a critical number of banks take risk. The rightmost term is the expected payoff from the risky asset when the possible negative realization is disregarded. In other words, the rightmost term is the expected payoff from the point of view of a shareholder with limited liability. Thus, given a sufficiently high level of risk-taking in the banking industry, the public and the shareholders have opposing interests. It is in the public's interest to invest in the safe asset rather than the risky one, whereas the shareholders have more to gain from investing in the risky asset. In Section 8 we provide a simple balance sheet-based derivation of such a payoff structure.

In what follows, we model the relationship between shareholders and managers as a principal-agent problem. Shareholders do not directly control investment decisions, but can pay managers depending on the returns generated on investments.

2.3 The Banking Game

Having described the economic environment in which banks operate, we now turn to decision-making within banks. Each bank $i = 1, \dots, n$ is owned by a single shareholder and run on his behalf by a manager. We will refer to the shareholder and the manager of bank i as shareholder i and manager i, respectively. The decision to invest in the risky or safe asset or to go out of business is the result of a strategic game (henceforth the *banking game*) played by the n shareholders and the n managers. More precisely, each period $t = 1, 2$ of the banking game proceeds as follows: First, all shareholders simultaneously offer a *wage scheme* to their respective manager. The wage scheme ω^{it} offered by shareholder i to manager i in period t is a triple $(\omega_g^{it}, \omega_{rf}^{it}, \omega_b^{it})$ specifying the manager's wage conditional on asset return. The shareholder can only use the asset return to finance the manager's wage. This budget constraint implies that the manager will earn zero if the asset return is negative. More formally, the set of possible wage schemes is $\Omega := \{\omega^{it} \in \mathbb{R}_+^3 | \omega^{it} \leq (x_g, x_{rf}, 0)\}$. Note that the shareholder cannot condition the wage on anything other than the realized asset return in the current period. More particularly, the remuneration of one manager cannot be conditioned on the performance of other managers.

Once the shareholders have made their wage offers, each manager i observes the wage scheme ω^{it} but not the wage schemes ω^{jt} offered to the other managers $j \in N \setminus \{i\}$. Then all managers choose simultaneously between three options: refuse the offered wage scheme and work outside the banking sector ("opt out"), accept the wage scheme and invest in the risky asset ("take risk"), or accept the wage scheme and invest in the safe asset ("invest safely"). If manager i does not opt out, then the instantaneous utilities of shareholder i

and manager i in period t are

$$u_s^{it} = \max(x_k - \omega_k, 0), \tag{3}$$

and

$$u_m^{it} = \omega_k \tag{4}$$

for $k \in \{g, rf, b\}$, respectively. If manager i does opt out, then the resulting instantaneous utilities are

$$u_s^{it} = 0 \tag{5}$$

and

$$u_m^{it} = D > 0. \tag{6}$$

We interpret D as the wage that the manager could earn outside the banking industry in each period. We make the following assumptions on D in relation to the other model parameters:

$$x_{rf} \geq D > (1-p)x_g + px_b. \tag{7}$$

These inequalities are related to the social desirability of investments by banks. The safe asset is at least as socially desirable as the manager's outside option. However, the value of the manager's outside option is greater than the expected payoff from the risky investment if risk-taking in the banking system is excessive. Note that the second inequality is satisfied whenever x_b is negative and sufficiently large in absolute value, that is, when the adverse consequences of a crisis are sufficiently bad.

During each period, no actions by a shareholder or a manager of one bank are observable to any other bank's shareholder or manager. After the first period, the investment choices of the first period become publicly observable. Moreover, all shareholders and managers can observe the occurrence of a crisis. A shareholder cannot make a credible commitment in the first period to a wage scheme he will offer in the second period. Moreover, we assume that a shareholder cannot replace the manager after the first period, even if the manager rejects the shareholder's wage offer for the second period.[6]

We have now described how each period of the game is played. In order to complete the

[6] This could be justified for instance by the human capital argument of Hart and Moore (1994). Once a manager is employed, the shareholder faces a loss when he replaces him, as the manager has acquired human capital to run the bank. Our current set-up with wage offers by the shareholders assigns all bargaining power to shareholders. The analysis can be performed for circumstances in which wage offers are made by managers.

formal description of the incentives in this game, we now specify the intertemporal utilities of shareholders and managers. For this purpose, we will now formally introduce the *Crisis Contract*: It is a remuneration rule for bank managers which stipulates that the manager's wage from the first period is taxed retroactively at a flat rate of $c \in [0,1]$ in the event of a crisis in the second period.[7] Hence a Crisis Contract leads to a kind of collective liability for bank managers.

Definition 2.2. *Suppose that manager i has earned wage ω_k^{i1} at $t = 1$, and suppose that a crisis occurs at $t = 2$. Then the manager will be charged a* crisis tax *of $c\ \omega_k^{i1}$, where $c \in [0,1]$ and $k \in \{g, rf, b\}$.*

Note that the Crisis Contract is only relevant if the manager has worked for the bank and obtained a strictly positive wage in the first period. A manager who takes the outside option of D in the first period will never be liable for crisis tax. We have already seen that the interests of the shareholder and the public diverge. A shareholder can use "performance pay" (i.e., condition wage payment on the return on investment) to align the manager's interests with his own. The introduction of a Crisis Contract may allow the government to distort the alignment of shareholder and manager interests in order to better protect the interests of the public.

We assume that the manager is risk-neutral with utility being linear in income. Thus intertemporal utility of a manager is additively separable into the instantaneous utilities of the two periods and the possible crisis tax. More specifically, manager i's intertemporal utility is given by

$$U_m^i = u_m^{i1} + \delta\ u_m^{i2} - \delta\ Z_2\ c\ u_m^{i1}, \tag{8}$$

where $\delta \in [0,1]$ is the discount factor. As we proceed, we will sometimes refer to $(1 - \delta\ Z_2\ c)u_m^{i1}$ as the manager's *net payoff* from the first period. Note that this net payoff depends on whether or not a crisis occurs in the second period. If a crisis is expected to occur in the second period with some probability $\tilde{p} \in [0,1]$ and u_m^{i1} is manager i's expected instantaneous payoff in the first period, then we will refer to the quantity $(1 - \delta\tilde{p}c)u_m^{i1}$ as the manager i's *expected net payoff* from the first period. In this context, we can also think of quantity $(1 - \delta\tilde{p}c)u_m^{i1} + \delta u_m^{i2}$ as manager i's *expected intertemporal utility.*

Intertemporal utilities of the shareholder are additively separable into instantaneous utilities, and all shareholders have the same time preferences as all managers. Accordingly,

$$U_s^i = u_s^{i1} + \delta u_s^{i2}. \tag{9}$$

[7] If the government is unsure about the feasibility of retroactive taxation, it may require the relevant share of the first period wage to be deposited in escrow or in a frozen account.

Of course, the linear specification above implies that all shareholders and all managers are risk-neutral. We can think of the wage payments as a direct transfer of utility from the shareholders to the managers.

2.4 Banking Equilibrium

To complete the description of the game, we now need to specify the notion of a strategy and the solution concepts. To begin with, we focus on second-period subgames of the banking game only. The *first-period history* of the banking game consists of the wage schemes offered in the first period, the investment decisions taken by the managers in the first period, and the realization of Z_1. We use h to denote such a first-period history of the banking game. The set of all first-period histories is denoted by $H \subset \Omega^n \times \{R, S, O\}^n \times \{0, 1\}$. We will only consider histories that are consistent with the rules of the game. For instance, a first-period history in which all managers have invested safely but a crisis has occurred is not consistent and hence does not belong to the set H.

From the information in a history h one can infer the amount of crisis tax each manager i would have to pay in the second period if a crisis were to happen. We refer to this amount as the *looming crisis tax* and treat it as a function $\tau^i(h)$ of the first-period history. We will refer to the subgame starting in the second period following first-period history h as the *h-subgame.* A strategy σ_s^{ih} for shareholder i in the h-subgame consists only of wage offer $\omega^{ih} \in \Omega$. A strategy σ_m^{ih} for manager i in the h-subgame is a partition of the set Ω into three subsets: the set of wage offers to which manager i responds by taking risk, those to which he responds by investing safely, and those to which he responds by opting out. Suppose σ^h is a strategy profile for the h-subgame. This strategy profile induces an activity profile $A^{-i2}(\sigma^h)$ indicating the second-period investment decisions of all banks other than bank i. Assuming that managers $j \in N \setminus \{i\}$ do indeed choose according to $A^{-i2}(\sigma^h)$, and given the wage he has realized in the first period, manager i can calculate the expected amount of crisis tax he will have to pay if he takes risk, or invests safely, or opts out.

If, in addition, he is given any second-period wage scheme $w \in \Omega$, manager i can compute his expected payoffs from risk-taking, investing safely, and opting out. If the strategy σ_m^{ih} partitions the set Ω in such a way that manager i chooses an action with maximal expected payoff, then we will say that manager i's strategy σ_m^{ih} is *a best response to $A^{-i2}(\sigma^h)$.* Note that the optimality of σ_m^{ih} relates only to the activity profile, not to the whole profile σ^h. This kind of optimal behavior is crucial for the equilibrium concept we use to solve the h-subgame and to which we refer as the *h-banking equilibrium.*

Definition 2.3. *A strategy profile* $\sigma^h = (\sigma_m^{1h}, \ldots, \sigma_m^{nh}, \sigma_s^{1h}, \ldots, \sigma_s^{nh})$ *in the* h*-subgame is an* h-banking equilibrium *if*

1. *For each manager* $i \in N$*, the strategy* σ_m^{ih} *is a best response to* $(A^{-i2}(\sigma^h), \tau^i(h))$*.*
2. *For each shareholder* $i \in N$*, the strategy* σ_s^{ih} *is a best response to* $A^{-i2}(\sigma^h)$ *and* σ_m^{ih}*.*

The next step is to introduce the strategies and the solution concept for the entire banking game. A strategy σ_m^i for manager i in the banking game consists of a strategy σ_m^{ih} in the h-subgame for every $h \in H$ and a partition of Ω into three subsets, the set of wage offers to which manager i responds by risk-taking, those to which he responds by investing safely, and those to which he responds by opting out in the first period. A strategy σ_s^i of shareholder i consists of a strategy σ_s^{ih} in the h-subgame for every $h \in H$ and a wage offer ω^{i1}.

Recall that we have assumed that at the end of the first period the shareholders and managers of bank i learn about the investment choices of the remaining banks $j \in N \setminus \{i\}$ but cannot condition their behavior in the second period on the wage offers ω^{j1} for $j \in N \setminus \{i\}$. We formalize this assumption indirectly as a restriction on the strategy space rather than directly on the information structure of the game. This is convenient, as it allows us to maintain a definition of the first-period history under which every such history is the root of a proper subgame.[8] In particular, we require the strategies of the shareholders and managers to satisfy the following restriction: If two first-period histories $h', h'' \in H$ involve the same first-period investment choices by all banks and the same realization of Z_1, and if for bank i the first-period wage payment to manager i under h' is equal to the first-period wage payment to manager i under h'', then $\sigma_m^{ih'} = \sigma_m^{ih''}$ and $\sigma_s^{ih'} = \sigma_s^{ih''}$.[9] A banking equilibrium is then defined as follows:

Definition 2.4. *A strategy profile* $\sigma = (\sigma_m^1, \ldots, \sigma_m^n, \sigma_s^1, \ldots, \sigma_s^n)$ *is a* banking equilibrium *if the following holds for every* $i \in \{1, ..., n\}$*:*

1. *For every* $h \in H$ *consistent with* σ*, the restriction of* σ *to the* h*-subgame is an* h*-banking equilibrium.*

[8]If the restriction at hand was dropped from the definition of the strategy space, then additional equilibria could arise in which the investment choice of one bank depends on earlier wage offers made in other banks. But the wage is a mere redistribution of payoffs between the shareholder and manager of a particular bank and need not in any way concern the shareholders or managers of other banks. Accordingly, such equilibria seem implausible.

[9]We note that histories h' and h'' can only differ with respect to wage offers and realized wage payments at banks other than bank i and with respect to the wage offer at bank i except for the realized wage in the first period.

2. *Given $A^{i2}(\sigma^h)$ for every $h \in H$ consistent with σ and given $A^{-i1}(\sigma)$, the partition of Ω prescribed by σ^i_m for the first period is optimal.*

3. *Given $A^{i2}(\sigma^h)$ for every $h \in H$ consistent with σ, given $A^{-i1}(\sigma)$, and σ^i_m, the wage offer prescribed by σ^i_s for the first period is optimal.*

The idea of a banking equilibrium is based on the subgame-perfect Nash equilibrium. The game-theoretic literature distinguishes between non-cooperative games with sequential and simultaneous moves. In a game with simultaneous moves, one finds Nash equilibria by fixed-point search. Each player's strategy must be a best response to the strategies of the other players. In games with sequential moves, one finds subgame-perfect Nash equilibria by backward induction. Replacing the ultimate decision nodes by the payoffs resulting from an optimal choice at these nodes, one obtains a reduced game with a "shorter" tree. This reduction is repeated until only the initial node remains. The banking game has both simultaneous and sequential move aspects. On the one hand, the decision by the manager of a bank always follows a decision by the shareholder of the same bank. On the other hand, shareholders decide simultaneously on wage schemes, and the manager of one bank is uninformed about the moves made by the shareholders and managers of the other banks. In addition, the game is played over two periods.

The procedure for finding the banking equilibria is backward induction nested in a fixed-point argument. Initially, we fix the activities of all but one bank. Given the activities of other banks, we can find a best-response correspondence for the manager of the one remaining bank. We then proceed by backward induction to the shareholder of that bank and determine his optimal decision. In this way, we find the optimal activity of the bank under consideration. That is, we have begun with a specification of bank activities by all but one bank and obtained an optimal bank activity for the bank under consideration. If we do this for every bank one at a time, we obtain a map from a profile of bank activities to a profile of bank activities. A fixed point of that map is an h-banking equilibrium. One important feature of the banking equilibrium is that it restricts the managers' behavior at non-singleton information sets. Loosely speaking, one could describe the banking equilibrium as being "information-set perfect" rather than "subgame perfect." [10]

[10] One could also think of the banking equilibrium as a Bayesian equilibrium with deterministic beliefs. We do not, however, define belief systems in this paper, since we do not focus on the whole set of Bayesian equilibria of the banking game.

3 Second-period Subgames

In this section we focus on the second period and deal first with the relationship between the shareholder and the manager of each bank. Later we deal with the strategic interaction between the different banks.

Fix a first-period history $h \in H$ and a strategy profile σ^h for the h-subgame. Let σ^h be such that manager i works for the bank (rather than taking the outside option) in the second period. In this case, we define manager i's *expected wage* in the second period as

$$\mu^{i2}(\sigma^h) = \begin{cases} \omega_g^{i2}(\sigma^h) & \text{if } n_2(\sigma^h) \leq \bar{n} - 1 \text{ and } A^{i2}(\sigma^h) = R, \\ (1-p)\omega_g^{i2}(\sigma^h) & \text{if } n_2(\sigma^h) \geq \bar{n} \text{ and } A^{i2}(\sigma^h) = R, \\ \omega_{rf}^{i2}(\sigma^h) & \text{if } A^{i2}(\sigma^h) = S. \end{cases}$$

This wage is "expected" in the sense that the realization of the possible move of nature in the second period is not yet known. Observe that the expected wage has only been defined for the case where the manager works for the bank and thus forgoes his outside option. Let us now suppose that manager i deviates from the strategy profile σ^h by opting out in the second period. This has three effects on his expected payoff. First, he loses the expected wage as defined above. Second, he gains the payoff D. Third, if $n_2(\sigma^h) = \bar{n}$ and $A^{i2}(\sigma^h) = R$, then his deviation to the outside option reduces the probability of a crisis from p to zero and the expected crisis tax bill from $p\tau^i(h)$ to zero. Whenever the latter two effects outweigh the loss of the expected wage, the deviation to the outside option is profitable. More formally, we say that manager i is *pivotal under* σ^h if $n_2(\sigma^h) = \bar{n}$ and $A^{i2}(\sigma^h) = R$. In words, manager i is considered pivotal if his decision to take risk is responsible for attaining threshold $\bar{n}$.

For a (not necessarily pivotal) manager i, we define his *reservation wage* as

$$\kappa^{i2}(\sigma^h) = \begin{cases} D + p\tau^i(h) & \text{if } n_2(\sigma^h) = \bar{n} \text{ and } A^{i2}(\sigma^h) = R, \\ D & \text{otherwise.} \end{cases}$$

From the construction of the reservation wage it follows that working for a bank is optimal for a manager only if the expected wage is greater than, or equal to, the reservation wage. In the sequel, our claim is that in equilibrium the expected wage is equal to the reservation wage. The intuition behind this claim is straightforward. If the expected wage were higher than the reservation wage, then it would be a profitable deviation for the shareholder to offer the manager an infinitesimally lower remuneration. This is a variant of an argument well-known in the literature on ultimatum and bargaining games, where equilibrium offers make the responding player exactly indifferent between accepting or declining the offer.

Lemma 3.1. *Fix a bank $i \in \{1, ..., n\}$. Suppose σ^h is a strategy profile in the h-subgame and $A^{i2}(\sigma^h) \neq O$. Also suppose the strategies σ_m^{ih} and σ_s^{ih} are best responses to σ^h. Then it holds that $\kappa^{i2}(\sigma^h) = \mu^{i2}(\sigma^h)$.*

Proof. Suppose σ^h is an h-banking equilibrium in the h-subgame and $A^{i2}(\sigma^h) \neq O$. Suppose $\mu^{i2}(\sigma^h) > \kappa^{i2}(\sigma^h)$. Let $\bar{\omega}^{i2}$ be the wage offer made under σ^h. For $\varepsilon > 0$, define the triple $\hat{\omega}^{i2}$ as follows:

$$\hat{\omega}^{i2} = \begin{cases} (\bar{\omega}_g^{i2} - \varepsilon, 0, 0) & \text{if } n_2(\sigma^h) \leq \bar{n} - 1 \text{ and } A^{i2}(\sigma^h) = R \\ (\bar{\omega}_g^{i2} - \frac{\varepsilon}{(1-p)}, 0, 0) & \text{if } n_2(\sigma^h) \geq \bar{n} \text{ and } A^{i2}(\sigma^h) = R \\ (0, \bar{\omega}_{rf}^{i2} - \varepsilon, 0) & \text{if } A^{i2}(\sigma^h) = S \end{cases}$$

Note that for sufficiently small $\varepsilon > 0$, the triple $\hat{\omega}^{i2}$ belongs to the set Ω and is therefore a wage scheme to which the strategy σ_m^{ih} must assign a response from $\{R, S, O\}$. We argue that it is the same response as the one assigned to wage scheme $\bar{\omega}^{i2}$. In order to see this, first observe that if manager i responds to the offer $\hat{\omega}^{i2}$ in the same way as to the offer $\bar{\omega}^{i2}$, then by the construction of $\hat{\omega}^{i2}$ he will obtain a payoff of $\mu^{i2}(\sigma^h) - \varepsilon > D$ for $\varepsilon > 0$ small enough. It is clear that manager i will not respond to $\hat{\omega}^{i2}$ by opting out. Now suppose, by way of contradiction, that strategy σ_m^{ih} responds to $\hat{\omega}^{i2}$ by working for the bank but taking a different asset than in response to $\bar{\omega}^{i2}$. Again, by the construction of $\hat{\omega}^{i2}$ this would lead to a zero payoff for manager i. However, $\mu^{i2}(\sigma^h) - \varepsilon > D > 0$, so it is clear that manager i will not respond to $\hat{\omega}^{i2}$ by working for the bank and choosing a different asset than in response to $\bar{\omega}^{i2}$. We have now established that strategy σ_m^{ih} prescribes the same response from manager i to both wage schemes $\bar{\omega}^{i2}$ and $\hat{\omega}^{i2}$. To complete the proof of the lemma, observe that for shareholder i it is a profitable deviation from strategy profile σ^h to offer $\hat{\omega}^{i2}$ instead of $\bar{\omega}^{i2}$. This deviation increases shareholder i's payoff by $\varepsilon > 0$. □

Lemma 3.1 demonstrates how the shareholder can align the manager's interests with his own: First, by setting the appropriate component of the wage scheme equal to zero, the shareholder can ensure that a manager who works for the bank will choose the asset preferred by the shareholder. Second, by choosing the remaining component of the wage scheme so as to equalize expected and reservation wages, the shareholder can ensure that a manager will indeed work for the bank. A manager who works for the bank always expects to receive D unless he is pivotal, in which case he obtains an additional transfer payment of $p\tau^i(h)$ compensating him for the cost of crisis tax.

If all banks take risk under the strategy profile σ^h, then we say that σ^h involves *full risk.* If exactly $\bar{n}-1$ banks take risk under strategy profile σ^h, then we say that σ^h involves *threshold risk.*

Lemma 3.2. *An h-banking equilibrium involves either full risk or threshold risk.*

Proof. The proof of Lemma 3.2 rests on two contradictions. Suppose first that σ^h is an h-banking equilibrium but $n_2(\sigma^h) < \bar{n}-1$. Then there is $i \in N$ such that $A^{i2}(\sigma^h) \neq R$. Clearly, manager i is not pivotal under σ^h, so his payoff is D. From the supposition that σ^h is an h-banking equilibrium we conclude that strategy σ_m^{ih} prescribes taking risk as a response to offer $(D+\varepsilon, 0, 0)$ for $\varepsilon > 0$ sufficiently small (and in particular $\varepsilon < (1-p)x_g - x_{rf}$). If shareholder i deviates from σ^h by making the offer $(D+\varepsilon, 0, 0)$, then he obtains a payoff of $x_g - D - \varepsilon$. Under σ^h, however, his payoff would be either $x_{rf} - D$ or zero. Since $x_g > x_{rf}$ and $x_g > D$, we see that shareholder i has a profitable deviation from σ^h. Hence we obtain a contradiction and conclude that $n_2(\sigma^h) \geq \bar{n}-1$.

Now suppose secondly that σ^h is an h-banking equilibrium but $n > n_2(\sigma^h) \geq \bar{n}$. Then there is $i \in N$ such that $A^{i2}(\sigma^h) \neq R$. Clearly, manager i is not pivotal under σ^h, so his payoff is D. From the supposition that σ^h is an h-banking equilibrium we conclude that strategy σ_m^{ih} prescribes taking risk as a response to the offer $(\frac{D+\varepsilon}{1-p}, 0, 0)$ for $\varepsilon > 0$ sufficiently small. If shareholder i deviates from σ^h by making the offer $(\frac{D+\varepsilon}{1-p}, 0, 0)$, he obtains a payoff of $(1-p)x_g - D - \varepsilon$. Under σ^h, however, his payoff would be either $x_{rf} - D$ or zero. Since by assumption $(1-p)x_g > x_{rf} + \varepsilon$, shareholder i has a profitable deviation from σ^h. Therefore we again obtain a contradiction, so $n_2(\sigma^h) = \bar{n}-1$ or $n_2(\sigma^h) = n$. □

One implication of Lemma 3.2 is that no manager is pivotal in an h-banking equilibrium. But if a manager is not pivotal, his expected and reservation wage will both be equal to his outside option.

Corollary 3.3. *In an h-banking equilibrium, the expected wage of each manager in the h-subgame is equal to D.*

Having excluded any other types of h-banking equilibria, we now turn to the conditions for the existence of h-banking equilibria involving full risk or threshold risk.

Theorem 3.4. *For every h-subgame, there is an h-banking equilibrium that involves full risk.*

The proof of Theorem 3.4 can be found in the Appendix.

Risk-taking by all banks in the second period can be supported by an h-banking equilibrium irrespective of the choice of the model parameters, first-period history, or a Crisis Contract. This result is driven by the fact that in the second period managers can only collectively eliminate the risk of a crisis tax. In other words, they need to coordinate on not crossing threshold $\bar{n}$. Due to the assumption that $\bar{n} \leq n-1$, no individual manager can insulate himself against the crisis tax. Indeed, once the threshold $\bar{n}$ is crossed, each individual manager has an incentive to take risk. In the first period, where each individual manager can eliminate the risk of the crisis tax for himself by opting out, no coordination among managers is necessary.

Let us define the following threshold for the looming crisis tax:

$$\tau^* = \frac{(1-p)x_g - x_{rf}}{p}.$$

It follows from our assumptions that threshold τ^* is strictly positive.

Lemma 3.5. *If there is an h-banking equilibrium involving threshold risk, then $\tau^i(h) \geq \tau^*$ for at least $n - \bar{n} + 1$ banks.*

Proof. Suppose that σ^h is an h-banking equilibrium and $n_2(\sigma^h) = \bar{n} - 1$. Then there are $n - \bar{n} + 1$ banks $i \in N$ such that $A^{i2}(\sigma^h) \neq R$. Take one such bank, say i. Observe that i is not pivotal under σ^h. Hence, from the supposition that σ^h is an h-banking equilibrium, we obtain that $\mu^{i2}(\sigma^h) = D$. We argue that strategy σ_m^{ih} prescribes taking risk as the response to offer $\hat{\omega}^{i2} = (\frac{D+\varepsilon+p\tau^i(h)}{1-p}, 0, 0)$. To see this, note that the expected payoff to manager i from taking risk in response to $\hat{\omega}^{i2}$ is equal to $D + \varepsilon + p\tau^i(h) - p\tau^i(h) = D + \varepsilon$, whereas the expected payoff to manager i from investing safely or opting out in response to $\hat{\omega}^{i2}$ is zero and D, respectively. Now suppose that shareholder i deviates from σ^h by making offer $\hat{\omega}^{i2}$. Then he will obtain an expected payoff of $(1-p)x_g - D - \varepsilon - p\tau^i(h)$, whereas his payoff under σ^h is $x_{rf} - D$. We conclude that shareholder i has a profitable deviation from σ^h if $(1-p)x_g - p\tau^i(h) > x_{rf}$, or, equivalently, if $\tau^i(h) < \tau^*$. Repeating the argument for every $i \in N$ with $A^{i2}(\sigma^h) \neq R$, we obtain the claim of the lemma. □

When $c = 0$, then we have $\tau^i(h) = 0$ for all $i \in N$ and $h \in H$. Since $\tau^* > 0$, we know that an h-banking equilibrium with threshold risk does not exist in the absence of Crisis Contracts.

Corollary 3.6. *In the absence of Crisis Contracts (i.e., $c = 0$) all h-banking equilibria in all h-subgames involve full risk.*

Lemma 3.5 states a necessary condition for an h-banking equilibrium involving threshold risk. We show next that the necessary condition is also sufficient.

Theorem 3.7. *For every $h \in H$, there is an h-banking equilibrium involving threshold risk if and only if $\tau^i(h) \geq \tau^*$ for at least $n - \bar{n} + 1$ banks.*

The proof of Theorem 3.7 can be found in the Appendix.

4 Managerial Pay in the First Period

We now turn to the first period. The expected wage of manager i in the first period can be defined analogously to the earlier definition of μ^{i2} as follows:

$$\mu^{i1}(\sigma) = \begin{cases} \omega_g^{i1}(\sigma) & \text{if } n_1(\sigma) \leq \bar{n} - 1 \text{ and } A^{i1}(\sigma) = R, \\ (1-p)\omega_g^{i1}(\sigma) & \text{if } n_1(\sigma) > \bar{n} - 1 \text{ and } A^{i1}(\sigma) = R, \\ \omega_{rf}^{i1}(\sigma) & \text{if } A^{i1}(\sigma) = S. \end{cases}$$

However, the realized wage may be subject to the crisis tax in the second period. Given that some strategy profile σ is played throughout the banking game, we can define the *ex ante* probability $\rho(\sigma)$ with which a crisis will occur in the second period. In this context, ex ante means that $\rho(\sigma)$ has not been updated with the possible move of nature in the first period. For example, if σ is such that all managers invest safely in the second period, then $\rho(\sigma) = 0$. If, on the contrary, σ is such that all managers take risk in the second period, then $\rho(\sigma) = p$. It is possible, however, for $\rho(\sigma)$ to take other values than 0 or p. For example, suppose that under strategy profile σ managers $1, \ldots, \bar{n} - 1$ always take risk in the second period, whereas managers $\bar{n}, \ldots, n$ take risk in the second period if and only if a crisis has occurred in the first period. Suppose further that under σ all managers take risk in the first period. In that case, we have $\rho(\sigma) = p^2$. Based on the probability $\rho(\sigma)$ and the expected wage of manager i in the first period, we can define manager i's *expected net wage* under σ from the first period as follows:

$$\nu^{i1}(\sigma) = (1 - \delta c \rho(\sigma))\, \mu^{i1}(\sigma).$$

To understand this expected net wage intuitively, suppose that by the end of the first period a manager has earned a certain wage, say w. Since the manager has worked for a bank in the first period, he may now have to pay crisis tax. So, the realized wage of w in the first period only increases his intertemporal utility in expected terms by the amount $(1 - \delta c\rho(\sigma))\, w$.

Lemma 4.1. *If σ is a banking equilibrium and $A^{i1}(\sigma) \neq O$, then $\nu^{i1}(\sigma) = D$.*

Proof. First suppose, by way of contradiction, that $\bar{\sigma}$ is a banking equilibrium and $\nu^{i1}(\bar{\sigma}) < D$. By Corollary 3.3, manager i's expected wage at $t = 2$ equals D. Hence, his expected intertemporal wage under $\bar{\sigma}$ is $\nu^{i1}(\bar{\sigma}) + \delta D < (1 + \delta)D$. However, by opting out in both periods, manager i could have obtained the intertemporal payoff of $(1 + \delta)D$, which is a contradiction.

Second, suppose now that $\bar{\sigma}$ is a banking equilibrium and $\nu^{i1}(\bar{\sigma}) > D$. Again, we show that this leads to a contradiction. Let $\bar{\omega}^{i1}$ and $\bar{\omega}^{i2}$ be the wage schemes offered by shareholder i associated with the supposed banking equilibrium $\bar{\sigma}$. Moreover, consider the following alternative wage schemes:

$$\hat{\omega}^{i1} = \begin{cases} (\bar{\omega}_g^{i1} - \frac{2\varepsilon}{1-\delta c\rho(\sigma)}, 0, 0) & \text{if } n_1(\sigma) \leq \bar{n} - 1 \text{ and } A^{i1}(\sigma) = R, \\ (\bar{\omega}_g^{i1} - \frac{2\varepsilon}{(1-p)(1-\delta c\rho(\sigma))}, 0, 0) & \text{if } n_1(\sigma) \geq \bar{n} \text{ and } A^{i1}(\sigma) = R, \\ (0, \bar{\omega}_{rf}^{i1} - \frac{2\varepsilon}{1-\delta c\rho(\sigma)}, 0) & \text{if } A^{i1}(\sigma) = S, \end{cases}$$

for the first period, and

$$\hat{\omega}^{i2} = \begin{cases} (\tilde{\omega}_g^{i2} + \varepsilon, 0, 0) & \text{if } A^{i2}(\bar{\sigma}) = R, \\ (0, \tilde{\omega}_{rf}^{i2} + \varepsilon, 0) & \text{if } A^{i2}(\bar{\sigma}) = S, \end{cases}$$

for the second period.

The proof strategy is to show that a unilateral deviation by shareholder i from $\bar{\sigma}$ to the alternative wage schemes $(\hat{\omega}^{i1}, \hat{\omega}^{i2})$ is profitable. In the first step below, we demonstrate that under banking equilibrium profile $\bar{\sigma}$, manager i will respond to both $\hat{\omega}^{i1}$ and $\bar{\omega}^{i1}$ with the same choice from $\{R, S, O\}$.

Step 1. Suppose that this is not the case. That is, suppose that strategy $\bar{\sigma}_m^i$ assigns different responses to wage offers $\hat{\omega}^{i1}$ and $\bar{\omega}^{i1}$. If manager i opts out in response to $\hat{\omega}^{i1}$, then his payoff from the first period is D. If manager i does not opt out in response to $\hat{\omega}^{i1}$, then, by construction of $\hat{\omega}^{i1}$, his payoff from the first period is zero. Since $\bar{\sigma}$ is a banking equilibrium, manager i's expected payoff in the second period is D. If he does not opt out in response to $\hat{\omega}^{i1}$, his intertemporal payoff is δD, but if he does opt out, it is

$(1+\delta)D$. We see that manager i reacts to $\hat{\omega}^{i1}$ by opting out. He obtains the intertemporal payoff of $(1+\delta)D$ in the banking game under $\bar{\sigma}$. However, if manager i did respond to $\hat{\omega}^{i1}$ and $\bar{\omega}^{i1}$ with the same action, then, by construction of $\hat{\omega}^{i1}$, his intertemporal payoff would be $\nu^{i1}(\bar{\sigma}) - 2\varepsilon + \delta D > (1+\delta)D$, where the inequality follows directly from the supposition that $\nu^{i1}(\bar{\sigma}) > D$ when $\varepsilon > 0$ is small enough. We see that manager i has a profitable deviation from $\bar{\sigma}$, which yields the desired contradiction.

We conclude that if $\bar{\sigma}$ is a banking equilibrium, then manager i responds by the same action to two wage offers $\hat{\omega}^{i1}$ and $\bar{\omega}^{i1}$.

Step 2. Now suppose that shareholder i deviates from $\bar{\sigma}$ by offering $\hat{\omega}^{i1}$ in the first period and $\hat{\omega}^{i2}$ in the second period. Clearly, shareholder i's payoff from the first period increases by $2\varepsilon > 0$. As we have shown, the deviation to $\hat{\omega}^{i1}$ does not have any effect on the investment choices of the banks in the first period. Hence the actions of shareholders and managers $j \in N \setminus \{i\}$ in the second period are unaffected by the deviation to $\hat{\omega}^{i1}$; consequently, banks $j \in N \setminus \{i\}$ will act according to activity profile $A^{-i2}(\bar{\sigma})$ in the second period.

Step 3. We now want to show that manager i responds to wage offer $\hat{\omega}^{i2}$ by the same investment choice with which he responds to $\bar{\omega}^{i2}$. Notice that under $\bar{\sigma}$, manager i's payoff from the second period is D. If he responds to $\hat{\omega}^{i2}$ with the same action as to $\bar{\omega}^{i2}$, then the resulting payoff will be $D + \varepsilon$. If manager i opts out in response to $\hat{\omega}^{i2}$, then his expected payoff from the second round is D. If he works for the bank, but makes a different investment choice than under $\bar{\sigma}$, then, by construction of $\hat{\omega}^{i2}$, his payoff is zero. We see that following the unilateral deviation by shareholder i to wage offers $(\hat{\omega}^{i1}, \hat{\omega}^{i2})$, activity profiles $A^{i1}(\bar{\sigma})$ and $A^{i2}(\bar{\sigma})$ remain unchanged.

Step 4. We have considered a unilateral deviation by shareholder i from the supposed banking equilibrium $\bar{\sigma}$ to wage offers $(\hat{\omega}^{i1}, \hat{\omega}^{i2})$. We have shown that this deviation leaves activity profiles $A^{i1}(\bar{\sigma})$ and $A^{i2}(\bar{\sigma})$ unchanged in both rounds. But then, by the construction of $(\hat{\omega}^{i1}, \hat{\omega}^{i2})$, the deviation increases shareholder i's net expected payoff from the first round by 2ε and decreases shareholder i's expected payoff from the second round by ε. Indeed, the deviation increases shareholder i's utility by the amount $2\varepsilon - \delta\varepsilon \geq \varepsilon > 0$; it is thus profitable. We have obtained the desired contradiction, and the proof of the lemma is complete. □

The above lemma shows that in the first period as well, a shareholder will extract all surplus created by the investment. A manager receives a payment that makes him indifferent between working for the bank and opting out. If a banking equilibrium is such that a manager has to pay crisis tax with positive probability, then a shareholder will compensate the manager for the expected crisis payment by an increase in wage in the first period. The expected intertemporal payoff for the manager is always equal to the payoff from taking the outside option in both periods.

Corollary 4.2. *In a banking equilibrium, each manager receives an expected intertemporal payoff equal to* $(1+\delta)D$.

Corollary 4.3. *In a banking equilibrium, each manager's instantaneous payoff in the first period is at least* D.

Suppose that manager i receives wage offer ω in the first period and accepts this offer. Observe that the manager's realized wage can then never exceed $\max\{\omega_g, \omega_{rf}\}$. We will say that a wage scheme $\omega \in \Omega$ is *insufficient* if $D > \max\{\omega_g, \omega_{rf}\}$. We know from the above corollary that in a banking equilibrium no manager works under an insufficient wage scheme, as accepting an insufficient wage scheme is a strictly dominated strategy for every manager. Hence from now on we restrict managers' behavior by the assumption that an insufficient wage scheme will not be accepted.[11]

Assumption 4.4. *A manager will not accept an insufficient wage scheme.*

Suppose that strategy profile σ is such that $A^{i1}(\sigma) \neq O$. If the banking game is played according to σ and if no crisis occurs in the first period, then this assumption guarantees that manager i will realize a wage of at least D in the first period. The importance of this assumption is technical. In the rest of the paper, we will discuss entire strategy profiles that are banking equilibria. Such strategy profiles must specify actions to be taken in the second period after any first-period history. However, we have introduced the restriction that the shareholders and managers of an individual bank cannot condition their behavior on the wage offers of other banks but only on their own investment choices. Without the assumption above, one would have to specify second-period actions for players $j \neq i$ after a history in which a manager i has worked for an insufficient wage, and these actions would not be allowed to differ from those taking place after a history in which the manager has received a sufficient wage.

[11] Note that whether or not a wage scheme is insufficient is defined purely on the basis of D, which is a primitive of the model. In particular, a wage scheme that leads to an expected wage lower than the reservation wage need not be insufficient. Conversely, an insufficient wage scheme always leads to an expected wage that is lower than the reservation wage.

Here we conclude our discussion of managerial pay in the banking game. In the next two sections, we will consider two specific types of banking equilibria, which from the social welfare point of view turn out to be the best and worst equilibria, respectively.

5 Full Risk Equilibria

We refer to a banking equilibrium as a *full risk equilibrium* if it involves full risk in both periods on the equilibrium path of play. In particular, a full risk equilibrium involves risk-taking by all managers in the second period, irrespective of the realization of Z_1. If σ is a full risk equilibrium, then we have $\rho(\sigma) = p$. In this section we give necessary and sufficient conditions for the existence of a full risk equilibrium. We will also be interested in the "uniqueness" of such equilibria.

To understand intuitively when a full risk equilibrium does or does not exist, let us consider an individual bank. If the bank is out of business in the first period, the manager's payoff is D and the shareholder's payoff is zero. Recall that the manager's outside payoff of D will never be subject to crisis tax, as a Crisis Contract applies only to wages earned in the banking sector in the first period. Now we turn to the case where the bank invests in the risky asset in both periods. Here the expected net payoff from bank activities in the first round is equal to $(1-p)x_g - \delta p(1-p)cx_g$, which is shared by the manager and the shareholder of the bank under consideration. The first term is simply the expected return on the risky asset in the first period. The second term is the expected utility loss from the crisis tax. A crisis tax will only have to be paid if there is no crisis in the first period but a crisis does occur in the second period. Given that all banks take the risky investment in both periods, the probability of this event is $p(1-p)$. If this event takes place, crisis tax cx_g has to be paid. It may now seem intuitive that a full risk equilibrium exists if $(1-p)x_g - \delta p(1-p)cx_g \geq D$ or, equivalently, if c does not exceed the threshold c' defined as

$$c' \quad = \quad \frac{(1-p)x_g - D}{\delta p(1-p)x_g}. \tag{10}$$

A formal analysis, however, is more involved, since our model includes strategic interaction between the shareholder and the manager of each bank and crisis tax is levied on the manager's realized wage, not on asset returns. Nevertheless, the following theorem confirms our first intuition, indeed it establishes that $c \leq c'$ is both a necessary and a sufficient condition for the existence of a full risk equilibrium.

Theorem 5.1. *A full risk equilibrium exists if and only if $c \leq c'$.*

Proof ($\Leftarrow$). Suppose $c \leq c'$. The proof is constructive. Consider a strategy profile $\bar{\sigma}$, under which every shareholder $i \in N$ makes wage offers

$$\begin{aligned}\bar{\omega}^{i1} &= \left(\frac{D}{(1-p)(1-\delta pc)}, 0, 0\right) \text{ and} \\ \bar{\omega}^{i2} &= \left(\frac{D}{1-p}, 0, 0\right).\end{aligned}$$

The managers' choices under $\bar{\sigma}$ are as follows:

- At $t = 1$, each manager $i \in N$ opts out in response to the wage scheme $\omega^{i1} \in \Omega$ if and only if $D > \max\{(1-p)(1-\delta pc)\omega_g^{i1}, (1-\delta pc)\omega_{rf}^{i1}\}$. Conditional on not opting out, manager i will take risk if and only if $(1-p)(1-\delta pc)\omega_g^{i1} > (1-\delta pc)\omega_{rf}^{i1}$.
- At $t = 2$, each manager $i \in N$ opts out in response to the wage scheme $\omega^{i2} \in \Omega$ if and only if $D > \max\{(1-p)\omega_g^{i2}, \omega_{rf}^{i2}\}$. Conditional on not opting out, manager i will take risk if and only if $(1-p)\omega_g^{i2} > \omega_{rf}^{i2}$.

Moreover, under $\bar{\sigma}$ each manager i will choose to work at his bank in the second period if $(1-p)\max(\bar{\omega}_g^{i2}, \bar{\omega}_{rf}^{i2}) \geq D$. If he works at his bank in the second period, he will take risk if and only if $\bar{\omega}_g^{i2} \geq \bar{\omega}_{rf}^{i2}$. In the first period, each manager i chooses to work at his bank if $(1-p)(1-\delta cp)\max(\bar{\omega}_g^{i1}, \bar{\omega}_{rf}^{i1}) \geq D$. If he works at his bank, he will take risk if and only if $\bar{\omega}_g^{i1} \geq \bar{\omega}_{rf}^{i1}$ and invest safely otherwise.

For any history $h \in H$, the relevant restriction $\bar{\sigma}^h$ of strategy profile $\bar{\sigma}$ is an h-banking equilibrium in the h-subgame. This has been demonstrated in the proof of Theorem 3.4. Hence we need only consider unilateral deviations in the first period. It is straightforward to see that the managers' decisions in the first period are optimal. We show that no shareholder has a profitable unilateral deviation from $\bar{\sigma}$ in the first period. Under $\bar{\sigma}$, shareholder i's payoff in the first period equals $(1-p)x_g - \frac{D}{1-\delta pc}$. Suppose that shareholder i deviates from $\bar{\sigma}$ by offering some $\tilde{\omega}^{i1}$ to which manager i responds by investing safely. Since manager i does not opt out, it must be true that $(1-\delta pc)\tilde{\omega}_{rf}^{i1} \geq D$. Then, however the shareholder' s payoff in the first period is bounded from above by $x_{rf} - \frac{D}{1-\delta pc}$. Since $(1-p)x_g > x_{rf}$, this deviation is not profitable. Now suppose that shareholder i deviates from $\bar{\sigma}$ by offering some $\tilde{\omega}^{i1}$ to which manager i responds by opting out. In that case, the payoff to shareholder i in the first period is zero. By rewriting the supposition that $c \leq c'$, it holds that $(1-p)x_g - \frac{D}{1-\delta pc} \geq 0$, so this deviation is again not profitable. Indeed, $\bar{\sigma}$ is a banking equilibrium.

Proof (⇒). Suppose that a full risk equilibrium σ exists. Then $\nu^{i1}(\sigma) = D$ implies $\mu^{i1}(\sigma) = \frac{D}{1-\delta pc}$. Hence the payoff of shareholder i from the first period is $(1-p)x_g - \frac{D}{1-\delta pc}$. Since shareholder i could guarantee a payoff of zero by proposing $(0,0,0)$ to manager i, it must be true that $(1-p)x_g - \frac{D}{1-\delta pc} \geq 0$. Indeed, rearranging this inequality yields $c \leq c'$. □

We now turn to the question whether the full risk equilibrium is "unique" in the sense that all banking equilibria are full risk equilibria. We have shown previously that, without Crisis Contracts, the banking equilibrium unambiguously predicts full risk in the second period. This is the result of Corollary 3.6. However, if full risk will always prevail in the second period, then we can conclude that the expected wage of a manager in the first period must equal $\frac{D}{1-\delta pc}$ irrespective of the investment choices made in the first period. This observation yields

Theorem 5.2. *In the absence of Crisis Contracts, all banking equilibria are full risk equilibria.*

Proof. Suppose that $c = 0$. Let $\bar{\sigma}$ be a banking equilibrium. By Corollary 3.6 it holds that $A^{i2}(\sigma^h) = R$ for all $h \in H$ and $i \in N$. Consequently, a deviation from $\bar{\sigma}$ in the first period has no effect on risk choices or wages in the second period.

Suppose, by way of contradiction, that there is $i \in N$ so that $A^{i1}(\bar{\sigma}) \neq R$. Either $A^{i1}(\bar{\sigma}) = O$ and then the shareholder will earn zero, or $A^{i1}(\bar{\sigma}) = S$ and then the shareholder will earn $x_{rf} - D > 0$. If $c = 0$ and if $\bar{\sigma}$ is a banking equilibrium but not a full risk equilibrium, then at least one shareholder will earn $x_{rf} - D$ in the first period.

Consider wage offer $\tilde{\omega}^{i1} = (\frac{D+\varepsilon}{1-p}, 0, 0)$. Manager i would respond to this proposal by taking risk. However, if shareholder i were to deviate from $\bar{\sigma}$ by proposing $\tilde{\omega}^{i1}$, then the resulting expected payoff for shareholder i would be either $x_g - \frac{D+\varepsilon}{1-p}$ or $(1-p)x_g - D - \varepsilon$. Clearly, the latter term is smaller than the former, so the deviation yields shareholder i a gain of at least $(1-p)x_g - D - \varepsilon - x_{rf} + D = (1-p)x_g - x_{rf} - \varepsilon$. This gain is positive when $\varepsilon > 0$ is chosen sufficiently small. Shareholder i then has a profitable unilateral deviation from $\bar{\sigma}$. This is a contradiction. □

Theorem 5.2 establishes a benchmark for our further analysis of Crisis Contracts. We have now demonstrated that, in the absence of Crisis Contracts, the banking equilibrium unambiguously predicts full risk, so a crisis will occur with positive probability in both periods.

6 Threshold Equilibria

We define a *threshold equilibrium* as a banking equilibrium in which threshold risk is played in both periods on the equilibrium path. Note that in a threshold equilibrium the probability of a crisis in either period is equal to zero. A strategy profile which is a threshold equilibrium may prescribe full risk in the second period after a crisis has occurred in the first period. Such first-period histories are not on the equilibrium path. Let

$$c'' = \frac{(1-p)x_g - x_{rf}}{pD}. \tag{11}$$

Theorem 6.1. *A threshold equilibrium exists only if $c \geq c''$.*

Proof. Suppose that $\bar{\sigma}$ is a threshold equilibrium. Let $\bar{h}$ be the first-period history induced by playing according to $\bar{\sigma}$ in the first period. Then, by the definition of a threshold equilibrium, we know that $\bar{\sigma}^{\bar{h}}$ is an h-banking equilibrium that involves threshold risk in the $\bar{h}$-subgame. By Theorem 3.7, this implies that $\tau^i(\bar{h}) \geq \tau^*$ for at least $n - \bar{n} + 1$ banks. Since under $\bar{\sigma}$ the crisis probability in either period is zero, we have $\mu^{i1}(\bar{\sigma}) = \nu^{i1}(\bar{\sigma}) = D$ for all $i \in N$, and the realized wage in the first period is D for all $i \in N$. Hence $\tau^i(\bar{h}) = cD$ for all $i \in N$. It follows that $cD \geq \tau^*$. Substituting from the definition of τ^* and rearranging this inequality, we find that $c \geq c''$, as desired. □

We have established a necessary condition for the existence of a threshold equilibrium. For the next theorem we shall now derive a set of sufficient conditions. Let us define $\hat{n} = (\bar{n} - 1)/n$. This ratio tells us which share of the banks in our banking sector can take the risky investment without running the risk of triggering a crisis. We can interpret $\hat{n}$ as a measure for the stability of the banking sector. We will also make use of the following condition:

$$x_{rf} \geq x_g \left(1 - p\left(\frac{1+\delta p}{1+\delta p - \delta}\right)\right). \tag{12}$$

Note that for any choice of parameters x_g, x_{rf}, and p, the above inequality is satisfied when δ is sufficiently close (or equal) to one.

Assuming that $\hat{n}$ is sufficiently large and that inequality (12) holds, we now construct a threshold equilibrium of the following kind: On the equilibrium path of play, exactly $\bar{n}-1$ banks take the risky asset in each period, while the remaining banks invest in the safe asset. All banks investing safely in the first period take the risky asset in the second period. (Clearly, this is only possible when $\hat{n} \geq \frac{1}{2}$.) If some bank deviates from the equilibrium path in the first period by investing in the risky rather than the safe asset, then the game enters into a "punishment mode." If a crisis occurs in the first period, then all banks take the risky asset in the second period. If the game is in the punishment mode, but no crisis has occurred in the first period, then the bank that has deviated from the equilibrium path in the first period is among those banks taking the safe asset in the second period. Since the punishment occurs in the second period, the shareholders and managers need to care sufficiently about the future payoff for the punishment to be effective. This makes it intuitively clear why inequality (12) is crucial for the result.

Theorem 6.2. *Suppose that $\hat{n} \geq \frac{1}{2}$, inequality (12) holds, and $c \geq c''$. Then a threshold equilibrium exists.*

Proof. Define a strategy profile $\bar{\sigma}$ as follows: In the first period, shareholders make the following wage offers:

$$\begin{aligned}\bar{\omega}^{i1} &= (D,0,0),\ i=1,\ldots,\bar{n}-1,\\ \bar{\omega}^{j1} &= (0,D,0),\ j=\bar{n},\ldots,n.\end{aligned}$$

Furthermore, under $\bar{\sigma}$ managers $i=1,\ldots,\bar{n}-1$ respond to the wage offer ω^{i1} by opting out if and only if $D > \max\{\omega_g^{i1}, \omega_{rf}^{i1}\}$. If a manager does not opt out, he will respond by taking risk if and only if $\omega_g^{i1} \geq \omega_{rf}^{i1}$ and invest safely otherwise. Furthermore, under $\bar{\sigma}$ manager $j=\bar{n},\ldots,n$ will react to the wage offer ω^{j1} in the following way: He opts out if and only if $D > \max\{(1-p)\omega_g^{j1}, \omega_{rf}^{j1}\}$. Conditional on not opting out, manager j will take risk if and only if $(1-p)\omega_g^{j1} > \omega_{rf}^{j1}$ and invest safely otherwise.

We now define the restriction $\bar{\sigma}^h$ of $\bar{\sigma}$ to the h-subgame for each $h \in H$. If all shareholders and all managers play according to $\bar{\sigma}$ in the first period, then there will be no crisis. That is, play according to $\bar{\sigma}$ induces a unique history, which we will denote henceforth by $\bar{h} \in H$. To define $\bar{\sigma}^h$, we distinguish three cases.

1. Suppose that h is a first-period history involving the same investment choices by all banks as in $\bar{h}$. By Assumption 4.4, it holds that $\tau^k(h) \geq cD$ for all $k \in N$. By the supposition that $c \geq c''$, it follows that $\tau^k(h) \geq \tau^*$ for all $k \in N$. There exists an

h-banking equilibrium involving threshold risk, in which all banks $j = \bar{n}, \dots, n$ take risk. Let $\bar{\sigma}^h$ be that h-banking equilibrium.

2. Suppose that history h is such that no bank has been out of business in the first period and $Z_1 = 0$. Moreover, suppose that the investment choices under h differ from those under $\bar{h}$ with regard to exactly one bank, say $k' \in N$. Again, by Assumption 4.4 it holds that $\tau^k(h) \geq cD$ for all $k \in N$. By the supposition that $c \geq c''$, it follows that $\tau^k(h) \geq \tau^*$ for all $k \in N$. There exists an h-banking equilibrium involving threshold risk in which bank k' makes the safe investment. Let $\bar{\sigma}^h$ be such an h-banking equilibrium.

3. For any other histories $h \in H$, let $\bar{\sigma}^h$ be an h-banking equilibrium involving full risk.

Now we need to show that $\bar{\sigma}$ is in fact a banking equilibrium. By construction, it holds for all $h \in H$ that $\bar{\sigma}^h$ is an h-banking equilibrium of the h-subgame. Hence, one only has to verify that there is no profitable unilateral deviation in the first period. Moreover, it holds by construction of $\bar{\sigma}$ that a unilateral deviation by shareholder k' or manager k' will lead to an h-banking equilibrium in which no crisis tax is to be paid by manager k'. This implies that manager k' expects a payoff of D in the second period, regardless of whether bank k' makes the first-period investment choice prescribed by $\bar{h}$ or whether bank k' is the only bank to make a different investment choice. Consequently, a deviation by a manager in the first-period can only be profitable if it increases that manager's instantaneous payoff in the first period. It is now straightforward to see that there is no profitable unilateral deviation from $\bar{\sigma}$ in the first period for any manager.

What is left to show is that no shareholder can gain from unilateral deviation from $\bar{\sigma}$ in the first period.

First, consider a shareholder $i = 1, \dots, \bar{n} - 1$. Clearly, wage offer $\bar{\omega}^{i1}$ is optimal among all those wage offers to which manager i responds by taking risk under $\bar{\sigma}$. Suppose that shareholder i deviates from $\bar{\sigma}$ by offering a wage scheme ω^{i1} to which manager i responds by investing safely. Then shareholder i's payoff is bounded from above by $(1+\delta)(x_{rf} - D)$. However, under $\bar{\sigma}$, shareholder i's payoff is $x_g - D + \delta(x_{rf} - D)$. Since $x_g > x_{rf}$, this deviation is not profitable. Now suppose that shareholder i deviates from $\bar{\sigma}$ by offering a wage scheme ω^{i1} to which manager i responds by opting out. Then shareholder i's payoff is $0 + \delta(x_{rf} - D)$. Since $x_g - D > 0$, this deviation is not profitable.

Now consider a shareholder $j = \bar{n}, \dots, n$. Clearly, wage offer $\bar{\omega}^{j1}$ is optimal among all those wage offers to which manager j responds by choosing the safe investment under $\bar{\sigma}$. Suppose that shareholder j deviates from $\bar{\sigma}$ by offering a wage scheme to which manager j responds by opting out. The resulting payoff for shareholder j is $0+\delta(x_{rf}-D)$. However, under $\bar{\sigma}$, shareholder j's payoff would be $x_{rf}-D+\delta(x_g-D)$. Since $x_g>D$, this deviation is not profitable. Finally, suppose that shareholder j deviates from $\bar{\sigma}$ by offering a wage scheme to which manager j responds by taking risk. Then shareholder j's payoff is bounded from above by

$$(1-p)x_g+\delta p(1-p)x_g+\delta(1-p)x_{rf}-(1+\delta)D.$$

To see this, recall that after a change in the investment choice of a single bank in the first period, no crisis tax will have to be paid by that bank in the second period. This explains the last term in the expression above. If bank j makes the risky investment in the first period, then a crisis will happen with probability p. So the expected asset return in the first period is $(1-p)x_g$, explaining the first term in the expression above. If a crisis does occur in the first period, then we have a situation where an h-banking equilibrium with full risk is played in the second period, hence the second term. If, by contrast, no crisis occurs in the first period, then we have a situation where the h-banking equilibrium with threshold risk is played in the second period and bank j makes the safe investment, hence the third term. In order to complete the proof, we need to show that

$$(1-p)x_g+\delta p(1-p)x_g+\delta(1-p)x_{rf}-(1+\delta)D\leq(x_{rf}-D)+\delta(x_g-D).$$

Rearranging this inequality yields (12), which holds by the supposition of the theorem. □

Theorem 6.2 shows that if an additional restriction on $\hat{n}$ is satisfied and if the discount factor is sufficiently high, $c\geq c''$ is a necessary and sufficient condition for the existence of a threshold equilibrium. We note that the threshold equilibrium constructed in the proof of Theorem 6.2 can coexist with a full risk equilibrium. In Theorem 6.3 below, we present an existence result that addresses these shortcomings, meaning that it does not require a condition on $\hat{n}$ and cannot coexist with a full risk equilibrium. We construct a threshold equilibrium that differs from the previous one. This threshold equilibrium exists irrespective of the choice of $\hat{n}$. It requires the inequality

$$x_g\leq\frac{x_{rf}(1+\delta)-D}{\delta(1-p)} \tag{13}$$

to be satisfied. Furthermore, the threshold equilibrium we are now going to construct requires that $c>c'$, so it can only exist when there is no full risk equilibrium. The idea behind the threshold equilibrium to be constructed is that in both periods the banks $1,\ldots,\bar{n}-1$

make the risky investment, while the other banks make the safe investment. If one of the banks $\bar{n}, \ldots, n$ deviates from the equilibrium path of play in the first period by choosing the risky investment, then all banks will invest in the risky asset in the second period as a "punishment." Note that the threat of such a punishment is always credible since, in any second period subgame, risk-taking by all banks is consistent with the banking equilibrium described in Theorem 3.4. This type of threshold equilibrium can be constructed even in the extreme case of $\bar{n} = 1$. The idea behind this threshold equilibrium is as follows: The punishment mechanism ensures that full risk will be played in the second period if more than $\bar{n} - 1$ managers take risk in the first period. Anticipating this punishment, a manager would only be willing to take risk in the first period if the shareholder offers him a compensation for the potential crisis tax payment. However, if the crisis tax rate is sufficiently high, then the shareholder cannot afford such a compensation.

Theorem 6.3. *Suppose that $c > c'$, $c \geq c''$, and that inequality (13) holds. Then a threshold equilibrium exists.*

Proof. The proof is constructive. Define the strategy profile $\bar{\sigma}$ as follows: In the first period, shareholders make the wage offers

$$\begin{aligned} \bar{\omega}^{i1} &= (D, 0, 0),\ i = 1, \ldots, \bar{n} - 1, \\ \bar{\omega}^{j1} &= (0, D, 0),\ j = \bar{n}, \ldots, n. \end{aligned}$$

A manager $i = 1, \ldots, \bar{n} - 1$ will opt out in response to wage offer ω^{i1} if and only if $D > \max\{\omega_g^{i1}, (1 - \delta pc)\omega_{rf}^{i1}\}$. Conditional on not opting out, he will select the risky investment if and only if $\omega_g^{i1} \geq (1 - \delta pc)\omega_{rf}^{i1}$.

A manager $j = \bar{n}, \ldots, n$ will opt out in response to wage offer ω^{j1} if and only if $D > \max\{(1-p)(1-\delta pc)\omega_g^{j1}, \omega_{rf}^{j1}\}$. Conditional on not opting out, he will take risk if and only if $(1 - \delta pc)(1 - p)\omega_g^{i1} \geq \omega_{rf}^{i1}$.

Now we define $\bar{\sigma}^h$ for every $h \in H$. Note that $\bar{\sigma}$ induces a unique first-period history, say $\bar{h}$. We distinguish two cases.

1. If h is a history involving the same investment choices by all banks in the first period as $\bar{h}$, then there has been no crisis. By Assumption 4.4, it holds that $\tau^k(\bar{h}) \geq cD$ for all $k \in N$. From the supposition that $c \geq c''$ it follows that $\tau^k(\bar{h}) \geq \tau^*$ for all $k \in N$. There exists an h-banking equilibrium involving threshold risk in which each shareholder makes the same wage offer at $t = 2$ as at $t = 1$. Let $\bar{\sigma}^h$ be that h-banking equilibrium.

2. If h is a history that does not involve the same investment choices by all banks in the first period as $\bar{h}$, then let σ^h be an h-banking equilibrium involving full risk.

Now we need to show that $\bar{\sigma}$ is a banking equilibrium. By construction, $\bar{\sigma}^h$ is an h-banking equilibrium for every $h \in H$. We need to verify the absence of any profitable unilateral deviation in the first period. Note that by construction of $\bar{\sigma}$, a crisis tax will only have to be paid if some bank $j = \bar{n}, \dots, n$ has made the risky investment in the first period but there has been no crisis in that period. This reveals that there is no profitable unilateral deviation from $\bar{\sigma}$ for any manager. It remains to be shown that no shareholder has a profitable deviation from $\bar{\sigma}$ in the first period.

First, consider a unilateral deviation by shareholder $i = 1, \dots, \bar{n} - 1$ in the first period. Clearly, $\bar{\omega}^{i1}$ is optimal among all those wage schemes to which manager i responds by taking risk. Suppose shareholder i deviates from $\bar{\sigma}$ by offering some wage scheme ω^{i1} to which manager i responds by investing safely. Shareholder i's payoff from this deviation is bounded from above by $x_{rf} - \frac{D}{1-\delta pc} + \delta(x_{rf} - D)$. However, his payoff under $\bar{\sigma}$ is $x_g - D + \delta(x_g - D)$. We see that the deviation is not profitable. Now suppose that shareholder i deviates from $\bar{\sigma}$ by offering some wage scheme ω^{i1} to which manager i responds by opting out. The resulting payoff to shareholder i is $0 + \delta(1-p)x_g - \delta D$. Again, we find that this is strictly less than the (expected) payoff $x_g - D + \delta(x_g - D)$ for shareholder i under $\bar{\sigma}$. Therefore the deviation is not profitable.

Now consider a unilateral deviation by shareholder $j = \bar{n}, \dots, n$. Clearly, $\bar{\omega}^{j1}$ is optimal among all those wage offers to which manager j responds by investing safely. Suppose shareholder j deviates from $\bar{\sigma}$ with a wage offer ω^{j1} to which manager j responds by opting out. Then shareholder j's payoff is $0 + \delta(1-p)x_g - \delta D$. But his payoff from $\bar{\sigma}$ is $(x_{rf} - D)(1+\delta)$. Rearranging inequality (13) yields $\delta(1-p)x_g - \delta D \leq (x_{rf} - D)(1+\delta)$. We see that the deviation is not profitable. Finally, suppose that shareholder j deviates from $\bar{\sigma}$ by offering a wage scheme ω^{j1} to which manager j responds by taking risk. But manager j will only respond to ω^{j1} by taking risk if $\omega_g^{j1} \geq \frac{D}{(1-p)(1-\delta pc)}$. By the shareholder's budget constraint we have $x_g \geq \omega_g^{j1}$, so $x_g \geq \frac{D}{(1-p)(1-\delta pc)}$. Appropriately rearranging this inequality, we find $c \leq c'$ - a contradiction to the supposition of the theorem. We conclude that there is no profitable unilateral deviation from $\bar{\sigma}$ for any shareholder in the first period. Hence $\bar{\sigma}$ is a banking equilibrium. □

One feature of the threshold equilibrium constructed in the proof of Theorem 6.3 is that it cannot coexist with a full risk equilibrium. By contrast, the threshold equilibrium constructed in the proof of Theorem 6.2 can coexist with a full risk equilibrium. This will be important for the analysis of welfare gains from Crisis Contracts in the next section.

7 Welfare Effects of Crisis Contracts

We now conduct a comparative statics analysis of the effects of Crisis Contracts. In particular, we will be interested in whether the introduction of Crisis Contracts enhances social welfare. Social welfare is defined as follows:

In period t ($t \in \{1,2\}$), *instantaneous social welfare* is given by

$$y^t = \sum_{i=1}^{n} \big\{ (u_s^{it} + u_m^{it}) + \min(x_k^{it}, 0) \big\}, \tag{14}$$

for $k \in \{g, rf, b\}$. The instantaneous utilities u_s^{it} and u_m^{it} are as specified in equations (3), (5), (4), and (6). The term $\min(x_k^{it}, 0)$ captures the social losses that occur in case of a crisis, which are neither internalized by the shareholders nor by the managers. It is clear from the above utility functions that y^t depends solely on the activity profile at t. As we have seen, for given investment choices the wage payments are pure redistributions from shareholders to managers; they do not affect social welfare accounting. We also consider tax revenue as part of social welfare, so the payment of crisis tax does not affect social welfare either. The above notion of social welfare implies that the managers' income is part of social welfare, whether or not they work in the banking sector. Hence, in the absence of any banking activity, the instantaneous social welfare level would be

$$y_0 = nD. \tag{15}$$

Under the above definition, instantaneous social welfare in period t is maximized when $\bar{n} - 1$ managers take risk whereas $n - \bar{n} + 1$ managers invest safely, i.e. social welfare is maximal under threshold risk. The maximal level of instantaneous social welfare is

$$\bar{y}^t = (\bar{n} - 1)x_g + (n - \bar{n} + 1)x_{rf}. \tag{16}$$

On the other hand, risk-taking by all managers leads to instantaneous expected social welfare given by

$$\underline{y}^t = n(1-p)x_g + npx_b. \tag{17}$$

Observe that $\bar{y}^t > y_0 > \underline{y}^t$. This reflects the fact that our model assumptions enable the banking sector both to enhance and to harm social welfare in expected terms as compared to a situation in which no bank activity takes place.

It remains to define aggregate social welfare over the two periods, denoted by Y. We assume that

$$Y = y^1 + \delta y^2. \tag{18}$$

We assume the same time preference for society at large as for the shareholders and managers. However, the results in the sequel will hold for any positive social discount factor.

Under the above notion of social welfare, we can view the full risk and threshold equilibria as, respectively, the worst and best outcomes of the banking game. So far in this paper, we have conducted a comparative statics analysis to see how the existence of these two types of equilibria depends on the model parameters, and in particular on crisis tax rate c. In what follows, we discuss how an appropriate choice of c by the regulator can improve social welfare.

Let $\mathcal{E}(c)$ be the set of banking equilibria when the crisis tax rate is $c \in [0,1]$. We use notations $\sigma' \succeq \sigma''$ to indicate that strategy profile σ' leads to at least as much expected social welfare as strategy profile σ'' and $\sigma' \succ \sigma''$ to indicate that social welfare under σ' is strictly greater than under σ''.

Definition 7.1. *A Crisis Contract with a tax rate of $c > 0$ is* weakly beneficial *if the following two conditions hold:*

1. *For all $(\sigma', \sigma'') \in \mathcal{E}(0) \times \mathcal{E}(c)$, it holds that $\sigma'' \succeq \sigma'$.*
2. *There is $\sigma' \in \mathcal{E}(c)$ such that $\sigma' \succ \sigma''$ for all $\sigma'' \in \mathcal{E}(0)$.*

The first part of the definition requires that the introduction of the Crisis Contract does not lead to an equilibrium which is worse that some equilibrium without Crisis Contracts. The second part of the definition requires that the introduction of the Crisis Contract leads to some equilibrium which is strictly better than any equilibrium without Crisis Contracts.

If an increase in social welfare can be obtained for any selection from the set of banking equilibria, then the Crisis Contract is considered to be *strictly beneficial,* as formalized in the next definition.

Definition 7.2. *A Crisis Contract with a tax rate of $c > 0$ is* strictly beneficial *if $\mathcal{E}(c) \neq \emptyset$ and $\sigma'' \succ \sigma'$ for all $(\sigma', \sigma'') \in \mathcal{E}(0) \times \mathcal{E}(c)$.*

Of course, a Crisis Contract that is strictly beneficial is also weakly beneficial.

We have seen that the existence of full risk and threshold equilibria depends on how the tax rate c relates to some threshold values c' and c''. In our model, the regulator is free to choose any tax rate c from the interval $[0, 1]$. Consequently, the power of the regulator to influence the existence of full risk and threshold equilibria hinges on whether c' and c'' fall into this interval. Using equations (10) and (11), we can express conditions $c' < 1$ and $c'' < 1$ as restrictions on x_g relative to the other model parameters. Intuitively, the regulator's ability to improve social welfare through Crisis Contracts requires that x_g be not too big relative to payoffs x_{rf} and D and to probability p. More precisely, the relevant restrictions on x_g are captured in the following two inequalities:

$$x_g \quad < \quad \frac{D}{(1-\delta p)(1-p)}, \tag{19}$$

$$x_g \quad < \quad \frac{pD + x_{rf}}{1-p}. \tag{20}$$

In general, neither of these inequalities implies the other.

Theorem 7.3. *If inequalities (13), (19), and (20) are satisfied, then there exists a strictly beneficial Crisis Contract. Moreover, under this Crisis Contract the socially optimal outcome of the banking game is an equilibrium.*

Proof. Note that, due to Theorem 5.2, all elements of $\mathcal{E}(0)$ are full risk equilibria and thus induce the strictly lowest social welfare among all equilibria. On the other hand, due to Theorem 5.1 and inequality (19), there exists $c \in (0, 1)$ such that $\mathcal{E}(c)$ does not contain a full risk equilibrium. Due to inequalities (13), (19), and (20), by Theorem 6.3 we may choose the value of c such that $\mathcal{E}(c)$ contains a threshold equilibrium, which leads to the socially optimal outcome of the banking game. Hence $\mathcal{E}(c) \neq \emptyset$.

□

Note that if p is close enough to 1, there will always exist a strictly beneficial Crisis Contract.

To illustrate Theorem 7.3, we provide the following numerical example:

Example 7.4. *Consider an example where the discount factor is $\delta = 1$ and the crisis probability $p = 0.5$. Let the payoffs be $D = 1$ and $x = (3,\ 1.4,\ -1.5)$.*

In the benchmark scenario with no Crisis Contracts, all banking equilibria are full risk equilibria. The social welfare is predicted to be $2n(1-0.5)3+(2n)(0.5)(-1.5) = 3n-1.5n = 1.5n$*. However, if there was no bank activity, the social welfare level of* $2n$ *could be reached. In the example at hand, the banking equilibrium unambiguously predicts a welfare loss from bank activity in the absence of Crisis Contracts.*

Note that inequality (13) holds in this example. Now suppose we introduce a Crisis Contract into the example. We calculate $c' = \frac{2}{3} \approx 0.66$ *and* $c'' = 0.2$*. Take a crisis tax rate of* $c = 0.67$*. The introduction of such a Crisis Contract will lead to the existence of a threshold equilibrium. The social welfare in this equilibrium is given by* $2((\bar{n}-1)3 + (n-\bar{n}+1)1.4) > 2n$*. Note that this holds independently of the value of* $\bar{n}$*. Finally note that the Crisis Contract rules out the full risk equilibria. Hence it is strictly beneficial.*

The following theorem establishes conditions under which a weakly beneficial equilibrium Crisis Contract exists, even if at least one of the inequalities (13) and (19) does not hold.

Theorem 7.5. *Suppose that* $\hat{n} \geq \frac{1}{2}$*. If inequalities (12) and (20) hold, then there is a weakly beneficial Crisis Contract. Moreover, under this Crisis Contract the socially optimal outcome of the banking game is an equilibrium.*

Proof. Note that, due to Theorem 5.2, all elements of $\mathcal{E}(0)$ are full risk equilibria. Due to inequalities (12) and (20), by Theorem 6.2, we may choose the value of c such that $\mathcal{E}(c)$ contains a threshold equilibrium, which leads to the socially optimal outcome of the banking game. Hence c fulfills the first condition in Definition 7.1. Since any full risk equilibrium induces the strictly lowest social welfare among all equilibria, c fulfills the second condition in Definition 7.1. Hence a Crisis Contract with tax rate c is weakly beneficial.

□

Note that if δ is sufficiently small, then in each of the above theorems at least one necessary condition does not hold. This means that Crisis Contracts are ineffective. This is intuitive, since a Crisis Contract can only have an impact if managers care enough about their second-period payoffs.

To illustrate Theorem 7.5, we provide the following numerical example:

Example 7.6. *Consider an example where the discount factor is* $\delta = 1$ *and the crisis probability* $p = 0.5$*. Let the payoffs be* $D = 1$ *and* $x = (3,\ 1.2,\ -1.5)$*.*

Note that inequality (13) does not hold in this example. Hence Theorem 7.3, which guarantees the existence of a strictly beneficial Crisis Contract, does not apply here. However, note that due to $\delta = 1$ inequality (12) holds. Assume in addition that the banking system is stable to some extent, namely such that $\tilde{n} \geq 0.5$ holds. It holds that $c'' = 0.6$. Then the Crisis Contract with tax rate $c = 0.61$ is weakly beneficial.

8 Capital Regulation and Crisis Contracts

In the previous section, we derived the key welfare result of our paper. The welfare-enhancing potential of Crisis Contracts crucially hinges on a set of three conditions on the model parameters. Each of these conditions imposes upper bounds on x_g, the return of the risky asset when no crisis occurs. We will now argue that a moderate value of x_g is suitably interpreted as resulting from stringent capital requirements. This allows us to draw conclusions about the interaction between capital requirements and our idea of Crisis Contracts in the effective regulation of financial sector risk. To do this, we first show how the incentive structure of our model can be derived from the banks' balance sheets.

To be more precise, consider a bank that finances its activities by deposits and by equity. Equity is given and we normalize the amount of equity to one. At the beginning of each period considered in our model, households deposit a total amount of d at the bank, and the bank promises them an interest rate of $r > 0$ over this period. Hence, at the end of a period, the bank owes $(1 + r)d$ to its depositors. We assume that the bank can invest its entire funds $1 + d$ in a risk-free asset, with interest rate r. Hence, the payoff that we have called x_{rf} in our model can be written as

$$x_{rf} = (1 + d)(1 + r) - d(1 + r) = 1 + r.$$

Alternatively, the bank may invest its entire funds in a risky asset which pays an interest rate of $r' > r$ if no crisis occurs, but which leads to a loss of fraction l ($l \in [0, 1]$) of the bank's total capital $1 + d$ in case of a crisis. When no crisis occurs the bank has an amount $(1 + r')(1 + d)$ at its disposal at the end of the period, while it owes $(1 + r)d$ to the depositors. After paying out the depositors, the remaining amount is available to equity holders. Then, the payoff that we have called x_g in our model can be written as

$$x_g = (1 + d)(1 + r') - d(1 + r) = 1 + r' + d(r' - r).$$

The payoff that we have denoted by x_b in our model can be written as

$$x_b = (1 - l)(1 + d) - (1 + r)d.$$

Suppose that $d \leq d^{crit} := \frac{1-l}{r+l}$ and thus $x_b \geq 0$. Then even in the worst case, bank equity would be sufficient to redeem all obligations towards depositors. If, however, $d > d^{crit}$ and thus $x_b < 0$, a bank which takes risk cannot honor all obligations towards its depositors in a crisis. The shortfall will have to be covered by a public-bailout fund. This is the scenario we have considered in our model.

We see that x_g tends to infinity, as d grows without bound. As equity is given, capital requirements impose an upper limit on d. Therefore, we can say that the inequalities which require x_g to be below certain bounds can be interpreted as requiring sufficiently strong capital requirements. As long as capital requirements are sufficiently strong in the aforementioned sense, however, Crisis Contracts can serve as a substitute to some extent. More precisely, recall that we have defined thresholds c' and c'' for the tax rate c, so that a Crisis Contract has to impose at least this threshold tax rate to be effective. Observe that these thresholds are increasing in x_g. If an effective Crisis Contract is in place, and one relaxes capital requirements somewhat, then the Crisis Contract can maintain its effectiveness if the associated tax rate c is increased. On the other hand, note that both inequalities (12) and (13) — each of which is a necessary condition for the existence of one type of threshold equilibrium — are fulfilled as long as x_g is not too large. Hence, the relationship between Crisis Contracts and capital requirements can be wrapped up as follows:

There exist thresholds $\underline{\phi}, \overline{\phi} \in \mathbb{R}_+$ such that $\overline{\phi} > \underline{\phi}$ and

1. if the banks' debt-equity ratio is below some threshold $\underline{\phi}$, risk-taking and banking crises do not necessitate public bailouts.

2. If the banks' debt-equity ratio is above threshold $\overline{\phi}$, then banks exhibit socially detrimental risk-taking which cannot be discouraged by Crisis Contracts.

3. If capital requirements are such that the banks' debt-equity ratios are within the interval $[\underline{\phi}, \overline{\phi}]$, then Crisis Contracts are welfare-enhancing. Within this interval, the regulator can achieve the welfare-enhancing effect by using different combinations of the tax rate associated with the Crisis Contract and the debt-equity ratio induced by the capital requirements. Stricter capital requirements allow a lower tax rate in the Crisis Contract, while a higher tax rate in the Crisis Contract allows more lenient capital requirements.

9 Hedging

9.1 Risk-neutral Players

We now examine whether the players could circumvent the effectiveness of Crisis Contracts by hedging themselves against the crisis tax by buying either insurance or a put option. Suppose that manager i, who has worked for a bank in the first period and has earned $\omega^{i1} > 0$, could buy insurance $I(Pr, R)$, which would pay him amount R in the case of crisis if he pays premium Pr in advance. It is natural to assume that the insurance contract will be signed and the insurance premium paid by the manager in the second period, after the wage offer has been made, and before the manager makes his investment decision. Payment from the insurance to the manager is conducted after Z_2 has been observed. We assume a competitive insurance market and that insurance firms offer actuarial fair insurance premiums.

Let us focus on the interesting case of a threshold equilibrium. First suppose that a fair insurance intends to enter the market and does not account for possible changes in investment decisions by managers. We argue that the insurance will suffer losses. Supposing that the threshold equilibrium is played, a fair insurance must have $Pr = 0$. So let us consider an insurance contract $I_0 = I(0, \tau^i(h))$, i.e. the insurance is free for the manager and pays his crisis tax in the case of a crisis occurring in the second period. We claim that I_0 cannot be part of an equilibrium in a game in which insurance is modeled explicitly. We do not consider such a game in detail, but provide the intuition for this case. The reason is that if the insurance I_0 is available, every shareholder will offer the second-period wage scheme $\omega = (\frac{D}{1-p}, 0, 0)$. Then each manager will choose risky investment and a crisis will occur with probability p in the second period. Hence the insurance expects losses. Accordingly, I_0 is not a fair insurance and cannot be part of an equilibrium in such a game.

Second, suppose that a fair insurance takes into account possible changes of investment decisions by managers.[12] Then, we argue, the threshold equilibrium cannot be destroyed by the insurance. Let us consider an insurance contract of the form $I(Pr, \tau^i(h))$. We use $\rho^{ih}(\sigma)$ to denote the probability with which a crisis will occur in the second period under strategy profile σ^{-i}, given first-period history h and supposing that manager i decides to take risk in the second period. Since $R = \tau^i(h)$, the expected claim payment to manager i is $\rho^{ih}(\sigma)\tau^i(h)$. Suppose that a bank deviates from the threshold equilibrium by choosing the risky investment instead of the safe investment. Then $\rho^{ih}(\sigma) = p$. Hence, since the insurance is assumed to be fair, manager i has to pay an insurance premium of $Pr = p\tau^i(h)$.

[12]The same argumentation also holds in the case of a put option rather than an insurance.

However, to prevent manager i from opting out shareholder i has to increase the expected wage payment by $p\tau^i(h)$. This is equivalent to increasing the wage payment by $\tau^i(h)$ and eliminating the insurance contract. But in a threshold equilibrium there is no profitable deviation for the shareholder that consists in increasing the manager's wage ω_g and thereby making the manager take risk. Hence, if a threshold equilibrium exists, it is not possible to annihilate it by insurance.

9.2 Differences in Risk Appetite

Even if managers were risk-averse, they would not negotiate an insurance contract that makes them worse off in terms of expected payoff, since they can always choose to opt out and obtain the riskless payoff. In fact, in equilibria with $n_t \geq \bar{n}$, shareholders would have to promise managers higher wages to retain them. We observe that manager risk neutrality is a conservative assumption in the sense that with risk-averse managers Crisis Contracts would be even more effective for establishing a threshold equilibrium.

10 Ramifications and Conclusions

In this section we provide a detailed discussion of further extensions, the role of contract triggers, and other aspects of Crisis Contracts. We start with some numerical examples.

10.1 Examples

Here we provide some additional numerical examples to illustrate the working of the model.

Example 10.1. *Consider an example where the discount factor is $\delta = 1$ and crisis probability $p = 0.4$. Let the payoffs be $D = 1$ and $x = (3,\ 1.7,\ -2.1)$.*

Note that $c' > 1$, so there exists no strictly beneficial Crisis Contract, since full risk equilibrium can not be ruled out by any Crisis Contract. However, $c'' = 0.25$, and due to $\delta = 1$ inequality (12) holds. If it holds that $\hat{n} \geq 0.5$, then a Crisis Contract with tax rate $c = 0.26$ is weakly beneficial.

Example 10.2. *Consider an example where the discount factor is $\delta = 0.5$ and crisis probability $p = 0.5$. Let the payoffs be $D = 1$ and $x = (2.5,\ 1.2,\ -1.5)$.*

Then $c' = 0.8$, $c'' = 0.1$, and inequality (13) holds. Hence a Crisis Contract with tax rate $c = 0.81$ is strictly beneficial. Note that inequality (12) holds in this example.

Example 10.3. *Consider an example where the discount factor is* $\delta = 1$ *and low crisis probability of* $p = 0.2$*. Let the payoffs be* $D = 1$ *and* $x = (1.5,\ 1.15,\ -1.1)$.

Then $c' = 0.83$, $c'' = 0.25$, *and inequality (13) holds. Hence a Crisis Contract with tax rate* $c = 0.84$ *is strictly beneficial.*

This example shows that if the difference between x_g *and* x_{rf} *(and hence the potential gains from investment in the risky asset in comparison to investment in the safe asset) is not too big Crisis Contracts may be beneficial even when crisis probability is low.*

10.2 Extensions

In this section we briefly discuss some potential extensions of the model. In reality, vulnerabilities in the financial sector seem to build up over time. In our model, one could assume that risk-taking in the first period will not trigger a crisis immediately but rather increase the risk of a crisis in the second period. This could be expressed by making the threshold $\bar{n}$ in the second period dependent on the number of banks that took risk in the first period. We conjecture that, in such a setup, Crisis Contracts can prevent both the build-up of vulnerabilities in the first period and a crisis in the second period. The qualitative gist of the results would probably carry over to such a model, which would incorporate the following features:

- Choosing risky investments by banks in the first period does not lead immediately to a banking crisis, but increases the probability of a future banking crisis.
- The probability of a banking crisis p if at least $\bar{n}$ banks take risks in the second period is higher, the more vulnerable the banking system is.[13] Formally, p is a monotonically increasing function of $n_1(\mathcal{A})$.

As long as the parameters fulfill the relevant conditions in both periods the qualitative results generalize to this set-up. In particular, appropriately designed Crisis Contracts can not only prevent a banking crisis in the second period but also preclude the build-up of vulnerabilities in the first period.

One more potential extension is to introduce some heterogeneity among the banks. We have assumed that all banks are of the same size and have the same impact on the triggering of a crisis. However, we could easily incorporate into our model a situation where some

[13] In addition or alternatively, threshold $\bar{n}$ in the second period could be a decreasing function of the number of banks that took risk in the first period.

banks are small and have no, or only a very small, impact on crisis probability and hence neglect it when making decisions. Indeed, suppose in addition to n large banks there is a continuum of small banks. First note that if we assume that they have no impact on the crisis probability, then our results obviously continue to hold. Second, assume that each small bank has an infinitesimal impact on crisis probability and that the aggregate impact of all small banks would be equal to the impact of m large banks if they all chose the risky investment. Obviously, all small banks will chose the risky investment, since each of them neglects its impact on the crisis probability. Then if $m < \bar{n}$, the results on the impact of Crisis Contracts on crisis probability and on risk-taking incentives of big banks in this set-up are the same as in the model without small banks and with a smaller threshold value $\bar{n}_{new} = \bar{n} - m$.

We have assumed that all shareholders benefit from excessive risk-taking by managers. Suppose that some shareholders are harmed by a banking crisis. For instance, they may have invested in other firms and hence suffer from a banking crisis.[14] Suppose that $\hat{n}$ shareholders (with $\hat{n} < n - \bar{n}$) are crisis-sensitive and internalize the losses x_b from such crisis. Then we do not expect changes to the equilibrium predictions. However, the banking industry does not choose full risk, as some shareholders select the safe investment.

If the abilities of bank managers in producing different returns to shareholders differ, Crisis Contracts might help to foster the socially efficient recruitment of bank managers.

So far, we have argued that Crisis Contracts are beneficial in that they can prevent crises that would lead to social losses. To assess social benefits and costs, we have considered a very simple additive social welfare function where payoffs to citizens, shareholders, and managers are treated equally. If one views the regulator as serving the interests of the public to a greater extent than those of shareholders or managers, Crisis Contracts are even more useful. In recent public discussion, it has often been felt to be unfair that the welfare of ordinary citizens has apparently not had enough weight in the regulator's considerations. Crisis Contracts do not suffer from this problem and may therefore be seen as a "fair" regulatory tool.

10.3 Contract Triggers

To implement a Crisis Contract, it is necessary to define some verifiable criteria for the occurrence of a banking crisis. There are several possibilities for defining such triggers of

[14]Some shareholders or bank managers may also incur non-pecuniary disutilities if their investment behavior triggers a banking crisis. This would also induce crisis sensitivity.

Crisis Contracts.

One possible trigger is bailing out a bank or banks e.g. by providing fresh equity or by guaranteeing the liabilities of banks. Using government bailout as a trigger for the execution of Crisis Contracts would probably have further effects. For instance, troubled banks may be more willing to opt for the bail-in of private debtors to avoid execution of the crisis tax. In turn, the threat to bail out may help the regulator to induce better bank equity capitalization in the banking system. Moreover, when only one or a few banks are troubled and may need to be rescued by the government, other banks may be more willing to play an active role in rescue activities.

An alternative trigger is the index of stock prices in the banking industry. Crisis Contracts are executed if the index falls below a certain threshold. A third possibility for defining a trigger is the (weighted) average of the debt-equity ratios in the banking industry. If this average exceeds a threshold, crisis taxes are due.

Each of these three possibilities defining triggers of Crisis Contracts has to be assessed in detail for the pros and cons and the further effects they may involve. The third trigger in particular is conceptually appealing, as it is rooted in the average debt-equity ratio in the banking industry. It relies on accounting information that regulators collect anyway and may be least susceptible to manipulation.

10.4 Concluding Remarks

We have presented an initial analysis of Crisis Contracts and have gauged their potential and limitations. This first pass of the analysis suggests that Crisis Contracts could be a useful tool in the design of a financial architecture that is significantly more resilient than in the past.

Numerous issues deserve scrutiny. While we have used a stylized model to study the functions of Crisis Contracts, in practice they have to be based on assessments of the extent of risk-taking and the likelihood of crisis in the banking industry in a calibrated model (see e.g. Chesney et al. (2012)). Erring on the conservative side will not undermine the efficacy of Crisis Contracts, but being too optimistic about the stability of the banking system will. Moreover, Crisis Contracts will likely have further effects.

Crisis Contracts may help to break peer effects when it is common in the banking system to motivate managers with high bonuses to take risks that collectively exceed socially desirable levels.[15] Crisis Contracts may also induce banks to become more prudent regarding their counterparties in the interbank market, which may promote stability.

[15] It is well-known that peer effects play a considerable role in banking. For instance, herding with regard to risk-taking is significant among the largest banks, see Bonfim and Kim (2012).

11 Appendix

Proof of Theorem 3.4

The proof is constructive. Let $\bar{\sigma}^h$ be the strategy profile for the h-subgame, where all shareholders $i \in N$ make offer $\bar{\omega}^{i2} = (\frac{D}{1-p}, 0, 0)$ to their managers. All managers will take risk in response to any wage offer $\omega \in \Omega$ that satisfies $(1-p)\omega_g \geq D$ and $(1-p)\omega_g \geq \omega_{rf}$. If the wage offer $\omega \in \Omega$ fails to satisfy one or both of these inequalities, then the manager will invest safely if and only if $\omega_{rf} > D$. Otherwise he will opts out. We show that $\bar{\sigma}^h$ is an h-banking equilibrium.

Fix a bank $i \in N$. Given $A^{-i2}(\bar{\sigma}^h)$, a crisis will occur in the second period with probability p. For every $\omega \in \Omega$, strategy $\bar{\sigma}_m^{ih}$ chooses an option from $\{R, S, O\}$ that leads to a weakly higher payoff for manager i than any other choice. It is straightforward to see that manager i does not have any profitable deviation from $\bar{\sigma}^h$.

If manager i opts out, then shareholder i will obtain a payoff of zero. By construction of $\bar{\sigma}_m^{ih}$, manager i only works for the bank if his expected wage is at least D. Accordingly, in the case where manager i invests safely, the expected payoff to shareholder i is bounded from above by $x_{rf} - D$. And in the case where manager i takes risk, the expected payoff to shareholder i is bounded from above by $(1-p)x_g - D$. Since $(1-p)x_g > x_{rf} > 0$, it follows that given $A^{-i2}(\bar{\sigma}^h)$ the expected payoff to shareholder i is bounded from above by $(1-p)x_g$. But this is the expected payoff of shareholder i under $\bar{\sigma}^h$. We see that shareholder i has no profitable deviation from $\bar{\sigma}^h$. □

Proof of Theorem 3.7

The "only if" part has been proved in Lemma 3.5. We now prove that an h-banking equilibrium involves threshold risk if $\tau^i(h) \geq \tau^*$ for at least $n - \bar{n} + 1$ banks.

The proof is constructive. Let $\bar{\sigma}^h$ be the strategy profile for the h-subgame where shareholders $i = 1, \ldots, \bar{n} - 1$ all make wage offer $\bar{\omega}^{i2} = (D, 0, 0)$ and shareholders $j = \bar{n}, \ldots, n$ make wage offer $\bar{\omega}^{j2} = (0, D, 0)$. Managers $i = 1, \ldots, \bar{n} - 1$ will take risk in response to wage offer $\omega \in \Omega$ if and only if $\omega_g \geq \omega_{rf}$ and $\omega_g \geq D$. Otherwise those managers will invest safely if $\omega_m \geq D$ and opt out if $\omega_{rf} < D$. Managers $j = \bar{n}, \ldots, n$ will respond to wage offer $\omega \in \Omega$ by taking risk if and only if $(1-p)\omega_g - p\tau^j(h) > \omega_{rf}$ and $(1-p)\omega_g - p\tau^j(h) > D$. Otherwise those managers will invest safely if $\omega_{rf} \geq D$ and opt out if $\omega_{rf} < D$.

Consider a manager $i = 1, \ldots, \bar{n}-1$. Given $A^{-i2}(\bar{\sigma}^h)$, no crisis occurs. So manager i chooses from the payoffs D (opting out), $\bar{\omega}^{ih}_{rf} = 0$ (investing safely), and $\bar{\omega}^{ih}_g = D$ (investing in the risky asset). Clearly, taking risk is optimal.

Consider a manager $j = \bar{n}, \ldots, n$. Given $A^{-j2}(\bar{\sigma}^h)$, there will be a crisis with probability p if j takes risk and with probability zero otherwise. Manager j chooses from the payoffs D (opting out), $\bar{\omega}^{jh}_{rf} = D$ (investing safely), and $(1-p)\bar{\omega}^{jh}_g - p\tau^j(h) \leq 0$ (taking risk). Investing safely is optimal.

Take a shareholder $i = 1, \ldots, \bar{n} - 1$. Under $\bar{\sigma}^h$, his payoff is $x_g - D > 0$. Shareholder i will only receive a positive payoff if manager i works. However, no manager works in the second period for an expected wage of less than D. Since x_g is the highest possible asset return, $x_g - D$ is an upper bound on the payoff for any shareholder in any h-subgame. In particular, shareholder i has no profitable deviation from $\bar{\sigma}^h$.

Finally, consider shareholder $j = \bar{n}, \ldots, n$. His payoff under $\bar{\sigma}^h$ is $x_{rf} - D \geq 0$. If shareholder j has a profitable deviation from $\bar{\sigma}^h$, it must involve manager j working for the bank. However, manager j will only invest safely for an expected wage of at least D and will only take risk for an expected wage of at least $(1-p)\omega^{jh}_g - p\tau^j(h)$. So the payoff for shareholder j from offering a wage scheme to which manager j responds by investing safely is bounded from above by $x_{rf} - D$. No such deviation can be profitable. The payoff for shareholder j from offering a wage scheme to which manager j responds by taking risk is bounded from above by $(1-p)x_g - D - p\tau^j(h)$. It follows that no profitable deviation from $\bar{\sigma}^h$ is possible for shareholder j if $(1-p)x_g - p\tau^j(h) \leq x_{rf}$. $\square$

Notation List

Symbol	Meaning
N	Set of banks
n	Number of banks, $n \geq 2$
i	Index of a bank
j	Index of a bank
t	Index of a period, $t \in \{1, 2\}$
A^{it}	Investment decision of bank i in period t, $A^{it} \in \{R, S, O\} = \{Risk, Safe, Outside\}$
$\mathcal{A}$	Activity profile, $\mathcal{A} = (A^{11}, ..., A^{n1}, A^{12}, ..., A^{n2})$
A^{-it}	Restriction of the activity profile to round t and banks $j \in N \backslash \{i\}$
$n_t(\mathcal{A})$	Number of banks choosing the risky asset in period t under the activity profile $\mathcal{A}$
$\bar{n}$	Threshold for triggering a positive probability of a banking crisis, $\bar{n} \in \{1, ..., n-1\}$
p	Probability of a crisis in period t under the activity profile $\mathcal{A}$, if $n_t(\mathcal{A}) \geq \bar{n}$
Z_t	Indicator with $Z_t = 1$ if a crisis occurs in period t and $Z_t = 0$ otherwise
$\mathcal{Z}$	Pair of indicators (Z_1, Z_2)
x_{rf}	Payoff from the safe investment in one period
x_g	Payoff from the risky investment in one period, if no crisis occurs in this period
x_b	Payoff from the risky investment in one period, if a crisis does occur in this period
k	Index of a payoff from an investment, $k \in \{g, rf, b\}$
ω^{it}	Wage scheme offered by shareholder i to manager i in period t
ω_g^{it}	Manager i's wage in period t, if the payoff from the investment is x_g The index ^{it} is omitted when redundant
ω_{rf}^{it}	Manager i's wage in period t, if the payoff from the investment is x_{rf} The index ^{it} is omitted when redundant
ω_b^{it}	Manager i's wage in period t, if the payoff from the investment is x_b. $\omega_b^{it} = 0$ The index ^{it} is omitted when redundant
Ω	Set of possible wage schemes, $\Omega = \{\omega^{it} \in \mathbb{R}^3 \vert \omega^{it} \leq (x_g, x_{rf}, 0)\}$
D	Outside wage of a manager in one period, $D > 0$
u_s^{it}	Instantaneous utility of shareholder i in period t. If manager i accepts the offer in period t, $u_s^{it} = x_k - \omega_k$ where $k \in \{g, rf, b\}$. Else $u_s^{it} = 0$
u_m^{it}	Instantaneous utility of manager i in period t. If manager i accepts the offer in period t, $u_m^{it} = \omega_k$ where $k \in \{g, rf, b\}$. Else $u_m^{it} = D$
c	Crisis Contract tax rate, $c \in [0, 1]$
δ	Discount factor, $\delta \in [0, 1]$
U_m^i	Manager i's intertemporal utility, $U_m^i = u_m^{i1} + \delta u_m^{i2} - \delta\ Z_2\ c\ u_m^{i1}$

Symbol	**Meaning**
h, h', h''	Variables to denote first-period histories
H	Set of all first-period histories, $H \subset \Omega^n \times \{R, S, O\}^n \times \{0, 1\}$
$\tau^i(h)$	Looming crisis tax given first-period history h
σ_s^{ih}	Strategy of shareholder i in the $h-$subgame
σ_m^{ih}	Strategy of manager i in the $h-$subgame
σ^h	Strategy profile for the $h-$subgame, $\sigma^h = (\sigma_m^{1h}, ..., \sigma_m^{nh}, \sigma_s^{1h}, ..., \sigma_s^{nh})$
σ_m^i	Strategy of manager i in the banking game
σ_s^i	Strategy of shareholder i in the banking game
σ	Strategy profile of the banking game, $\sigma = (\sigma_m^1, ..., \sigma_m^n, \sigma_s^1, ..., \sigma_s^n)$
$\mu^{i2}(\sigma^h)$	Manager i's expected wage in the second period, given strategy profile σ^h, under which manager i works for the bank in the second period
$\kappa^{i2}(\sigma^h)$	Manager i's reservation wage
τ^*	Threshold for the looming tax for existence of an $h-$banking equilibrium with threshold risk
$\mu^{i1}(\sigma)$	Manager i's expected wage in the first period under strategy profile σ
$\rho(\sigma)$	Ex ante probability with which a crisis will occur in the second period under the strategy profile σ(not updated with the possible move of nature in the first period)
$\nu^{i1}(\sigma)$	Manager i's expected net wage from the first period under strategy profile σ
c'	Lower bound for c excluding the existence of a full risk equilibrium
c''	Lower bound for c not excluding the existence of a threshold equilibrium
$\hat{n}$	Measure for the stability of the banking system, $\hat{n} = (\bar{n} - 1)/n$
y^t	Instantaneous social welfare in period t
Y	Aggregate social welfare
$\mathcal{E}(c)$	Set of banking equilibria when the crisis tax rate is c
$\rho^{ih}(\sigma)$	Probability with which a crisis will occur in the second period under strategy profile σ^{-i}, given first-period history h and supposing that manager i decides to take risk in the second period
e	Equity of a shareholder
d	Amount of deposits of a bank
r'	Interest rate on risky asset if no crisis occurs
r	Interest rate on deposits and on risk-free asset
l	Loss of the bank's total capital in the case of crisis
$\underline{\phi}$	Lower threshold on the bank's debt-equity ratio
$\overline{\phi}$	Upper threshold on the bank's debt-equity ratio
$I(Pr, R)$	Insurance with premium Pr and payoff R in the case of crisis

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